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TABLES FOR STATISTICIANS AND BIOMETRICIANS

PART I

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TABLES FOR STATISTICIANS AND BIOMETRICIANS

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PART I

THIRD EDITION

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PREFACE TO FIRST EDITION

I AM very conscious of the delay which has intervened between the announcement of the publication of these Tables and their appearance. This delay has been chiefly due to two causes. First the great labour necessary, which largely fell on those otherwise occupied, and secondly the great expense involved (*a*) in the calculation of the Tables, and (*b*) in their publication. This matter of expense is one which my somewhat urgent correspondents, I venture to think, have entirely overlooked. It is perfectly true that only one single Table in this volume has been directly paid for, but a very large part of the labour of calculation has been done by the Staff of the Biometric Laboratory, whose very existence depends on the generous grant made to that laboratory by the Worshipful Company of Drapers. Our staff is not a large one and it has many duties, so that the progress of calculation has of necessity been slow. Even now I am omitting projected Tables, which I can only hope may be incorporated in a later edition of this work, e.g. Tables of the Incomplete B- and Γ-functions, and the Table needed to complete Everitt's work on High Values of Tetrachoric r when r lies between $-\cdot80$ and $-1\cdot00$. It would only satisfy my ideal of what these Tables should be, had I been able to throw into one volume with the present special tables, extensive tables of squares, of square roots, of reciprocals and of the natural trigonometric functions tabled to decimals of a degree Logarithmic tables are relatively little used by the statistician to-day, which is the age of mechanical calculators, and he is perfectly ready to throw aside the fiction that there is any gain in the cumbersome notation of minutes and seconds of angle—a system which would have disappeared long ago, but for the appalling 'scrapping' of astronomical apparatus it would involve. But the ideal of one handy book for the statistician cannot be realised until we have a body of scientific statisticians far more numerous than at present. Statisticians must for the time being carry about with them not only this volume but a copy of *Barlow's Tables*, and a set of Tables of the Trigonometrical Functions.

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Beside the cost of calculating these Tables, to which I have referred, must be added the cost of printing them. I had to do this slowly as opportunity offered in my Journal *Biometrika*, and the Tables as printed were moulded, in order that stereos might be taken for reproduction. Even as it is, there are a number of Tables in this volume, either printed here for the first time (e.g. Tables of the Logarithm of the Factorial and of the Fourth Moment), or published here for the first time (e.g. Tables of the $G(r, \nu)$ Integrals), the setting up of which has naturally been very expensive.

From the beginning of this work in 1901* when the first of these Tables was published and moulded, I have had one end in view, the publication, as funds would permit, of as full a series of Tables as possible. It is needless to say that no anticipation of profit was ever made, the contributors worked for the sake of science, and the aim was to provide what was possible at the lowest rate we could. The issue may appear to many as even now costly; let me assure those inclined to cavil, that to pay its way with our existing public double or treble the present price would not have availed; we are able to publish because of the direct aid provided by initial publication in *Biometrika* and by direct assistance from the Drapers' Company Grant. Yet a few years ago when a reprint of these Tables in America was only stopped by the threat to prevent the circulation of the book in which they were to appear entering any country with which we had a reasonable copyright law, I was vigorously charged with checking the progress of science and acting solely from commercial ends! Meanwhile without any leave, large portions of these tables have been reprinted, sometimes without even citing the originals, in American psychological text-books. Two Russian subjects have reissued many of these Tables in Russian and Polish versions, and copies of their works in contravention of copyright are carried into other European countries. Some of the Tables have also been reproduced in Germany without thought of asking permission. It does not seem to have occurred to these men of science that there was anything blameworthy in depriving *Biometrika* of such increased circulation as it obtained from being the sole *locus* of these Tables, nor did they see in their actions any injury to science as a whole resulting from lessening my power to publish other work of a similar character. It is a singular phase of modern science that it steals with a plagiaristic right hand while it stabs with a critical left.

The *Introduction* gives a brief description of each individual table; it is by no means intended to replace actual instruction in the use of the tables such as

* When issuing their prospectus in the spring of 1901 the Editors of *Biometrika* promised to provide "numerical tables tending to reduce the labour of statistical arithmetic."

is given in a statistical laboratory, nor does it profess to provide an account of the innumerable uses to which they may be put, or to warn the reader of the many difficulties which may arise from inept handling of them. Additional aid may be found in the text which usually accompanies the original publication of the tables.

In conclusion here I wish to thank the loyal friends and colleagues—Dr W. F. Sheppard, Mr W. Palin Elderton, Dr Alice Lee, Mr P. F. Everitt, Miss Julia Bell, Miss Winifred Gibson, Mr A. Rhind, Mr H. E. Soper and others—whose unremitting exertions have enabled so much to be accomplished, if that much is indeed not the whole we need. I have further to acknowledge the courtesy of the Council of the British Association, who have permitted the republication of the Tables of the $G(r, \nu)$ Integrals, originally published in their *Transactions*.

To the Syndics of the Cambridge Press I owe a deep debt of gratitude for allowing me the services of their staff in the preparation of this work. Pages and pages of these Tables were originally set up for *Biometrika*, or were set up afresh here, without the appearance of a single error. To those who have had experience of numerical tables prepared elsewhere, the excellence of the Cambridge first proof of columns of figures is a joy, which deserves the fullest acknowledgement.

KARL PEARSON.

Biometric Laboratory,
February 7, 1914.

PREFACE TO THIRD EDITION

A further edition of Part I of this work is demanded before the computing of certain tables which I desire to issue in Part II has been completed. The labour of calculation has been considerably in excess of my anticipations but it is now well advanced, and I have great hopes of bringing out Part II this summer. Accordingly Part I is now reissued alone.

The computation of the Incomplete B-functions which has been many years in progress is now nearly finished, and if a publisher can be found should also be issued this year.

I draw attention to the fact that Tables of the Incomplete Γ-functions were published by H.M. Stationery Office in 1922 in a volume of the same format as the present one and with an Introduction by the same Editor.

KARL PEARSON.

Biometric Laboratory,
January, 1930.

When you can measure what you are speaking about and express it in numbers, you know something about it, but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meagre and unsatisfactory kind.

LORD KELVIN.

La théorie des probabilités n'est au fond que le bon sens réduit au calcul; elle fait apprécier avec exactitude ce que les esprits justes sentent par une sorte d'instinct, sans qu'ils puissent souvent s'en rendre compte.

LAPLACE.

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* The Roman figures to the pages refer to the Introduction, where the Table is discussed, the Arabic to the Table itself.

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INTRODUCTION TO THE USE OF THE TABLES

For this introduction to the use of the Tables I have largely drawn on the prefaces to the original papers in *Biometrika*, and record here my acknowledgements to the authors of the same.

Interpolation.

(1) A word must first be said as to interpolation. Let a function u be tabled for the argument x proceeding by differences $\Delta x = h$. Then the scheme of such a table with the differences of u is:

x_{-3}	u_{-3}					
		Δu_{-3}				
x_{-2}	u_{-2}		$\Delta^2 u_{-3}$			
		Δu_{-2}		$\Delta^3 u_{-3}$		
x_{-1}	u_{-1}		$\Delta^2 u_{-2}$		$\Delta^4 u_{-3}$	
		Δu_{-1}		$\Delta^3 u_{-2}$		
x_0	u_0		$\Delta^2 u_{-1}$		$\Delta^4 u_{-2}$	
		Δu_0		$\Delta^3 u_{-1}$		
x_1	u_1		$\Delta^2 u_0$		$\Delta^4 u_{-1}$	etc., etc.
		Δu_1		$\Delta^3 u_0$		
x_2	u_2		$\Delta^2 u_1$		$\Delta^4 u_0$	
		Δu_2		$\Delta^3 u_1$		
x_3	u_3		$\Delta^2 u_2$		$\Delta^4 u_1$	
		Δu_3		$\Delta^3 u_2$		
x_4	u_4		$\Delta^2 u_3$			
		Δu_4				
x_5	u_5					

where:

$$\Delta u_s = u_{s+1} - u_s,$$
$$\Delta^2 u_s = \Delta u_{s+1} - \Delta u_s,$$
$$\Delta^3 u_s = \Delta^2 u_{s+1} - \Delta^2 u_s \text{ etc., etc.}$$

Now there are three interpolation formulae which it is desirable to remember. If the function be required for the value $x_0 + \theta h$ and this value be termed $u_0(\theta)$, then we have:

$$u_0(\theta) = u_0 + \theta \Delta u_0 - \frac{\theta(1-\theta)}{2!}\Delta^2 u_0 + \frac{\theta(1-\theta)(2-\theta)}{3!}\Delta^3 u_0 - \ldots\ldots\ldots\text{(i)},$$

$$u_0(\theta) = \phi u_0 + \theta u_1 - \frac{\theta(1-\theta^2)}{3!}\Delta^2 u_0 - \frac{\phi(1-\phi^2)}{3!}\Delta^2 u_{-1} \quad \ldots\ldots\ldots\ldots\text{(ii)},$$

where $\phi = 1 - \theta$. This is Everett's formula*. And lastly:

$$u_0(\theta) = u_0 + \theta \tfrac{1}{2}(\Delta u_0 + \Delta u_{-1}) + \frac{\theta^2}{2!}\Delta^2 u_{-1} - \frac{\theta(1-\theta^2)}{3!}\tfrac{1}{2}(\Delta^3 u_{-1} + \Delta^3 u_{-2}) \ldots \text{(iii)},$$

where we work with the differences on or adjacent to the horizontal through x_0.

* *Journal of the Institute of Actuaries*, Vol. xxxv, p. 452.

It is very rarely indeed that we need go beyond second differences, often the first will suffice. Not infrequently the inverse problem arises, namely we are given $u_0(\theta)$ and have to determine θ from it. If we only go as far as second differences, either (i) or (iii) gives us a quadratic to find θ and the root will generally be obvious without ambiguity. Usually it suffices to find

$$\theta' = \{u_0(\theta) - u_0\}/\Delta u_0,$$

and then determine θ from

$$\theta = \{u_0(\theta) - u_0\}/\Delta u_0 + \frac{\theta'(1-\theta')}{2!}\Delta^2 u_0/\Delta u_0 \quad \text{.....(iv);}$$

or to find $$\theta' = \{u_0(\theta) - u_0\}/\tfrac{1}{2}(\Delta u_0 + \Delta u_{-1})$$

and then $$\theta = \{u_0(\theta) - u_0\}/\tfrac{1}{2}(\Delta u_0 + \Delta u_{-1}) - \frac{\theta'^2}{2!}\frac{\Delta^2 u_{-1}}{\tfrac{1}{2}(\Delta u_0 + \Delta u_{-1})} \quad \text{.....(v).}$$

Very often good results are readily obtained by applying Lagrange's interpolation formula which for three values of u reduces to

$$u_0(\theta) = (1-\theta^2)u_0 - \tfrac{1}{2}\theta(1-\theta)u_{-1} + \tfrac{1}{2}\theta(1+\theta)u_1 \quad \text{.....(vi).}$$

Or, we may use the mean of two such formulae and take

$$u_0(\theta) = (1-\theta)(1-\tfrac{1}{4}\theta)u_0 + \tfrac{1}{4}\theta(5-\theta)u_1 - \tfrac{1}{4}\theta(1-\theta)(u_{-1}+u_2) \quad \text{...(vii).}$$

The resulting quadratics are respectively:

$$\theta^2\{\tfrac{1}{2}(u_1+u_{-1}) - u_0\} + \theta\tfrac{1}{2}(u_1 - u_{-1}) + u_0 - u_0(\theta) = 0 \quad \text{.....(vi)}^{\text{bis}},$$

and $$\theta^2\tfrac{1}{4}(-u_0 - u_1 + u_{-1} + u_2) + \theta\tfrac{1}{4}(5u_1 - 3u_0 - u_{-1} - u_2) + u_0 - u_0(\theta) = 0 \quad \text{...(vii)}^{\text{bis}}.$$

This equation may often be effectively dealt with by finding the value of:

$$\theta_0 = \frac{u_0 - u_0(\theta)}{\tfrac{1}{4}(3u_0 + u_{-1} + u_2 - 5u_1)},$$

and then calculating

$$\theta = \theta_0\left\{1 + \theta_0\left(\frac{-u_0 - u_1 + u_{-1} + u_2}{3u_0 + u_{-1} + u_2 - 5u_1}\right)\right\}.$$

(2) There are some tables in this book which are of double entry, e.g. those for the Tetrachoric Functions and for the $G(r, \nu)$ Integrals. The simplest solid interpolation formula, using second differences, is:

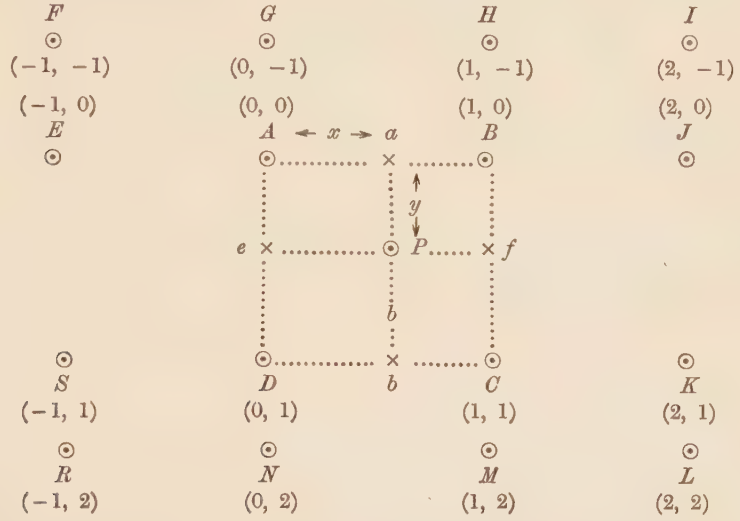
$$u_{x,y} = u_{0,0} + x\Delta u_{0,0} + y\Delta' u_{0,0} + \tfrac{1}{2}\{x(x-1)\Delta^2 u_{0,0} + 2xy\Delta\Delta' u_{0,0} + y(y-1)\Delta'^2 u_{0,0}\} \quad \text{.....(viii),}$$

where Δ denotes a difference with regard to x, and Δ' with regard to y. But if we consider $u_{x,y}$ to be the ordinate of a surface, and the figure, p. xv, to represent the xy plane of such a surface, then it is clear that, if P be the point (x, y), and A, B, C, D, &c. the adjacent points at which the ordinates are known from the table of double entry, only the points A, B, C, D, J, and N are used by the above formula; and of these points, not equal weight is given to the fundamental points A, B, C, D, for C only appears in a second difference. If another point of the fundamental square other than A be taken as origin, we get a divergent, occasionally a widely divergent result. If we use only four points—A, B, C, D—to determine the value of the function at P, then we might take the ordinate

at P of the plane which (by the method of least squares) most nearly passes through the four points of the surface vertically above A, B, C, D. We have then

$$u_{x,y} = \tfrac{1}{4}(u_{0,0} + u_{1,0} + u_{0,1} + u_{1,1}) + \tfrac{1}{2}(u_{1,0} - u_{0,0} + u_{1,1} - u_{0,1})(x - \cdot 5) + \tfrac{1}{2}(u_{0,1} - u_{0,0} + u_{1,1} - u_{1,0})(y - \cdot 5) \quad \ldots\ldots(\text{ix}),$$

but by trial it has been found that this formula gives occasionally worse results than that for first differences, using only three points. To find by the methods of simple interpolation (with first or first and second differences) the points a and b, and then interpolate P between them, generally gives a fairly good result; but this result usually differs somewhat from that obtained by first simply



interpolating e and f and then interpolating between e and f*. Various other methods for interpolation in n-dimensioned space will be found discussed by Palin Elderton in *Biometrika*†. The ideal method can hardly yet be said to be known, and it may well vary from table to table and from one part of the same table to another‡.

I consider now the individual tables.

Table I (p. 1)

Table of Deviates of the Normal Curve for each Permille of Frequency. (Calculated by Sheppard and published by Galton in *Biometrika*, Vol. v. p. 405.)

If N be the total number in a population, $z\delta x$ the frequency between x and $x + \delta x$, σ the standard-deviation, then the frequency curve of the population assuming its distribution to be Gaussian or normal will be:

$$z = \frac{N}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\frac{x^2}{\sigma^2}} \quad \ldots\ldots(\text{ix})$$

* *B. A. Report*, Dover 1899. Tables of $G(r, \nu)$-Integrals, Report of the Committee (Drawn up by K. Pearson).

† Vol. vi. p. 94.

‡ The reader may consult *Tracts for Computers*, No. III, which deals with interpolation into Bivariate Tables.

the origin being the mean. Table I. gives the value of x/σ for each thousandth of the area of this curve,—each 'permille'—reckoned from left to right.

In entering the table we enter from the left-hand column and top row if the permille be less than 500. For example, if the frequency below a particular value were 387 per thousand, the corresponding deviate would be $-0{\cdot}2871$, the number placed at the intersection of the ·38 row from left and ·007 column from top. The negative sign is always to be given when reading permilles below 500, because the deviate will be in defect of the mean, supposing increasing variates to be plotted as usual from left to right.

On the other hand if the permille be greater than 500 we enter the table from the right-hand column and bottom row. For example, if the permille be 748, the deviate is $+0{\cdot}6682$, the number placed at the intersection of the ·74 row from right and ·008 column from bottom of the table. The plus sign must be given, as the deviation is in excess of the mean, if the convention as to plotting variables has been observed.

Illustration: The following observations were made on the nature of the degree taken by 1011 Cambridge undergraduates measured at the Anthropological Society's Laboratory:

Poll	487	Second Class	182
Third Class	189	First Class	153

Find the deviates of these on a normal or Gaussian scale.

The sums from the lowest to each class top are 487, 676, 858 and 1011 respectively. If we term with Francis Galton the one man in a thousand of surpassing intelligence or special ability a "genius," we have on multiplying by ·0009891197 the reciprocal of 1011, the series for entering Table I. Thus we find:

		·4817	·6686	·8487	and ·9990
Hence:	(·481)	·0476	(·668)·4344	(·848) 1·0279	(·999) 3·0902
	(·482)	·0451	(·669)·4372	(·849) 1·0322	—
	Δ	·0025	Δ ·0028	Δ ·0043	—
	Δ × ·7	·00175	Δ × ·6 ·00168	Δ × ·7 ·00301	—
Deviates:		− ·0458	+ ·4361	+1·0309	+3·0902

Supposing with Pearson* that 100 units of intelligence ("mentaces") separate the lowest man of the First Class from the highest man of the Poll, we have $+1{\cdot}0309-(-{\cdot}0458)=100/\sigma$, where σ is the standard deviation of intelligence. Thus $\sigma=100/1{\cdot}0767=92{\cdot}88$ mentaces. Hence we conclude that the range of Third Class men is from $-4{\cdot}25$ (i.e. $92{\cdot}88\times(-{\cdot}0458)$) below to $+40{\cdot}50$

* *Biometrika*, Vol. v. p. 109.

(i.e. 92·88 × (+ ·4361)) above the average undergraduate. The range of Second Class men is from + 40·50 to + 95·75 mentaces above the average undergraduate, and the range of First Class men all those with more than 95·75 mentaces above the average. The "genius" corresponds to an excess of no less than 287·02 mentaces. If we suppose that one individual in 1000 is completely feeble-minded or practically wanting in all intelligence, we should credit roughly the average man with 300 mentaces, and we should then have our range of intelligence on a Gaussian scale:

Poll: below 296 mentaces;

Third Class: above 296 and below 340 mentaces;

Second Class: above 340 and below 396 mentaces;

First Class: above 396 mentaces;

"Genius": above 587 mentaces.

In rough numbers: Poll, below 300; Third Class, 300 to 350; Second Class, 350 to 400; First Class, over 400; "Genius," over 600.

Of course there is much that is hypothetical here, but the numbers give us some appreciation of the distribution of ability, and they serve to illustrate the construction of a Gaussian or normal scale. When more than three or four significant figures are needed Tables II and III must be used.

Tables II and III (pp. 2—10)

Tables of the Probability Integral: Area and Ordinate of the Normal Curve in terms of the Abscissa; and Abscissa and Ordinate in Terms of Difference of Areas. (Calculated by Dr W F. Sheppard, and published in *Biometrika*, Vol. II. pp. 174—190.)

"Sheppard's Tables" were the first to express the Gaussian* or normal probability integral in terms of the standard deviation; they are so familiar to statisticians that it would almost seem a work of supererogation to explain their 'use,' which is further too manifold for full description. We can only give a few sample illustrations.

It is most important when using these tables to pay attention to the signs of the differences recorded at the tops of the columns.

Illustration (i). The mean length of cubit in 1063 adult English males is recorded as 18″·31 ± ·019 and of their 1063 adult sons as 18″·52 ± ·021. Determine the odds against these two measurements being really identical, i.e. random samples from the same population. We assume that the deviation of means and their differences follow the normal law. The difference is 0″·21 and the probable error of this difference $= \sqrt{(\cdot 019)^2 + (\cdot 021)^2} = 0''\cdot 0283$. Since the probable error

* The term is usual, but inaccurate. Laplace had reached the probability integral and suggested its tabulation several years before Gauss.

$= \cdot 67449 \times$ standard deviation, we have the standard deviation of the difference $= 0''\cdot 04196$. Hence the deviation in terms of the standard deviation

$$= 0\cdot 21/(0\cdot 04196) = 5\cdot 0048.$$

Table II, p. 8, gives the area $\frac{1}{2}(1+\alpha)$ of the normal curve up to the abscissa x/σ. Noting the remark at the foot of the table, we have

$$\begin{aligned} x/\sigma = 5\cdot 00, \quad & \tfrac{1}{2}(1+\alpha) = \cdot 999,999,7133, \\ x/\sigma = 5\cdot 01, \quad & \tfrac{1}{2}(1+\alpha) = \cdot 999,999,7278, \\ & \Delta = \quad 145, \\ & \Delta \times 48 \quad 70, \\ x/\sigma = 5\cdot 0048, \quad & \tfrac{1}{2}(1+\alpha) = \cdot 999,999,7203. \end{aligned}$$

Hence $\tfrac{1}{2}(1-\alpha) = \cdot 000,000,2797.$

Accordingly if we suppose the deviation as likely to be in defect as in excess, the probability that we shall reach the observed deviation, or exceed it, is $2 \times \frac{1}{2}(1-\alpha)$, and that we shall not is $\frac{1}{2}(1+\alpha) - \frac{1}{2}(1-\alpha)$, or the odds against the result on a pure random sampling chance are $\cdot 999,999,4406$ to $\cdot 000,000,5594$, or 1,787,629 to 1, i.e. overwhelming odds. Thus we may reasonably argue that sons in the professional classes in 1900 were substantially differentiated from their fathers by a longer forearm of about $\frac{1}{5}''$.

Illustration (ii). Find the value in mentaces of the mean intelligence of Pollmen, First, Second and Third Class men as given by the numbers in the *Illustration* to Table I.

The equation to the normal or Gaussian curve being

$$z = \frac{N}{\sqrt{2\pi}\sigma}\, e^{-\frac{1}{2}(x/\sigma)^2},$$

we easily find that if there be 'tabled' ordinates z_1 and z_2* at the abscissae x_1 and x_2, which cut off an area n_{12}, then the mean $\bar{x}_{12}$ of this area is given by

$$\bar{x}_{12} = \sigma \,.\, (z_1 - z_2)/(n_{12}/N) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(\mathrm{x}).$$

It will be sufficient to take the values of the abscissae already found, i.e.

$$\begin{aligned} x_1/\sigma &= - \cdot 0458, \quad x_2/\sigma = + \cdot 4361, \\ x_3/\sigma &= + 1\cdot 0309, \quad x_4/\sigma = + 3\cdot 0902. \end{aligned}$$

We require the z's for these. For example:

$$\begin{aligned} x = \cdot 04, \quad & z = \cdot 398,6233 \\ \cdot 05, \quad & z = \cdot 398,4439 \\ \theta = \cdot 58, \quad & \Delta_1 \quad -1793 \\ & \Delta_2 \quad -397. \end{aligned}$$

* The symbol z here used is that of the Tables, i.e. $\frac{1}{\sqrt{2\pi}}\, e^{-\frac{1}{2}(x/\sigma)^2}$.

Therefore by formula (i) p. xiii:

$$z_1 = \cdot 398{,}6233 - \cdot 58\,[1793] + \frac{\cdot 58 \times \cdot 42}{2}\,[397]$$

$$\left.\begin{aligned} &= \cdot 398{,}6233 \\ &\quad - 1040 \\ &\quad + \ \ 48 \end{aligned}\right\} = \cdot 398{,}5241.$$

Or, we might proceed as follows: for the Poll-men $\frac{1}{2}(1-\alpha) = \cdot 4817$, hence $\alpha = \cdot 0366$. But from Table III, p. 9, which gives z for α:

$$\begin{aligned} \alpha &= \cdot 03, \quad z = \cdot 398{,}6603 \\ \alpha &= \cdot 04, \quad z = \cdot 398{,}4408 \\ \theta &= \cdot 66, \quad \Delta_1 = \ \ -2194 \\ &\qquad\ \ \Delta_2 = \ \ \ \ -627. \end{aligned}$$

Hence by formula (i):

$$z_1 = \cdot 398{,}6603 - \cdot 66\,[2194] + \frac{\cdot 66 \times \cdot 34}{2}\,[627]$$

$$\left.\begin{aligned} &= \cdot 398{,}6603 \\ &\quad - 1448 \\ &\quad + \ \ 70 \end{aligned}\right\} = \cdot 398{,}5225.$$

We conclude therefore that z would be correct to five figures with second differences, and that for four figures, first differences from either Table II or Table III will suffice.

If we use formula (ii) p. xiii—Everitt's formula—we find from Table II:

$$z_1 = \cdot 398{,}6233 - \cdot 58\,[1793] + \frac{\cdot 58 \times \cdot 6636}{6}\,[397] + \frac{\cdot 42 \times \cdot 8236}{6}\,[398]$$

$$\left.\begin{aligned} &= \cdot 398{,}6233 \\ &\quad - 1040 \\ &\quad + \ \ 13 \\ &\quad + \ \ \ 5 \end{aligned}\right\} = \cdot 398{,}5211,$$

and from Table III:

$$z_1 = \cdot 398{,}6603 - \cdot 66\,[2194] + \frac{\cdot 66 \times \cdot 5644}{6}\,[627] + \frac{\cdot 34 \times \cdot 8844}{6}\,[627]$$

$$\left.\begin{aligned} &= \cdot 398{,}6603 \\ &\quad - 1448 \\ &\quad + \ \ 37 \\ &\quad + \ \ 31 \end{aligned}\right\} = \cdot 398{,}5223.$$

Working with formula (iii), Table II gives us $z_1 = \cdot 398{,}5242$ and Table III $z_1 = \cdot 398{,}5225$ with second differences. We shall not therefore without higher differences get from any of our formulae closer than ·398,522 with a possible error

of 1 or 2 in the last place. This is, of course, amply sufficient for statistical purposes, where four figures as a rule would be sufficient.

Using formula (i) p. xiii we obtain:

$$z_1 = \cdot 39852, \quad z_3 = \cdot 23450,$$
$$z_2 = \cdot 36275, \quad z_4 = \cdot 00337.$$

Whence:

$$\bar{x}_{\infty 1} = + \frac{0 - \cdot 39852}{\cdot 4817}\sigma = - \cdot 8273\sigma = - 76 \cdot 84 \text{ mentaces},$$

$$\bar{x}_{12} = + \frac{\cdot 39852 - \cdot 36275}{\cdot 1869}\sigma = + \cdot 1925\sigma = + 17 \cdot 88 \text{ mentaces},$$

$$\bar{x}_{23} = + \frac{\cdot 36275 - \cdot 23450}{\cdot 1801}\sigma = + \cdot 7121\sigma = + 66 \cdot 14 \text{ mentaces},$$

$$\bar{x}_{34} = + \frac{\cdot 23450 - \cdot 00337}{\cdot 1503}\sigma = + 1 \cdot 5378\sigma = + 142 \cdot 83 \text{ mentaces},$$

$$\bar{x}_{4\infty} = + \frac{\cdot 00337 - 0}{\cdot 0010}\sigma = + 3 \cdot 3700\sigma = + 313 \cdot 01 \text{ mentaces}.$$

Assuming as before the average man to have 300 mentaces of intelligence we find:

Average Poll-man has 223 mentaces.

Average Third Class man has 318 mentaces.

Average Second Class man has 366 mentaces.

Average First Class man has 443 mentaces.

Average man of "genius" has 613 mentaces.

Thus the average First Class Honours man is twice as able as the average Poll-man, and the average "genius" has not quite twice the ability of the average Third Class Honours man.

Illustration (iii). It is required to determine normal curve frequencies corresponding to the following frequencies of the cephalic index in Bavarian skulls.

Here the mean and standard deviation found by moments in the usual way are

$$m = 83 \cdot 069, \quad \sigma = 3 \cdot 432.$$

The deviations from the mean were next expressed in terms of the standard deviation, i.e. these deviations are

$$- 13 \cdot 569, \quad - 12 \cdot 569, \quad \ldots - 0 \cdot 569, \quad + \cdot 431, \quad + 1 \cdot 431, \quad + 2 \cdot 431, \quad \ldots + 14 \cdot 431,$$

and they are multiplied on a calculator by the reciprocal of the standard deviation, whence the column x/σ is found. Table II gives us $\frac{1}{2}(1 + \alpha)$ knowing x/σ; this has been calculated by first differences only. We shall consider as an illustration to Table XII, whether the normal distribution thus reached is to be considered a good fit to the observations.

Index	Observed	x/σ	$\frac{1}{2}(1+\alpha)$	Calculated Frequency
69·5—70·5	1	−3·9539	·99996	Under 70·5 ·1
70·5—71·5	1	−3·6625	·99988	·2
71·5—72·5	—	−3·3711	·99963	·6
72·5—73·5	2·5	−3·0797	·99896	1·5
73·5—74·5	1·5	−2·7883	·99735	3·3
74·5—75·5	3·5	−2·4969	·99374	6·7
75·5—76·5	12·5	−2·2055	·98629	12·7
76·5—77·5	17	−1·9141	·97219	22·1
77·5—78·5	37	−1·6228	·94768	35·3
78·5—79·5	55	−1·3314	·90846	51·9
79·5—80·5	71·5	−1·0400	·85082	70·1
80·5—81·5	82	− ·7486	·77294	87·0
81·5—82·5	116	− ·4572	·67623	99·4
82·5—83·5	98	− ·1658	·56584	104·2
83·5—84·5	107	·1256	·54997	100·5
84·5—85·5	82	·4170	·66165	89·1
85·5—86·5	74	·7084	·76064	72·6
86·5—87·5	58	·9998	·84129	54·3
87·5—88·5	34·5	1·2912	·90167	37·4
88·5—89·5	19	1·5825	·94324	23·7
89·5—90·5	10	1·8739	·96953	13·8
90·5—91·5	8	2·1653	·98482	7·4
91·5—92·5	3	2·4567	·99299	3·6
92·5—93·5	1·5	2·7481	·99700	1·6
93·5—94·5	2	3·0395	·99882	·7
94·5—95·5	1·5	3·3309	·99957	·3
95·5—96·5	—	3·6223	·99985	Over 95·5 ·1
96·5—97·5	—	3·9137	·99995	—
97·5—98·5	1	4·2050	·99999	—
Totals ...	900	—	—	900·2

TABLE IV (p. 11)

Extension of the Table of the Probability Integral $F = \frac{1}{2}(1-\alpha)$. (Calculated by Julia Bell, M.A., *Drapers' Research Memoirs*, Biometric Series, VIII, p. 27.)

It has been found needful occasionally to determine probabilities for deviations exceeding considerably the limit $x/\sigma = 6$ of Sheppard's Table II.

Illustration. If $x/\sigma = 34{\cdot}31$, determine to two significant figures the probability of a deviation occurring as large or larger than this.

The table gives us:

33	238·39135		
		14·56180	
34	252·95315		·43393
		14·99573	
35	267·94888		·43394
		15·42967	
36	283·37855		

Hence using formula (ii) p. xiii:

$$(-\log F) = 252{\cdot}95315 + {\cdot}31\,[14{\cdot}99573] - \frac{{\cdot}31 \times {\cdot}9039}{6} \times {\cdot}43393 - \frac{{\cdot}69 \times {\cdot}5239}{6} \times {\cdot}43394$$

$$\left.\begin{aligned} &= 252{\cdot}95315 \\ &+ 4{\cdot}64868 \\ &- \ {\cdot}04641 \end{aligned}\right\} = 257{\cdot}55542.$$

Hence $\log F = -257{\cdot}55542 = \overline{258}{\cdot}44458,$

$$F = 2{\cdot}7834/10^{258},$$

which measures the improbability required.

Table V (pp. 12—18) and Table VI (p. 18)

Probable Errors of Means, Standard Deviations and Coefficients of Variation. (Table V calculated by Winifred Gibson, B.Sc.; Table VI by Dr Raymond Pearl and T. Blakeman, M.A. *Biometrika*, Vol. IV. pp. 385—393.)

If m be a mean, σ a standard deviation and $V = 100\sigma/m$ a coefficient of variation, for a population of n, we have

Probable Error of Mean

$$= {\cdot}6744898\sigma/\sqrt{n} = \chi_1\sigma \quad \text{.....(xi),}$$

Probable Error of Standard Deviation

$$= {\cdot}6744898\sigma/\sqrt{2n} = \chi_2\sigma \quad \text{.....(xii),}$$

Probable Error of the Coefficient of Variation

$$= {\cdot}6744898V \times \left\{1 + 2\left(\frac{V}{100}\right)^2\right\}^{\frac{1}{2}} \Big/ \sqrt{2n} \quad \text{.....(xiii),}$$

$$= {\cdot}6744898/\sqrt{2n} \times \psi$$

$$= \chi_2 \times \psi \quad \text{.....(xiv).}$$

Table V gives χ_1 and χ_2 for each value of n up to 1000, Table VI gives ψ for each value of V proceeding by units from 0 to 50.

When the frequency n is greater than 1000, the tables may still be used by taking out a square factor, which can be divided out at sight.

Illustration (i). $n = 2834 = 4 \times 708{\cdot}5.$

$n = 708,\ \chi_1 = {\cdot}02535;\quad n = 709,\ \chi_1 = {\cdot}02533.$

$\therefore\ n = 708{\cdot}5,\ \chi_1 = {\cdot}02534,$ and $\therefore$ for $n = 2834,$

we have $\chi_1 = {\cdot}01267.$

Illustration (ii). In the case of the 900 Bavarian crania of the *Illustration* (iii) to Table II the values

$$m = 83{\cdot}069, \quad \sigma = 3{\cdot}432,$$

and therefore $V = 4{\cdot}1315$ were found. It is required to find the probable errors of these values.

For 900, $\chi_1 = {\cdot}02248$ and $\chi_2 = {\cdot}01590$, hence the probable errors of m and σ are

$$\text{p.e. of } m = \chi_1 \sigma = {\cdot}077,$$
$$\text{p.e. of } \sigma = \chi_2 \sigma = {\cdot}055.$$

Next for $V = 4{\cdot}1315$,

$$\psi_1 = 4{\cdot}00639 + {\cdot}1315\,[1{\cdot}00609] - \tfrac{1}{2}({\cdot}1315)({\cdot}8685) \times [299]$$
$$= 4{\cdot}13852.$$
$$\text{p.e. of } V = \chi_2 \times \psi = {\cdot}01590 \times 4{\cdot}13852$$
$$= {\cdot}0658.$$

Hence our results should be recorded as

$$m = 83{\cdot}069 \pm {\cdot}077,$$
$$\sigma = 3{\cdot}432 \pm {\cdot}055,$$
$$V = 4{\cdot}1315 \pm {\cdot}0658.$$

TABLE VII (p. 19)

Abac for Probable Errors of r. (Calculated by Dr David Heron, drawn by H. Gertrude Jones, *Biometrika,* Vol. VII. p. 411.)

The probable error of a coefficient of correlation

$$= \frac{{\cdot}67449}{\sqrt{n}}(1 - r^2).$$

To ascertain the value of this function approximately, turn the page horizontal, enter with the proper frequency on the scale at base, follow the corresponding vertical until the sloping line with the given correlation is reached, then move along the horizontal to the left until the scale of probable error is reached, which will give the required approximate probable error of the correlation in a population of the given size.

Illustration. $r = {\cdot}671$ and $n = 415$.

Probable error of $r = {\cdot}018$. The actual value is ·0182.

TABLE VIII (pp. 20—21)

Values of $1 - r^2$. (Calculated by H. E. Soper, M.A.)

Illustration. As in the last example let

$$r = {\cdot}671 \text{ and } n = 415.$$

Then probable error of $r = \chi_1 \times (1 - r^2)$

$$= {\cdot}549{,}759 \text{ (from Table VIII)}$$
$$\times {\cdot}03311 \text{ (from Table V)}$$
$$= {\cdot}0182.$$

The value of $1 - r^2$ for r to four instead of to three figures can be obtained by interpolation.

TABLE IX (pp. 22—23)

Values of the Incomplete Normal Moment Functions. (Calculated by Dr Alice Lee, *Biometrika*, Vol. VI. p. 59.)

The nth incomplete normal moment function is defined to be

$$\mu_n(x) = \frac{1}{\sqrt{2\pi}} \int_0^x x^n e^{-\frac{1}{2}x^2}\, dx \quad \text{.....(xv).}$$

We take

$$\left.\begin{aligned} m_n(x) &= \mu_n(x)/\{(n-1)(n-3)(n-5)\dots 1\} \text{ if } n \text{ be even} \\ &= \mu_n(x)/\{(n-1)(n-3)(n-5)\dots 2\} \text{ if } n \text{ be odd} \end{aligned}\right\} \quad \text{.....(xvi),}$$

and $m_n(x)$ is the function tabled.

In multiple correlation (supposed normal), the frequency surface is

$$\left.\begin{aligned} z &= \frac{N}{(2\pi)^{\frac{1}{2}n}\sigma_1\sigma_2\dots\sigma_n\sqrt{R}} e^{-\frac{1}{2}\chi^2} \\ \text{where} \quad \chi^2 &= \frac{1}{R}\{S(R_{pp}x_p^2/\sigma_p^2) + 2S'(R_{pq}x_px_q/\sigma_p\sigma_q)\} \end{aligned}\right\} \quad \text{.....(xvii),}$$

and

$$R = \begin{vmatrix} 1, & r_{12}, & r_{13} \dots r_{1n} \\ r_{21}, & 1, & r_{23} \dots r_{2n} \\ \dots & \dots & \dots \\ \dots & \dots & \dots \\ r_{n1}, & r_{n2}, & r_{n3} \dots 1 \end{vmatrix} \quad \text{.....(xviii),}$$

while R_{pp} and R_{pq} are the usual minors.

$\chi^2 =$ constant is the "ellipsoid" of equal frequency in n-dimensional space.

The total frequency, i.e. the volume of the surface, inside any ellipsoid χ is

$$I_\chi = \int_0^\chi z\, dV$$

and

$$\left.\begin{aligned} I_\chi/N &= \frac{\sqrt{2\pi}\,\mu_{n-1}(\chi)}{2.4.6\dots(n-2)} \text{ if } n \text{ be even} \\ &= \frac{2\mu_{n-1}(\chi)}{1.3.5\dots(n-2)} \text{ if } n \text{ be odd} \end{aligned}\right\} \quad \text{.....(xix).}$$

Thus a knowledge of the incomplete normal moment functions enables us to predict for multiple variables whether an outlying observation consisting of a system of n variate values is or is not reasonably probable.

If $I_{\chi_0}/N = \frac{1}{2}$, we obtain the 'ellipsoidal' contour χ_0 within which half the frequency lies. This χ_0 is the "generalised probable error" of Pearson and Lee.

Values of the "generalised probable error" coefficients are given in Table X for $n = 1$ to 11, and by means of a smooth curve the results may probably be extended to $n = 15$. The values found for this extension are:

	$n=12$	$n=13$	$n=14$	$n=15$
χ_0	3·367	3·513	3·654	3·791

Illustration (i). Let us consider long bone data for Frenchmen. $1 = F =$ femur, $2 = H =$ humerus, $3 = T =$ tibia, $4 = R =$ radius*, then by formula (xviii) p. xxiv:

$$R = \begin{vmatrix} 1, & \cdot8421, & \cdot8058, & \cdot7439 \\ \cdot8421, & 1, & \cdot8601, & \cdot8451 \\ \cdot8058, & \cdot8601, & 1, & \cdot7804 \\ \cdot7439, & \cdot8451, & \cdot7804, & 1 \end{vmatrix}$$

Further in cms:

$$m_1 = 45{\cdot}23, \quad \sigma_1 = 2{\cdot}372,$$
$$m_2 = 33{\cdot}01, \quad \sigma_2 = 1{\cdot}538,$$
$$m_3 = 36{\cdot}81, \quad \sigma_3 = 1{\cdot}799,$$
$$m_4 = 24{\cdot}39, \quad \sigma_4 = 1{\cdot}170.$$

What is the chance that the following individual may be considered French?

$$F' = 36{\cdot}97, \quad H' = 26{\cdot}82, \quad T' = 30{\cdot}56, \quad R' = 20{\cdot}68.$$

The deviations in terms of their standard deviations are:

$$x_1 = (F' - m_1)/\sigma_1 = -3{\cdot}482, \quad x_2 = (H' - m_2)/\sigma_2 = -4{\cdot}059,$$
$$x_3 = (T' - m_3)/\sigma_3 = -3{\cdot}474, \quad x_4 = (R' - m_4)/\sigma_4 = -3{\cdot}171.$$

Further:

$$\frac{R_{11}}{R} = 3{\cdot}7810, \quad \frac{R_{22}}{R} = 6{\cdot}5496, \quad \frac{R_{33}}{R} = 4{\cdot}8406, \quad \frac{R_{44}}{R} = 3{\cdot}6508,$$
$$\frac{R_{12}}{R} = 2{\cdot}0231, \quad \frac{R_{13}}{R} = 1{\cdot}1404, \quad \frac{R_{14}}{R} = 0{\cdot}2130, \quad \frac{R_{23}}{R} = 2{\cdot}1946,$$
$$\frac{R_{24}}{R} = 2{\cdot}3175, \quad \frac{R_{34}}{R} = 0{\cdot}6842.$$

Whence $\chi^2 = 16{\cdot}741{,}035$ and $\chi = 4{\cdot}0916$,

n is even, hence: $$I_\chi/N = \frac{\sqrt{2\pi} \cdot \mu_3 (4{\cdot}0916)}{2} = \sqrt{2\pi} \cdot m_3,$$

* For particulars of these length measurements the reader must consult *R. S. Proc.* Vol. 61, pp. 343 *et seq.* and *Phil. Trans.* Vol. 192, A, p. 180.

and from the Table, p. 22, we have by formula (i), p. xiii:

$$m_3(4{\cdot}0916) = {\cdot}397{,}7378 + {\cdot}916[3650] - \tfrac{1}{2}({\cdot}916)({\cdot}084)[1043]$$
$$= {\cdot}398{,}0682.$$

Hence $$I_\chi/N = \sqrt{2\pi} \times {\cdot}398{,}0682 = {\cdot}9978.$$

Thus the odds are 9978 to 22, say 454 to 1 against a deviation-complex as great as or greater than this occurring in a French male skeleton, i.e. the bones very improbably were those of a Frenchman. Actually they were those of a male of the Aino race.

Illustration (ii). The following are the ordinates of a frequency distribution for the speed of American trotting horses*. It is assumed that they form a truncated normal curve, and we require to determine (i) the mean of the whole population, (ii) its standard deviation, and (iii) what fraction the 'tail' is of the whole population.

The values of frequency in an arbitrary scale are:

Seconds	Frequency	Seconds	Frequency
29—28	92·8	*20—19*	45·8
28—27	100·4	*19—18*	38·4
27—26	95·0	*18—17*	27·8
26—25	71·2	*17—16*	19·8
25—24	67·6	*16—15*	10·7
24—23	61·3	*15—14*	15·8
23—22	61·4	*14—13*	7·9
22—21	44·8	*13—12*	5·0
21—20	44·5	*12—11*	2·1
		11—10	5·6

Taking the working origin at 20—19 seconds, we find

$$\nu_1' = -3{\cdot}9214, \quad \nu_2' = 32{\cdot}545{,}666$$

for raw moment coefficients. Hence, if d be the distance from 29 seconds, i.e. the stump of the tail from the mean, and Σ the standard deviation of the tail about its mean:

$$d = 9{\cdot}5 - 3{\cdot}9214 = 5{\cdot}5786 \text{ secs.},$$
$$\Sigma^2 = \nu_2' - \nu_1'^2 = 17{\cdot}168{,}288,$$

and accordingly $$\Sigma^2/d^2 = {\cdot}5517.$$

If this value be compared with those for ψ_1 in Table XI, p. 25, it will be seen that we have got slightly more than the half of a normal curve, i.e. not a true tail. We cannot therefore use Table XI, but must fall back on Table IX.

* Galton, *R. S. Proc.* Vol. 62, p. 310. See for another method of fitting, Pearson, *Biometrika*, Vol. II. p. 3.

Let x be the distance from stump to centre of curve, n equal the area of truncated portion, and N be whole population. Then

$$n/N = \int_0^\infty + \int_0^{x/\sigma} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x'^2}\, dx' = \tfrac{1}{2} + m_0\,(x/\sigma) \qquad \text{(xx)};$$

$$n\bar{x} = N\sigma \left\{ \int_0^\infty + \int_{-x/\sigma}^0 \frac{x'}{\sqrt{2\pi}} e^{-\frac{1}{2}x'^2}\, dx' \right\},$$

$$= N\sigma \left\{ \frac{1}{\sqrt{2\pi}} - \int_0^{x/\sigma} \frac{x'}{\sqrt{2\pi}} e^{-\frac{1}{2}x'^2}\, dx' \right\}$$

$$= N\sigma \left\{ \frac{1}{\sqrt{2\pi}} - m_1\,(x/\sigma) \right\} \qquad \text{(xxi)};$$

$$n\mu_2' = N\sigma^2 \left\{ \int_0^\infty + \int_{-x/\sigma}^0 \frac{x'^2}{\sqrt{2\pi}} e^{-\frac{1}{2}x'^2}\, dx' \right\}$$

$$= N\sigma^2 \left\{ \tfrac{1}{2} + \int_0^{x/\sigma} \frac{x'^2}{\sqrt{2\pi}} e^{-\frac{1}{2}x'^2}\, dx' \right\}$$

$$= N\sigma^2 \left\{ \tfrac{1}{2} + m_2\,(x/\sigma) \right\} \qquad \text{(xxii)}.$$

Now $d = \bar{x} + x$, and $\Sigma^2 = \mu_2' - \bar{x}^2$.

Hence
$$\frac{d}{\sigma} = \frac{\dfrac{1}{\sqrt{2\pi}} - m_1\,(x/\sigma) + \dfrac{x}{\sigma}\left\{\tfrac{1}{2} + m_0\,(x/\sigma)\right\}}{\tfrac{1}{2} + m_0\,(x/\sigma)} \qquad \text{(xxiii)},$$

$$\frac{\Sigma^2}{\sigma^2} = \frac{\left\{\tfrac{1}{2} + m_2\,(x/\sigma)\right\}\left\{\tfrac{1}{2} + m_0\,(x/\sigma)\right\} - \left\{\dfrac{1}{\sqrt{2\pi}} - m_1\,(x/\sigma)\right\}^2}{\left\{\tfrac{1}{2} + m_0\,(x/\sigma)\right\}^2} \qquad \text{(xxiv)},$$

and
$$\frac{\Sigma^2}{d^2} = \frac{(\tfrac{1}{2} + m_2)(\tfrac{1}{2} + m_0) - \left(\dfrac{1}{\sqrt{2\pi}} - m_1\right)^2}{\left\{\dfrac{1}{\sqrt{2\pi}} - m_1 + x'\,(\tfrac{1}{2} + m_0)\right\}^2} \qquad \text{(xxv)},$$

say, for brevity.

Here m_1 and m_2 are given by Table IX and $\frac{1}{2} + m_0$ is the $\frac{1}{2} + \frac{1}{2}\alpha$ of Table II.

Formula (xxv) has not yet been tabled for different values of x', as it occurs much more rarely than the corresponding function for a true tail.

If we take three values $x' = 0$, 0·1 and 0·2, we have, from Tables II and IX,

$x' = 0$,	$\frac{1}{2} + m_0 = \cdot 500{,}0000$,	$\frac{1}{2} + m_2 = \cdot 500{,}0000$,	$\dfrac{1}{\sqrt{2\pi}} - m_1 = \cdot 398{,}9423$,
$x' = \cdot 1$,	„ $= \cdot 539{,}8278$,	„ $= \cdot 500{,}1325$,	„ $= \cdot 396{,}9526$,
$x' = \cdot 2$,	„ $= \cdot 579{,}2597$,	„ $= \cdot 501{,}0512$,	„ $= \cdot 391{,}0427$.

Whence from formula (xxv) for the three values of x

$$\Sigma^2/d^2 = \cdot 5708, \quad \cdot 5528 \text{ and } \cdot 5345.$$

But our value of Σ^2/d^2 is ·5517. Thus we find by interpolation

$$x' = \cdot 1060.$$

It remains to determine m_0, m_1 and m_2 for this value of x', or simpler $\frac{1}{2} + m_0$, $\frac{1}{2} + m_2$ and $\frac{1}{\sqrt{2\pi}} - m_1$ from the above values for $x' = \cdot 1$ and $x' = \cdot 2$. We find

$$\tfrac{1}{2} + m_0 = \cdot 542{,}194, \quad \tfrac{1}{2} + m_2 = \cdot 500{,}184, \quad \frac{1}{\sqrt{2\pi}} - m_1 = \cdot 396{,}598.$$

Whence
$$d/\sigma = \frac{\cdot 396{,}598}{\cdot 542{,}194} + \cdot 1060 = \cdot 8375.$$

Thus
$$\sigma = 5 \cdot 5786/(\cdot 8375) = 6 \cdot 6610 \text{ secs.},$$
$$x = x' \times \sigma = \cdot 7061 \text{ secs.},$$
$$n/N = \cdot 5422.$$

This gives a mean of 28·29 secs. with a variability measured by 6·66 secs. Those actually registered as trotters are 54 °/₀ of the population.

Table X (p. 24)

See under Table IX, p. xxv.

Table XI (p. 25)

Constants of Normal Curve from Moments of Tail about Stump. (Pearson and Lee, *Biometrika*, Vol. VI. pp. 65 and 68.)

This Table may be of service in cases of the following kind:

(*a*) In some cases a record is actually truncated as in the case of the American Trotters dealt with on p. xxvi. Or, again, we may take a record of stature obtained by measuring all men who exceed 69″, or a record of mental capacity found by measuring all persons with low intelligence in a community.

(*b*) When we recognise heterogeneity, e.g. when we have a mixture of male and female bones, or two strains of trypanosomes, we can occasionally get a rough approximate analysis by supposing the tails to represent homogeneous material and then fitting them with normal curves. From the tails we get two components, and if their compound agrees fairly well with the observed total we have performed an analysis far more rapidly than by using the nonic equation*. Difficulties arise owing to deviations from Gaussian frequency being not infrequent; different dichotomic lines may give different results, and owing to paucity of material in the 'tails' and corresponding irregularity there will be large probable errors.

Cases under (*a*) and (*b*) will be treated by exactly the same process, but our Table supposes that the distribution considered is less than half the normal distribution. The rules for determining the mean, standard deviation and total frequency of the untruncated population are given on p. 25 under Table XI itself.

* Pearson, *Phil. Trans.* Vol. 185, A, p. 84.

Illustration (i). The following distribution represents the 'tail' of a group of 301 mentally defective children measured by G. Jaederholm using Binnet-Simon test methods*:

Mental Defect in Years.

	−3·45—−3·95	−3·95—−4·45	−4·45—−4·95	−4·95—−5·45	−5·45—−5·95	−5·95—−6·45	−6·45—−6·95	−6·95—−7·45	Total
Frequency ...	34	18	13	3	4	4	1	1	78

We find, with origin at −3·45,

$$d = 1{\cdot}8077 \text{ in } \tfrac{1}{2} \text{ years and } \Sigma^2 = 2{\cdot}7258.$$

Hence $$\psi_1 = \Sigma^2/d^2 = {\cdot}834.$$

The Table (p. 25) gives us $h' = 2{\cdot}186$ and $\psi_2 = 2{\cdot}833$. Hence the mean is at distance from −3·45 on left $= 2{\cdot}186 \times \sigma$, and for the standard deviation

$$\sigma = \psi_2 \times d = 2{\cdot}833 \times 1{\cdot}8077 = 5{\cdot}1212 \tfrac{1}{2} \text{ years} = 2{\cdot}5606 \text{ years},$$

whence mean is at distance $h = 5{\cdot}597$ years from −3·45.

Now $h' = 2{\cdot}186$ corresponds by Table II to $\frac{1}{2}(1+\alpha) = {\cdot}98559$

$$\therefore\ n/N = {\cdot}01441, \text{ or } N = 78/({\cdot}01441) = 5413.$$

Thus the distribution is the tail of a population of 5413 individuals with a mean at 2·15 years of mental *excess*, and a standard deviation of 2·56 years.

The example is merely illustrative and of no importance in itself.

Illustration (ii). Assuming that the correlation data follow the normal law, determine indirectly the 'plural' partial correlation† of habits of mother and health of baby for the limited universe of unclean homes.

The correlations between the various characters found by tetrachoric tables are

Habits of mother and health of baby: $r_{12} = {\cdot}3060$,

Habits of mother and cleanliness of home: $r_{13} = {\cdot}7958$,

Health of baby and cleanliness of home: $r_{23} = {\cdot}2578$.

There are 947 out of 2931 homes which are not clean. Data from Bradford.

Here $$n/N = 947/2931 = {\cdot}3231 = \tfrac{1}{2}(1-\alpha)\cdot$$

hence $\alpha = {\cdot}3538$, and by Table III

$$x' = x/\sigma = {\cdot}459{,}049.$$

* *Mendelism and the Problem of Mental Defect.* II. *On the Continuity of Mental Defect.* Dulau & Co., 37, Soho Square, W.

† *Biometrika*, Vol. IX. p. 289.

Now on the assumption of a normal distribution:

$$n\mu'_s = \int_x^\infty x^s \frac{N}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}(x^2/\sigma^2)}\, dx$$
$$= N\sigma^s \int_{x'}^\infty x'^s e^{-\frac{1}{2}x'^2}\, dx'$$
$$= N\sigma^s \{\mu_s(\infty) - \mu_s(x')\}.$$

Here Table IX (p. 22) shows that if s be odd, $m_s(\infty) = \cdot 398{,}9423$, i.e. $1/\sqrt{2\pi}$, and (p. 23) if s be even, $m_s(\infty) = \cdot 500{,}0000$.

Hence in obtaining the moment coefficients of the tail, about the mean of the whole population, $m_s(x')$ should be subtracted from $\cdot 398{,}9423$ or from $\cdot 500{,}0000$ before the results are multiplied by $(s-1)(s-3)\ldots 2$ or $(s-1)(s-3)\ldots 1$, when s is odd or even respectively. It is convenient to term $\mu_s(\infty) - \mu_s(x')$ the complementary incomplete moment function of order s*.

For $s = 1$ and $s = 2$, we have

$$n\mu_1' = \sigma N \{m_1(\infty) - m_1(x')\},$$
$$n\mu_2' = \sigma^2 N \{m_2(\infty) - m_2(x')\},$$

for in this case the multiplying factors to proceed from $m_s(x')$ to $\mu_s(x')$ are both unity.

Now $x' = x/\sigma$ can be found when n is known from Tables II or III. Hence we have for the distance of centroid of tail from its stump, and for the square of its standard-deviation about its centroid:

$$\left.\begin{aligned} d = \mu_1' - h'\sigma &= \sigma\left[\frac{N}{n}\{m_1(\infty) - m_1(x')\} - h'\right] \\ \Sigma^2 &= \sigma^2 \frac{N}{n}\{m_2(\infty) - m_2(x')\} - \mu_1'^2 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (\text{xxvi}).$$

Of course $m_1(\infty) - m_1(x')$ is the z of Sheppard's Tables II and III.

Returning to our numerical example, we have from Table IX (p. 22):

$$m_1(\cdot 45905) = \cdot 030{,}6721 + \cdot 5905\,[162049] - \tfrac{1}{2}(\cdot 5905)(\cdot 4095)\,[26358]$$
$$= \cdot 039{,}9222,$$
$$m_1(\infty) - m_1(\cdot 45905) = \cdot 359{,}02.$$

Found directly from Sheppard's Tables, it equals $\cdot 35905$.

Similarly from Table IX (p. 23):

$$m_2(\cdot 45905) = \cdot 008{,}1136 + \cdot 5905\,[73{,}162] - \tfrac{1}{2}(\cdot 5905)(\cdot 4095)\,[30661]$$
$$= \cdot 012{,}0630,$$

and $$m_2(\infty) - m_2(\cdot 45905) = \cdot 487{,}9370.$$

* It is the function used by Dr Alice Lee and myself, *Biometrika*, Vol. VI. p. 65 to form Table XI, p. 25, but by an oversight not adequately distinguished in symbol from $\mu_s(x')$ of p. 60 of the same memoir.

We thus reach $d = \sigma\,(\cdot35902/\cdot3231 - \cdot45905)$
$= \cdot65212\sigma,$

and $\Sigma^2 = \sigma^2\,\{\cdot48794/\cdot3231 - (1\cdot11117)^2\}$
$= \cdot274{,}786\sigma^2.$

Or the distance of centroid from stump, and the standard deviation of the tail are respectively

$$d = \cdot65212\sigma \text{ and } \Sigma = \cdot52426\sigma.$$

We now find $\lambda = \sqrt{1 - \Sigma^2/\sigma^2} = \cdot85155.$

The formula for the 'plural' correlation coefficient is*

$$_3r_{12} = \frac{r_{12} - r_{13}'\,r_{23}'}{\sqrt{1 - r_{13}'^2}\sqrt{1 - r_{23}'^2}} \quad \text{.....(xxvii),}$$

where $r_{13}' = \lambda r_{13}, \quad r_{23}' = \lambda r_{23},$
$= \cdot2195, \quad = \cdot6777.$

Thus $_3r_{12} = \cdot2191.$

Actually taking out the universe of not clean homes, the correlation of habits of mother and health of child is ·1615. The difference is considerable, but $_3r_{12}$ is deduced from the entire population of 2931 homes, while ·1615 depends on only a third of this number. The 'singular' partial correlation is $_3r_{12} = \cdot1723$, i.e. the average relation between habits of mother and health of child for each individual grade of cleanliness of home.

Table XII (p. 26)

Tables for testing Goodness of Fit. (W. Palin Elderton, *Biometrika*, Vol. I. pp. 155—163.)

The theory of testing frequency distributions for goodness of fit was first given by Pearson† and may be summed up as follows:

If a frequency distribution or table contains n' 'cells' and the contents of these cells be $m_1', m_2', m_3' \ldots m_n'$ in number, while $m_1, m_2, m_3 \ldots m_n$ be the numbers that would occur in these cells on any theory; then calculate

$$\chi^2 = S\left\{\frac{(m_r' - m_r)^2}{m_r}\right\}$$
$$= \text{sum}\left(\frac{\text{(square of difference of theoretical and observed frequencies)}}{\text{theoretical frequency}}\right) \text{..(xxviii),}$$

and the probability that random sampling would lead to as large or larger deviation between theory and observation is

$$P = \sqrt{\frac{2}{\pi}}\int_\chi^\infty e^{-\frac{1}{2}\chi^2}\,d\chi + \sqrt{\frac{2}{\pi}}e^{-\frac{1}{2}\chi^2}\left(\frac{\chi}{1} + \frac{\chi^3}{1.3} + \frac{\chi^5}{1.3.5} + \ldots + \frac{\chi^{n'-3}}{1.3.5\ldots(n'-3)}\right) \quad \text{if } n' \text{ be even}$$
$$= e^{-\frac{1}{2}\chi^2}\left(1 + \frac{\chi^2}{2} + \frac{\chi^4}{2.4} + \frac{\chi^6}{2.4.6} + \ldots + \frac{\chi^{n'-3}}{2.4.6\ldots(n'-3)}\right) \quad \text{if } n' \text{ be odd}$$
$$\text{.....(xxix).}$$

* Pearson, *Phil. Trans.* Vol. 200, A, p. 25.

† *Phil. Mag.* Vol. L. pp. 157—175, 1900.

Short provisional tables of P were given in the article referred to and were replaced in the following year by the present standard tables of Palin Elderton.

In using the test for goodness of fit, due regard should be paid to the conditions under which it is deduced. It is assumed that the frequencies form a normal system of variates. This is legitimate only when in the binomial $(p+q)^n$, q is not very small as compared with p. If q be not very small as compared with p, even for n finite, the binomial approaches closely to the normal curve. Accordingly in using the test it is desirable to club together small frequencies at the tails of curves or margins of surfaces. The difficulty becomes very obvious when theory can go by fractions, but observations only by units.

The theory can be extended to cover much ground in all sorts of sampling*.

Illustration. The following data for observed frequencies of cephalic index in Bavarian crania and for corresponding frequencies of a fitted Gaussian curve have already been considered on p. xx. Test the goodness of fit.

	Observed (m')	Gaussian (m)	$m'-m$	$\frac{(m'-m)^2}{m}$
Under *75·5*	9·5	12·4	− 2·9	·68
75·5—76·5	12·5	12·7	− 0·2	·00
76·5—77·5	17	22·1	− 5·1	1·18
77·5—78·5	37	35·3	+ 1·7	·08
78·5—79·5	55	51·9	+ 3·1	·19
79·5—80·5	71·5	70·1	+ 1·4	·03
80·5—81·5	82	87·0	− 5·0	·29
81·5—82·5	116	99·4	+16·6	2·77
82·5—83·5	98	104·2	− 6·2	·37
83·5—84·5	107	100·5	+ 6·5	·42
84·5—85·5	82	89·1	− 7·1	·57
85·5—86·5	74	72·6	+ 1·4	·03
86·5—87·5	58	54·3	+ 3·7	·25
87·5—88·5	34·5	37·4	− 2·9	·22
88·5—89·5	19	23·7	− 4·7	·93
89·5—90·5	10	13·8	− 3·8	1·05
90·5—91·5	8	7·4	+ 0·6	·05
Over *91·5*	9	6·3	+ 2·7	1·16
Totals ...	900	900·2	18 Groups	$\chi^2=10·27$

* A word of caution must be given about a recent extension by Slutsky (see *Journal of Royal Statistical Society*, Vol. LXXVII. pp. 78—84) who has applied it to test the goodness of fit of regression curves. In such cases the means and standard deviations of each array should, I think, be deduced from the theoretical surface, and the method would then agree with that illustrated on pp. xxiv—xxvi, i.e. on the probability of a given complex of variates differing from the run of values of a given population significantly. Slutsky after assuming that the observed frequencies and standard deviations of the arrays may replace the theoretical values, deduces his P from Elderton's Tables instead of from the incomplete normal moment tables. He finds for the fit of a straight regression line, used to predict the probable price of rye at Samara from the price a month previously, $\chi^2=22·2$, giving $P=·02$, a bad fit. Had he, however, used the theoretical standard-deviation of an array, i.e. $\sigma\sqrt{1-r^2}$, instead of the very irregular observed standard deviations of individual arrays, he would have found $\chi^2=8·84$, leading to $P=·64$ an excellent fit, which would probably have been still further improved by the use of theoretical total frequencies for the arrays, based, say, on a Gaussian distribution.

Taking from the column for $n' = 18$ (p. 27) the values for $\chi^2 = 10$ and 11, we interpolate $P = \cdot 891$ for $\chi^2 = 10{\cdot}27$ by first differences, and conclude that in 89 out of 100 trials we should get in random sampling a fit as bad or worse than that observed, if the real distribution were Gaussian. Accordingly we say that a Gaussian curve describes excellently the distribution of Bavarian cephalic indices.

Tables XIII—XVI (pp. 29—30)

Auxiliary Tables provided by W. Palin Elderton (*Biometrika*, Vol. I. pp. 162—163), *useful for calculating values of P for* χ^2 *outside the range of the existing table.*

For such cases we must turn back to the fundamental formulae (xxix) of p. xxxi, and the numerical values of considerable portions of these formulae will be found evaluated in these auxiliary tables.

Illustration. Find P for $n' = 11$ and $\chi^2 = 78$, $\frac{1}{2}\chi^2 = 39$, hence by formula we have

$$\begin{aligned} P &= e^{-39}\left(1 + 39 + \tfrac{1}{2}(39)^2 + \frac{(39)^3}{3!} + \frac{(39)^4}{4!}\right) \\ &= e^{-39}(40 + 760{\cdot}5 + 9886{\cdot}5 + 96393{\cdot}375) \\ &= e^{-39} \times 107080{\cdot}375, \end{aligned}$$

where the powers of 39 are taken out of Table XXVII (p. 38).

Hence using Table XIII,

$$\log P = \bar{1}7{\cdot}0625{,}1520 + 5{\cdot}0297{,}0988 = \bar{1}2{\cdot}0922{,}2508,$$

which gives us $P = 1{\cdot}23659/10^{12}$.

As a rule we can select n' to be odd, but, if it is necessarily even, there is more trouble, not in the determination of the series, but in the evaluation of the integral

$$I = \sqrt{\frac{2}{\pi}} \int_{\chi}^{\infty} e^{-\frac{1}{2}\chi^2} d\chi.$$

A table of the values of $F = \frac{1}{2}I$ for $\chi = 5$ to 500 has been given as Table IV (p. 11). This gives $\chi^2 = 25$ to 250000 but the intervals are large.

If greater accuracy be required then Schlömilch's formula*

$$\begin{aligned} I = \sqrt{\frac{2}{\pi}} \int_{\chi}^{\infty} e^{-\frac{1}{2}\chi^2} d\chi = \sqrt{\frac{2}{\pi}}\, \chi e^{-\frac{1}{2}\chi^2} \Bigg\{ &\frac{1}{\chi^2} - \frac{1}{\chi^2(\chi^2+2)} + \frac{1}{\chi^2(\chi^2+2)(\chi^2+4)} \\ &- \frac{5}{\chi^2(\chi^2+2)(\chi^2+4)(\chi^2+6)} + \frac{9}{\chi^2(\chi^2+2)(\chi^2+4)(\chi^2+6)(\chi^2+8)} \\ &- \frac{129}{\chi^2(\chi^2+2)(\chi^2+4)(\chi^2+6)(\chi^2+8)(\chi^2+10)} + \ldots \Bigg\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (\text{xxx}) \end{aligned}$$

must be used.

Here $\sqrt{\frac{2}{\pi}}\, \chi e^{-\frac{1}{2}\chi^2}$ will be found in Table XIII, and the series converges fairly rapidly.

* *Compendium der höheren Analysis*, Bd. II. S. 270, Braunschweig, 1879.

TABLE XVII (p. 31)

Values of $(-\log P)$ *corresponding to given values of* χ^2 *in a fourfold table.* (K. Pearson: On a Novel Method of regarding the Association of two Variates classed solely in Alternate Categories. *Drapers' Company Research Memoirs, Biometric Series,* VIII. Dulau & Co.)

If individuals be classed by the characters into A and not-A, B and not-B, we form a tetrachoric table of the form

	A	Not-A	Totals
B ...	a	b	$a+b$
Not-B	c	d	$c+d$
Totals	$a+c$	$b+d$	N

For such a table:

$$\chi^2 = \frac{N(ad-bc)^2}{(a+b)(c+d)(b+d)(a+c)} \quad \text{.....(xxxi),}$$

gives a measure of the probability of independence, and, if the two attributes are highly associated, χ^2 will be large and P the probability of independence very small and largely outside Palin Elderton's Table XII. Table XVII provides for such cases.

Illustrations. The following tables are given by Mr G. U. Yule in his *Theory of Statistics** His conclusions with regard to them are

1. *Datura*: "No Association."
2. Eye Colour in Father and Son: "Shows the tendency to resemblance."
3. Houses in course of erection, Urban and Rural: "Distinct Positive Association."
4. Imbecility and Deaf-Mutism: "High Degree of Association."
5. Developmental Defects and Dullness: "Very high indeed."

It is required to measure the degree of probability that the variates in these five cases are independent.

(1) *Datura.*

Colour of Flower.

Fruit.	Violet	White	Totals
Prickly ...	47	21	68
Smooth ...	12	3	15
Totals ...	59	24	83

(2) *Eye-Colour in Father and Son.*

Father.

Son.	Light	Not Light	Totals
Light ...	471	148	619
Not Light ...	151	230	381
Totals ...	622	378	1000

* Pp. 37, 34, 62, 33, 34 and 45 respectively.

(3) *Houses in course of erection in Urban and Rural Districts (in 1000 houses as unit).*

	Built	Building	Totals
Urban ...	4960	50	5010
Rural ...	1749	12	1761
Totals ...	6709	62	6771

(4) *Imbecility and Deaf-Mutism.*

	Imbecile	Non-Imbecile	Totals ...
Deaf Mute ...	451	14,795	15,246
Non-Deaf Mute ...	48,431	32,464,323	32,512,754
Totals ...	48,882	32,479,118	32,528,000

(5) *Developmental Defects and Dullness.*

	With Defects	Without	Totals
Dull ...	888	1186	2074
Not-Dull ...	1420	22,793	24,213
Totals ...	2308	23,979	26,287

ϕ^2 the mean square contingency $= \chi^2/N$ and ϕ is the product-moment coefficient on the assumption that the 'presence of the character' is to be considered as a concrete unit*. The coefficient of mean square contingency $C_2 = \sqrt{\phi^2/(1+\phi^2)}$.

The following table gives the values of χ^2, ϕ^2, ϕ and C_2, and the values of P deduced.

	χ^2	ϕ^2	ϕ	C_2	P
(1) Datura	·7080	·085,301	·2921	·2803	·8713
(2) Eye-Colour	133·3265	·133,327	·3651	·3430	$1·035/10^{28}$
(3) Houses	1439·30	·000,2125	·0146	·0146	$8·730/10^{312}$
(4) Imbecility and Deaf-Mutism	8014·62	·000,2464	·0157	·0157	$3·179/10^{1739}$
(5) Defects and Dullness ...	3256·797	·123,894	·3519	·3320	$2·846/10^{706}$

For example, from Table XVII by Formula (i) we have for $\chi^2 = 133·3265$:

$$(-\log P) = 20·809 + \frac{33·3265}{50}[10·770] - \tfrac{1}{2}\left(\frac{33·3265}{50}\right)\left(\frac{16·6735}{50}\right)[·026]$$

$$= 27·985,$$

and therefore $\log P = \overline{28}·015,$

which leads to $P = 1·035/10^{28}.$

Or again, for $\chi^2 = 8014·62$:

$$-\log P = 1735·324 + \frac{14·62}{500}[108·561] - \tfrac{1}{2}\left(\frac{14·62}{500}\right)\left(\frac{485·38}{500}\right)[·000]$$

$$= 1738·498,$$

* Pearson and Heron, *Biometrika*, Vol. IX. p. 167.

and therefore $\log P = \overline{1739}{\cdot}502,$

and $P = 3{\cdot}179/10^{1739}.$

If we have a χ^2 less than 70 a different treatment must be used. For example if $\chi^2 = 4{\cdot}6329$ we use Table XII.

We have for $n' = 4$:

$$P = {\cdot}261464 + {\cdot}6329\,[-89667] - \tfrac{1}{2}({\cdot}6329)({\cdot}3671)\,[+29480]$$
$$= {\cdot}2013.$$

Had we worked from Table XVII by Formula (i), we should have had

$$P = {\cdot}2009.$$

For $\chi^2 = {\cdot}7080$, we can use Table XII, remembering that for $\chi^2 = 0$, $P = 1$. We have

$$P = 1{\cdot}000{,}000 + {\cdot}708\,[-198{,}747] - \tfrac{1}{2}({\cdot}708)({\cdot}292)\,[-30{,}099]$$
$$= {\cdot}8624.$$

Had we worked from Table XVII by Formula (i), we should have had $P = {\cdot}865$, close enough for practical purposes.

The true value of P worked from

$$P = 2\left\{\frac{1}{\sqrt{2\pi}}\int_{\chi}^{\infty} e^{-\frac{1}{2}\chi^2}\,d\chi + \frac{1}{\sqrt{2\pi}}\,e^{-\frac{1}{2}\chi^2}\,\chi\right\}$$

by using Table II is $P = {\cdot}8713$. See p. xxxviii.

Examining the values of P we see that having regard to the errors of random sampling we can only say that there is a significant but very slight relation between rural and urban districts and houses building or built; there is again a significant but very slight association of imbecility and deaf-mutism. There is no association on the given material in the *Datura* characters. The other two cases have clearly fairly marked association, quite independent of any influence of random sampling. If we regard these four tables the order of ascending association judged by either ϕ or C_2 is (3), (4), (5), (2), as against Mr Yule's (2), (3), (4), (5). If we disregard the non-significance and take merely intensity of association, without regard to random sampling, the order is (3), (4), (1), (5), (2), as against Mr Yule's order (1), (2), (3), (4), (5).

The best method of inquiry at present for relative association in the case of four-fold tables is, I hold, first to investigate P and throw out as not associated those cases like the "Fruit and Colour in *Datura*" above. Then to use either "tetrachoric r_t" or C_2 according as we are justified in considering the variates as continuous or not. r_p (see p. xxxvii) may be used as control.

Tables XVIII—XX (pp. 31—32)

Tables for determining the Equiprobable Tetrachoric Correlation r_p. (Pearson and Bell: On a Novel Method of regarding the Association of two Variates classed

solely in Alternate Categories. *Drapers' Company Research Memoirs, Biometric Series*, VIII. Dulau & Co.)

We have seen under the discussion of the previous Table how to find a measure of the improbability of two variates being independent, when they are classed in alternate categories. The difficulty in such cases is to appreciate the relative importance of very large inverse powers of 10. The object of the present tables is to enable us to deduce a tetrachoric correlation, r_t, of which the improbability is the same as that of the given system supposing it to arise, when the two variates have the same marginal frequencies but are really independent. In order to do this we have to determine ${}_0\sigma_r$ for the given marginal frequencies, i.e. the standard deviation of r_t on the assumption that r is really zero. This may be easily found from Abac Diagram XXI or from Table XXIV (see below). Table XVIII then gives us the value of $(-\log P)$ for each value of r_t and ${}_0\sigma_r$. If we now turn to our original table and calculate its χ^2, this as we have seen will correspond to a given $(-\log P)$. We now make the $(-\log P)$ from our χ^2 correspond to the $(-\log P)$ from our r_t and ${}_0\sigma_r$, this gives us a value of r_t which has the same degree of improbability as our observed table. In other words, instead of trying to appreciate the meaning of inverse high powers of 10, we say that a table of the same marginal frequency would be as improbable if it had a tetrachoric correlation r_t arising from random sampling of independent variates. Thus we read our improbability on a scale of tetrachoric correlation. We use our correlation merely as a scale to measure probability on.

As $\log \chi^2$ provides a more satisfactory basis for interpolation, and as many readers use logarithm tables and not calculators, $\log \chi^2$ will be the form in which χ^2 will be often presented. Table XX provides the value of r_t corresponding to given ${}_0\sigma_r$ and given $\log \chi^2$.

We will assume for the present that ${}_0\sigma_r$ can be readily found from the marginal totals: see p. xli below.

Illustration. Obtain the values of r_P for the five tables given above on pp. xxxiv—v.

The values of $\log \chi^2$ and ${}_0\sigma_r$ are as follows:

	$\log \chi^2$	${}_0\sigma_r$
(1) Datura	$\bar{1}\cdot8500$	·1941
(2) Eye-Colour, Father and Son ...	2·1249	·0514
(3) Houses in Course of Erection ...	·1582	·0634
(4) Imbecility and Deaf-Mutism ...	3·9039	·0175
(5) Developmental Defects and Dullness	3·5128	·0201

Of the values here recorded for $\log \chi^2$ and ${}_0\sigma_r$, those for the first and third cases lie beyond the limits of our Table XX. But if the point ${}_0\sigma_r = ·0634$ and $\log \chi^2 = ·1582$ be noted on the Abac XXII, p. 34, it will be seen that, having due

regard to the spacings of the correlation curves, the value of the equiprobable correlation is under ·03, say ·027. In other words no significant association can be asserted.

In the case of ${}_0\sigma_r = ·1941$ we are thrown back on the original formulae*. In the first place we must find P for the given value of χ^2, i.e. ·7080 (see p. xxxv). But for $n' = 4$ from formula (xxix),

$$P = 2\left\{\frac{1}{\sqrt{2\pi}}\int_{\chi}^{\infty} e^{-\frac{1}{2}\chi^2}\,d\chi + \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\chi^2}\chi\right\}$$
$$= 2\{·200{,}0578 + ·280{,}0088 \times ·84142\}$$
$$= ·871{,}3256.$$

To obtain r we have to use the formula below, where ${}_0\sigma_r = ·1941$, and $m = \frac{1}{2}\left(\frac{1}{{}_0\sigma_r{}^2} - 3\right)$, the μ_0, μ_4, μ_8 being the normal moment functions of Table IX.

$$P = \frac{2}{{}_0\sigma_r\sqrt{2m}}\left[\{\mu_0(\sqrt{2m}) - \mu_0(\sqrt{2m}r)\} - \frac{1}{8m}\{\mu_4(\sqrt{2m}) - \mu_4(\sqrt{2m}r)\}\right.$$
$$\left. - \frac{1}{24m^2}\{\mu_6(\sqrt{2m}) - \mu_6(\sqrt{2m}r)\} + \frac{1}{128m^2}\{\mu_8(\sqrt{2m}) - \mu_8(\sqrt{2m}r)\}\right]$$
.....(xxxii).

Substituting the values of ${}_0\sigma_r = ·1941$ and $\sqrt{2m} = 4·852{,}107$, we have for

$$r = ·03, \quad P = ·90550,$$
$$r = ·04, \quad P = ·86501.$$

Whence for $P = ·87133$, we have $r = ·038$.

We now turn to the three cases which fall inside Table XX.

(2) *Eye-colour, Father and Son.*

$$\log\chi^2 = 2·1249 \qquad {}_0\sigma_r = ·0514,$$

${}_0\sigma_r = ·05$	$r = 0·5$	$\log\chi^2 = 2·0942$
	$r = 0·6$	$\log\chi^2 = 2·2748$
${}_0\sigma_r = ·06$	$r = 0·6$	$\log\chi^2 = 2·1239$
	$r = 0·7$	$\log\chi^2 = 2·2935.$

Linear differences will suffice

$$\sigma_r = ·05 \qquad r = 0·5 + \frac{·0307}{·1806}[·1] = 0·517,$$

$${}_0\sigma_r = ·06 \qquad r = 0·6 + \frac{·0010}{·1696}[·1] = 0·601.$$

Hence ${}_0\sigma_r = ·0514$ gives

$$r = ·517 + \frac{14}{100} \times ·084$$
$$= ·517 + ·012 = ·529.$$

* *Drapers' Company Research Memoirs.* Biometric Series VII. "A Novel Method," etc.: see pp. 12, 13.

Interpolating for ${}_0\sigma_r$ first,

$$r = \cdot5 \qquad {}_0\sigma_r = \cdot0514 \qquad \log\chi^2 = 2\cdot0737,$$
$$r = \cdot6 \qquad {}_0\sigma_r = \cdot0514 \qquad \log\chi^2 = 2\cdot2537.$$

Hence for $\log\chi^2 = 2\cdot1249$:

$$r_P = \cdot5 + \frac{\cdot0512}{\cdot1800}[\cdot1] = \cdot528.$$

We conclude that the equiprobable correlation is ·53.

(4) *Imbecility and Deaf-mutism.*

$$\log\chi^2 = 3\cdot9039 \qquad {}_0\sigma_r = \cdot0175,$$
$$r = 0\cdot95,\ {}_0\sigma_r = \cdot01,\ \log\chi^2 = 4\cdot3673;\quad {}_0\sigma_r = \cdot02,\ \log\chi^2 = 3\cdot7660.$$

Hence: $r = 0\cdot95,\ {}_0\sigma_r = \cdot0175,\ \log\chi^2 = 3\cdot9163.$

Again:

$$r = 0\cdot90,\ {}_0\sigma_r = \cdot01,\ \log\chi^2 = 4\cdot2207;\quad {}_0\sigma_r = \cdot02,\ \log\chi^2 = 3\cdot6197.$$

Hence: $r = 0\cdot90 \quad {}_0\sigma_r = \cdot0175,\ \log\chi^2 = 3\cdot7699.$

Interpolating $\log\chi^2 = 3\cdot9039$ between 3·9163 and 3·7699, we find

$$r_P = 0\cdot946.$$

(5) *Developmental Defects and Dullness.*

$$\log\chi^2 = 3\cdot5128, \quad {}_0\sigma_r = \cdot0201.$$
$$r = 0\cdot8,\ {}_0\sigma_r = \cdot02,\ \log\chi^2 = 3\cdot4097;\quad {}_0\sigma_r = \cdot03,\ \log\chi^2 = 3\cdot0598.$$

Hence: $${}_0\sigma_r = \cdot0201, \quad \log\chi^2 = 3\cdot4062.$$
$$r = 0\cdot9,\ {}_0\sigma_r = \cdot02,\ \log\chi^2 = 3\cdot6197;\ {}_0\sigma_r = \cdot03,\ \log\chi^2 = 3\cdot2690.$$

Hence: $\log\chi^2 = 3\cdot6162$, for ${}_0\sigma_r = \cdot0201.$

Thus, by interpolating $\log\chi^2 = 3\cdot5128$ between 3·4062 and 3·6162, we find

$$r_P = \cdot851.$$

We have accordingly the following results:

	C_2	P	r_P	r_t	Q
(1) Datura	·2803	·8713	·038	$-\cdot188 \pm \cdot140$	$-\cdot282$
(2) Eye-Colour	·3430	$1\cdot035/10^{28}$	·529	$\cdot550 \pm \cdot027$	·581
(3) Houses	·0146	·6948	·027	$-\cdot081 \pm \cdot043$	$-\cdot190$
(4) Imbecility and Deaf-Mutism	·0157	$3\cdot179/10^{1739}$	·946	$\cdot330 \pm \cdot012$	·907
(5) Defects and Dullness ...	·3320	$2\cdot846/10^{706}$	·851	$\cdot652 \pm \cdot009$	·846

It will be seen that equiprobable r_P confirms generally the results from P, i.e. the tables for 'Datura' and 'Houses' give no sensible association. r_t also confirms this view and shows that 'Houses' is even lower in the scale than 'Datura.' The order of r_P is the same as that of Yule's coefficient of association Q, but neither r_P, r_t, C_2, P or Q support the conclusions stated to flow from the percentages on

p. xxxiv. Both r_P and Q give very high results for (4) and (5), and this is in accordance with the view elsewhere expressed that for extreme dichotomies Q is not to be trusted. It may further be doubted, whether for such dichotomies the theory of the distribution of deviations on which r_P is based can in its turn be accepted. On the whole r_t seems to me the most satisfactory coefficient of association, to be controlled by results for r_P in the cases where neither the dichotomies are extreme, nor the numbers so large or so small as to fall outside the moderate range of Tables XVIII—XX or Abacs XXI and XXII.

ABACS XXI AND XXII (pp. 33—34).

See after Tables XXIII and XXIV.

TABLES XXIII AND XXIV

Tables for determining approximately the probable error of a tetrachoric correlation. (Pearson, *Biometrika*, Vol. IX. pp. 22—27. Tables calculated by Julia Bell, M.A.)

Given a tetrachoric table

a	b	$a+b$
c	d	$c+d$
$a+c$	$b+d$	N

so arranged that $a+c>b+d$ and $a+b>c+d$,

then if $\frac{1}{2}(1+\alpha_1)=(a+b)/N$, $\frac{1}{2}(1+\alpha_2)=(a+c)/N$,

and r_t be the correlation, we have approximately:

$$\text{Probable error of } r_t = \chi_1 \cdot \chi_{r_t} \cdot \chi_{\alpha_1} \cdot \chi_{\alpha_2},$$

where $$\chi_1 = \cdot 67449/\sqrt{N},$$

and is tabled in Table V, p. 12,

$$\chi_{\alpha_1}=\frac{\sqrt{\frac{1}{2}(1+\alpha_1)\frac{1}{2}(1-\alpha_1)}}{H}, \quad \chi_{\alpha_2}=\frac{\sqrt{\frac{1}{2}(1+\alpha_2)\frac{1}{2}(1-\alpha_2)}}{K} \quad \text{...(xxxiii),}$$

H and K being found from the z column of Table II, p. 2, and

$$\chi_{r_t}=\sqrt{1-r_t^2}\sqrt{1-\left(\frac{\sin^{-1}r_t}{90^\circ}\right)^2} \quad \text{.....(xxxiv),}$$

$\sin^{-1}r_t$ being read in degrees. χ_{α_1} and χ_{α_2} are tabled in Table XXIV and χ_r in Table XXIII (p. 35).

This value of the probable error is only approximate and may diverge considerably from the true value* for extreme dichotomies. In such cases the full formula must be used.

* *Phil. Trans.* Vol. 195, p. 14. χ_0 in formula (*l*) should of course not be included under the radical.

When r_t is zero in the population and not in the sample, the standard deviation ${}_0\sigma_r$ of $r = 0$ is given accurately by $\frac{1}{\sqrt{N}}\chi_{a_1}\chi_{a_2}$.

Illustration (i). Tetrachoric r_t for the Table

22,793	1,420	24,213
1,186	888	2,074
23,979	2,308	26,287

is ·652. Find approximately its probable error.

From Table XXIII:

$$r = \cdot 65,\ \chi_r = \cdot 6785\,;\quad r = \cdot 66,\ \chi_r = \cdot 6675.$$

$$\therefore\ \chi_r = \cdot 6785 - \cdot 0110 \times \cdot 2 = \cdot 6763.$$

Now $\quad \frac{1}{2}(1+\alpha_1) = \cdot 9211,\quad \frac{1}{2}(1+\alpha_2) = \cdot 9122.$

Hence from Table XXIII,

$$\chi_{a_1} = 1\cdot 8249 + \cdot 11\,[754] = 1\cdot 8332,$$

$$\chi_{a_2} = 1\cdot 7623 + \cdot 22\,[626] = 1\cdot 7761,$$

$$\chi_{a_1}\chi_{a_2} = 3\cdot 2559.$$

χ_1 cannot be found from Table V in this case as N is beyond its range. But it equals

$$\cdot 67449/\sqrt{26287} = \cdot 67449/162\cdot 13 = \cdot 00416.$$

Thus finally $\quad$ p.e. of $r_t = \cdot 00416 \times \cdot 6763 \times 3\cdot 2559$

$$= \cdot 009.$$

Illustration (ii). Find the value of ${}_0\sigma_r$ for the table:

471	148	619
151	230	381
622	378	1000

Here $\quad \frac{1}{2}(1+\alpha_1) = \cdot 619$ and $\frac{1}{2}(1+\alpha_2) = \cdot 622.$

$$\chi_{a_1} = 1\cdot 2712 + \cdot 9\,[36] = 1\cdot 2744,$$

$$\chi_{a_2} = 1\cdot 2748 + \cdot 2\,[39] = 1\cdot 2756.$$

$${}_0\sigma_r = \chi_{a_1}\chi_{a_2}/\sqrt{1000} = \cdot 0514.$$

In a similar manner the values for all the ${}_0\sigma_r$'s in the table on p. xxxvii were found.

Abac XXI (p. 33)

For determination of the standard-deviation of the correlation coefficients obtained by random sampling from a four-fold table in which the correlation is zero. (*Drapers' Company Research Memoirs,* Biometric Series, VIII. G. H. Soper's Abac.)

Method of use: Enter with the total frequency of the sample on the left-hand scale, and with the first value of $\frac{1}{2}(1+\alpha)$ on the bottom scale. The horizontal through the former and the vertical through the latter meet at a point. At this point pass up the diagonal to the left-hand scale again. Where you meet that scale pass along the horizontal until you meet the vertical through the second value of $\frac{1}{2}(1+\alpha)$. Then from this point pass along the diagonal again to the left-hand scale, whence traverse the horizontal to the right-hand scale and there the required value of ${}_0\sigma_r$ may be read off.

Illustration (i). Find the value of ${}_0\sigma_r$ for the case just given of

$$N = 1000, \quad \tfrac{1}{2}(1+\alpha_1) = \cdot 619, \quad \tfrac{1}{2}(1+\alpha_2) = \cdot 622.$$

The vertical through ·619 meets the 1000 horizontal in a point whose diagonal reaches the left-hand scale almost exactly in 620. Whence passing horizontally we reach the vertical through ·622 in a point about midway between two diagonal lines. Passing up midway between these two diagonals, we reach almost exactly the 380 line on the left-hand scale. Passing across to the right-hand scale along this line, we see that we are slightly above the middle of the division between ·050 and ·052, say ·0512. The actual value of ${}_0\sigma_r$ is ·0514.

Illustration (ii). Let

$$N = \cdot 6771, \quad \tfrac{1}{2}(1+\alpha_1) = \cdot 7399, \quad \tfrac{1}{2}(1+\alpha_2) = \cdot 9908.$$

A similar process gives first 450 on left-hand scale and then about 248, whence crossing to right-hand scale we find ${}_0\sigma_r = \cdot 0635$ instead of ·0634 actual.

Abac XXII (p. 34)

Abac to determine from $\log \chi^2$ *and* ${}_0\sigma_r$ *the value of the equiprobable correlation* r_p, *for a fourfold table.* (*Drapers' Company Research Memoirs,* Biometric Series, VIII. G. H. Soper's Abac.)

The rule is very simple: Enter the Abac with the proper value of ${}_0\sigma_r$ on the scale at the foot and rise on the vertical till the horizontal through the proper value of $\log \chi^2$ on the left-hand scale is reached. Then follow the curve through the meet of these two lines to the right-hand scale, where the requisite correlation will be found inscribed.

Illustration. Take the Table for Eye Colour in Father and Son given on p. xxxiv. Here, as just shewn, ${}_0\sigma_r = \cdot 0514$ and (p. xxxvii) $\log \chi^2 = 2 \cdot 1249$. If we enter with the vertical through ·0514 on the scale at the bottom, and the horizontal through 2·1249 on the left-hand scale, the curve through their point of intersection reaches the right-hand scale just below the ·53 mark, say ·529. This agrees with the correlation found above (p. xxxviii) by interpolation from Table XX.

TABLE XXV (p. 36)

Value of the probability that the mean of a small sample of n, drawn at random from a population following the normal law, will not exceed (in the algebraic sense) the mean of that population by more than z times the standard deviation of the sample. ("Student": *Biometrika*, Vol. VI. p. 19.)

When n is greater than 10, it will be sufficient as a rule to use the approximate result

$$P = \frac{\sqrt{n-3}}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{(n-3)x^2}{2}} dx \quad \text{.....(XXXV)}$$

as a measure of the probability. This may be found from Table II.

Illustration (i). Experiments of A. R. Cushney and A. R. Peebles on the difference in effect of Dextro-hyoscyamine hydrobromide and Laevo-hyoscyamine hydrobromide*.

Patient	Additional Hours of Sleep (Laevo – Dextro)
1	+1·2
2	+2·4
3	+1·3
4	+1·3
5	0
6	+1·0
7	+1·8
8	+0·8
9	+4·6
10	+1·4
Mean	+1·58
Standard Deviation	1·17

$$z = \frac{+1{\cdot}58}{1{\cdot}17} = +1{\cdot}35.$$

Table XXV shows that for $z = 1{\cdot}35$:

$$P = {\cdot}99854,$$

or the odds are 684 to 1 that laevo- is a better soporific than dextro-hyoscyamine hydrobromide.

Illustration (ii). Difference in weight of crops of potatoes grown by Dr Voelcker with (i) sulphate of potash and (ii) kainite as artificial manure.

* *Journal of Physiology*, 1904.

Gain by sulphate of potash.

1904 (*a*) 10 cwt. 3 qr. 20 lbs.
„ (*b*) 1 ton 10 cwt. 1 qr. 26 lbs.
1905 (*a*) 6 cwt. 0 qr. 3 lbs.
„ (*b*) 13 cwt. 2 qr. 8 lbs.

Average gain=15·25 cwt., and the standard deviation=9 cwt., $z=15{\cdot}25/9=1{\cdot}694$.

Here $n=4$, and Table XXV gives us

$$P=\cdot9653+0{\cdot}94\times[46]=\cdot9696,$$

or the odds are about 32 to 1 that the sulphate of potash is a better dressing than kainite for potatoes.

Illustration (iii). Test whether it is of advantage to kiln-dry barley seed before sowing. The following table gives price of head corn in shillings per quarter for 11 sowings, the first seven in 1899 and the last four in 1900.

	Not Kiln-dried	Kiln-dried	Δ
1899	26·5	26·5	0
	28	26·5	1·5
	29·5	28·5	1
	30	29	1
	27·5	27	0·5
	26	26	0
	29	26	3
1900	29·5	28·5	1
	28·5	28	0·5
	30	29	1
	28·5	28	0·5
Mean		—	·91
Standard Deviation			·79

The Gaussian curve gives

$$P=\sqrt{\frac{8}{2\pi}}\int_{-\infty}^{x}e^{-\frac{8}{2}x^2}\,dx.$$

Here $x=\cdot91/\cdot79=1{\cdot}1519,$

and if $x'=x/\sqrt{1/8}=1{\cdot}1519\times2{\cdot}8284=3{\cdot}258,$

$$P=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{3\cdot258}e^{-\frac{1}{2}x'^2}\,dx',$$

which evaluated by Table II, p. 6, gives

$$P=\cdot99944,$$

or the odds are 1785 to 1 in favour of not kiln-drying seed barley.

If we had actually worked with the non-approximate formula, we should have found

$$P = \cdot 9976,$$

or odds of 416 to 1, considerably less than the approximate formula provide, but not enough difference to vitiate any conclusion likely to be drawn in practice*.

TABLE XXVI (p. 37)

Table for use in plotting Type III Curves, i.e.

$$y = y_0 e^{-p\frac{x}{a}} \left(1 + \frac{x}{a}\right)^p \quad \text{.....(xxxvi)}$$

(W. P. Elderton, *Biometrika*, Vol. II. p. 270.)

Rule: Taking p for the curve, multiply the values in the Table by p in succession on the machine with p on as multiplier. Then *subtract* the results from the logarithm of y_0, and we have the logarithms of the ordinates of the curve at the abscissae found by multiplying X in the first column of the Table by a of the curve. The curve can then be plotted. Its origin will be the mode. It is usually quite unnecessary to use the whole series of ordinates, either alternate ordinates will suffice, or we cut off one or both tails at a considerable distance from their tabulated values.

Illustration. The frequency curve of barometric heights at Dunrobin Castle is given by the curve

$$y = 39 \cdot 140 e^{-22 \cdot 9323 \frac{x}{17 \cdot 7661}} \left(1 + \frac{x}{17 \cdot 7661}\right)^{22 \cdot 9323}.$$

The range $X = -\cdot 65$ to $+\cdot 90$ is easily seen to be sufficient. Column (i) of the accompanying table gives aX for these values, the second gives

$$22 \cdot 9323 \times (\log_{10}(1+X) - X \log_{10} e);$$

* The three illustrations above are drawn from "Student's" original paper. He gives (*l. c.* p. 19) the values for P as drawn from the Gaussian for $n=10$ to compare with those obtained from the full formula. They are,—corrected for slips:

z	Full Formula	Gaussian	z	Full Formula	Gaussian
·1	·61450	·60411	1·1	·99539	·99819
·2	·71835	·70159	1·2	·99713	·99925
·3	·80422	·78641	1·3	·99819	·99971
·4	·86961	·85520	1·4	·99885	·99989
·5	·91607	·90691	1·5	·99926	·99996
·6	·94730	·94375	1·6	·99951	·99999
·7	·96744	·96799	1·7	·99968	—
·8	·98005	·98285	1·8	·99978	—
·9	·98780	·99137	1·9	·99985	—
1·0	·99252	·99592	2·0	·99990	—

Clearly even for $n=10$, the Gaussian ascends too rapidly in P, and this must be borne in mind in deducing conclusions for $z=1$ and upwards when $n=11$ to 20, say.

actually these values are *negative* and must be *subtracted* from log y_0, i.e. 1·592,621; the resulting values are given in the third column. In column (iv) are given the antilogarithms of the numbers in column (iii), and these must be plotted to the values in column (i) to obtain the graph of the curve which is a good fit.

(i)	(ii)	(iii)	(iv)
$x=aX$	$p[\log_{10}(1+X)-X\log e]$*	$\log y$	y
− 9·77	2·474,991	− ·882,370	·13
− 8·88	1·923,630	− ·331,009	·47
− 7·99	1·472,368	·120,253	1·32
− 7·11	1·103,755	·488,866	3·08
− 6·22	·804,557	·788,064	6·14
− 5·33	·564,456	1·028,165	10·67
− 4·44	·375,287	1·217,334	16·49
− 3·55	·230,493	1·362,128	23·02
− 2·67	·124,683	1·467,938	29·37
− 1·78	·053,386	1·539,235	34·61
− 0·89	·012,888	1·579,733	38·00
0·00	·000,000	1·592,621	39·14
0·89	·012,039	1·580,582	38·07
1·78	·046,713	1·545,908	35·15
2·67	·101,957	1·490,664	30·95
3·55	·176,074	1·416,547	26·09
4·44	·267,482	1·325,139	21·14
5·33	·374,828	1·217,793	16·51
6·22	·496,920	1·095,701	12·47
7·11	·632,702	·959,919	9·12
7·99	·781,189	·811,432	6·48
8·88	·941,509	·651,112	4·48
9·77	1·112,905	·479,716	3·02
10·66	1·294,689	·297,932	1·99
11·55	1·486,196	·106,425	1·28
12·44	1·686,831	− ·094,210	·80
13·32	1·896,111	− ·303,490	·50
14·21	2·113,510	− ·520,889	·30
15·10	2·338,613	− ·745,992	·18
15·99	2·570,963	− ·978,342	·11

Once the reader is used to the process it will be found to work readily, and the same multipliers are kept on the mechanical calculator throughout.

Tables XXVII and XXVIII (pp. 38—41)

Tables of the Powers and Sums of the Powers of the natural numbers from 1 *to* 100. (W. Palin Elderton, *Biometrika*, Vol. II. p. 474.)

These tables can be used in a great variety of ways, for example in finding the roots of equations, or in fitting parabolae of various orders to curves.

Illustration (i). Find the positive root of the equation:

$$\phi(r) = ·002,726r^7 + ·057,149r^6 + ·017,192r^5 + ·083,578r^4 + ·088,331r^3 + ·134,717r^2 + r - ·560,386 = 0.$$

* Actually these values are negative, and are therefore *subtracted* from log y_0 to give (iii).

The positive root is less than ·56, but the term in r^2 shows that it must be less than ·52. Take ·52 and ·50 as trials. From Table XXVII we have

1st	·520,000	and	·500,000,
2nd	·270,400	„	·250,000,
3rd	·140,608	„	·125,000,
4th	·073,116	„	·062,500,
5th	·038,020	„	·031,250,
6th	·019,771	„	·015,625,
7th	·010,281	„	·007,813.

Multiply out by the coefficients of $\phi(r)$, retaining the products always on the arithmometer. We find

$$\phi(\cdot 52) = +\cdot 016{,}384.$$
$$\phi(\cdot 50) = -\cdot 008{,}990.$$

Interpolating $r = \cdot 52 - \frac{16384}{25374} \times 2 = \cdot 5071,$

which is correct to last figure.

Illustration (ii). Fit a cubic parabola to the data below, giving the average age of husband to each age of wife in Italy (see *Biometrika*, Vol. II. p. 20). We will suppose each observation to be of equal weight,—this is of course not the fact, but it will illustrate the general method of fitting parabolic curves. In the paper just cited illustrations are given up to parabolae of the sixth order. The object here is to show the use of Table XXVII.

Age of Bride	Probable Age of Groom	Age of Bride	Probable Age of Groom	Age of Bride	Probable Age of Groom
15·5	25·0	25·5	27·0	35·5	36·0
16·5	25·2	26·5	27·5	36·5	37·0
17·5	25·4	27·5	28·0	37·5	38·5
18·5	25·5	28·5	29·0	38·5	39·5
19·5	25·5	29·5	30·0	39·5	41·5
20·5	25·5	30·5	32·0	40·5	41·5
21·5	25·75	31·5	33·0	41·5	42·5
22·5	26·0	32·5	33·5	42·5	43·5
23·5	26·0	33·5	34·0	43·5	43·5
24·5	26·8	34·5	34·5	44·5	43·5
—	—	—	—	45·5	43·5

The ages of groom have been taken as approximate means. Now we can take our axis of x, the age of bride through 30·5, and the age of groom to be measured from 32·0. x will accordingly range from -15 to $+15$, and the age $32+y$ of groom will range from $y=-7$ to $y=11{\cdot}5$. We can now re-arrange the above table in a form suitable for working on the following table. Then the squares, cubes, and if necessary, higher powers of x are taken from Table XXVII, p. 38, and are given as Columns (iii) and (iv) below. The entries in Column (i) are then multiplied by those in (ii), (iii) and (iv) by *continuous* process on the machine, and

it is not needful to enter separate products, the sums being reached which are placed at the foot. Next from Table XXVIII we read off

$$S(x)=0, \quad S(x^2)=2(S(15^2)), \quad S(x^3)=0, \quad S(x^4)=2(S(15^4)),$$
$$S(x^5)=0, \quad S(x^6)=2(S(15^6)).$$

These give us:

$$S(x^2)=2480, \quad S(x^4)=356{,}624, \quad S(x^6)=6096{,}5840.$$

We have now all the numerical data for a solution. Let the required cubic be

$$y=c_0+c_1x+c_2x^2+c_3x^3.$$

Then we must make $u=S(y-c_0-c_1x-c_2x^2-c_3x^3)^2$ a minimum. The resulting equations are

$$S(y) = c_0S(1)+c_1S(x)+c_2S(x^2)+c_3S(x^3),$$
$$S(xy) = c_0S(x)+c_1S(x^2)+c_2S(x^3)+c_3S(x^4),$$
$$S(x^2y) = c_0S(x^2)+c_1S(x^3)+c_2S(x^4)+c_3S(x^5),$$
$$S(x^3y) = c_0S(x^3)+c_1S(x^4)+c_2S(x^5)+c_3S(x^6).$$

(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)
y	x	x^2	x^3	xy	x^2y	x^3y
− 7·0	−15	225	−3375	—	—	—
− 6·8	−14	196	−2744	—	—	—
− 6·6	−13	169	−2197	—	—	—
− 6·5	−12	144	−1728	—	—	—
− 6·5	−11	121	−1331	—	—	—
− 6·5	−10	100	−1000	—	—	—
− 6·25	− 9	81	− 729	—	—	—
− 6·0	− 8	64	− 512	—	—	—
− 6·0	− 7	49	− 343	—	—	—
− 5·2	− 6	36	− 216	—	—	—
− 5·0	− 5	25	− 125	—	—	—
− 4·5	− 4	16	− 64	—	—	—
− 4·0	− 3	9	− 27	—	—	—
− 3·0	− 2	4	− 8	—	—	—
− 2·0	− 1	1	− 1	—	—	—
0	0	0	0	—	—	—
1·0	1	1	1	—	—	—
1·5	2	4	8	—	—	—
2·0	3	9	27	—	—	—
2·5	4	16	64	—	—	—
4·0	5	25	125	—	—	—
5·0	6	36	216	—	—	—
6·5	7	49	343	—	—	—
7·5	8	64	512	—	—	—
9·5	9	81	729	—	—	—
9·5	10	100	1000	—	—	—
10·5	11	121	1331	—	—	—
11·5	12	144	1728	—	—	—
11·5	13	169	2197	—	—	—
11·5	14	196	2744	—	—	—
11·5	15	225	3375	—	—	—
$S(y)=23{\cdot}65$	—	—	—	$S(xy)=1833{\cdot}45$	$S(x^2y)=4560{\cdot}35$	$S(x^3y)=248{,}807{\cdot}85$

Write $b_0 = c_0, \quad b_1 = 10c_1, \quad b_2 = 100c_2, \quad b_3 = 1000c_3.$

Then our equations are

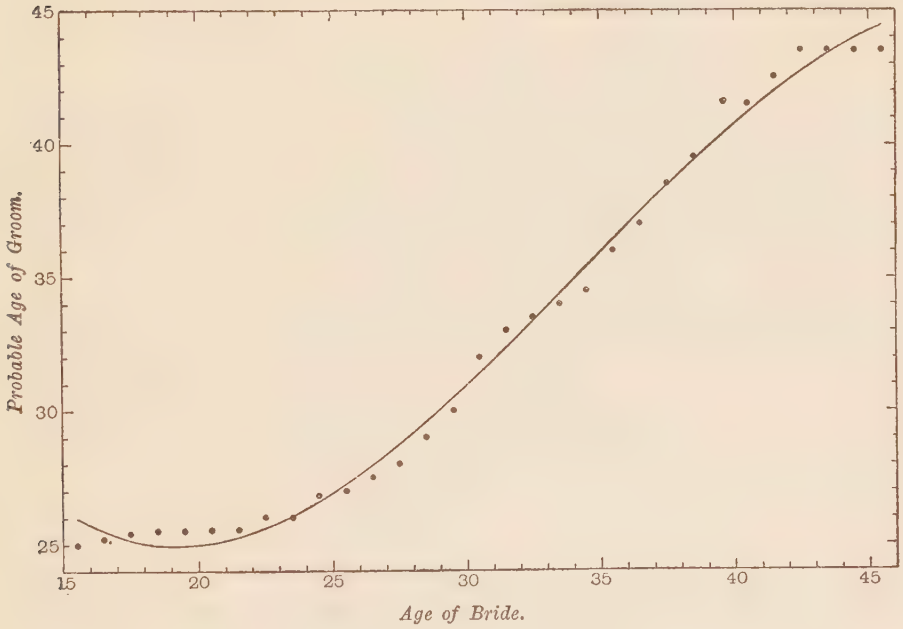
$$\begin{aligned}
\cdot23650 &= b_0 \times \cdot31000 &&+ b_2 \times \cdot24800,\\
1\cdot83345 &= b_1 \times \cdot24800 &&+ b_3 \times \cdot35662,\\
\cdot45603 &= b_0 \times \cdot24800 &&+ b_2 \times \cdot35662,\\
2\cdot48808 &= b_1 \times \cdot35662 &&+ b_3 \times \cdot60966;
\end{aligned}$$

giving

$$\begin{aligned}
b_0 &= -\cdot58626, & \therefore\ c_0 &= -\cdot58626,\\
b_2 &= 1\cdot686453, & c_2 &= \cdot016,8645,\\
b_1 &= 9\cdot59613, & c_1 &= \cdot959,613,\\
b_3 &= -1\cdot532,144, & c_3 &= -\cdot001,532,144,
\end{aligned}$$

and the required cubic is

$$y = -\cdot58626 + \cdot959,613x + \cdot016,8645x^2 - \cdot001,532,144x^3.$$



The graph of the cubic and the observations are given in the accompanying diagram. If X and Y be the actual ages of bride and groom, then

$$Y = 61\cdot30457 - 4\cdot344,941X + \cdot157,0553X^2 - \cdot001,53214X^3.$$

For higher parabolic curves fitted to the same data, see *Biometrika*, Vol. II. pp. 21—22.

TABLE XXIX (pp. 42—51)

Tables of the Tetrachoric Functions. (P. F. Everitt, *Biometrika*, Vol. VII. pp. 437—451.)

The purpose of these tables is to expedite the calculation of tetrachoric r_t, the correlation coefficient from a four-fold table, when we suppose the variates to be Gaussian in the law of their frequency.

Let the table be

a	b	$a+b$
c	d	$c+d$
$a+c$	$b+d$	N

where a is the quadrant in which the mean falls, then $b+d$ and $c+d$ are clearly each less than $\frac{1}{2}N$. Let

$$\tau_0 = (b+d)/N = \tfrac{1}{2}(1-\alpha_1), \quad \tau_0' = (c+d)/N = \tfrac{1}{2}(1-\alpha_2),$$

then
$$d/N = \tau_0\tau_0' + \tau_1\tau_1' r + \tau_2\tau_2' r^2 + \ldots + \tau_n\tau_n' r^n + \ldots \ldots\ldots\ldots\text{(xxxvii)}$$

is the equation to determine r the tetrachoric correlation, and Table XXIX gives the values for given τ_0, i.e. $\frac{1}{2}(1-\alpha)$ of the following six tetrachoric functions $\tau_1, \tau_2 \ldots \tau_6$, and further of h, the ratio of the abscissa of the dichotomic line to the standard deviation of the corresponding variate.

It is occasionally needful to go beyond the first six tetrachoric functions. In this case the following finite difference formula is available:

$$\tau_n = hp_n\tau_{n-1} - q_n\tau_{n-2} \ldots\ldots\ldots\ldots\ldots\ldots\text{(xxxviii)},$$

where
$$p_n = 1/\sqrt{n}, \quad q_n = (n-2)/\sqrt{n(n-1)} \ldots\ldots\ldots\ldots\text{(xxxix)}.$$

The following table gives the values of p_n and q_n from $n=7$ to 24.

n	p_n	q_n	n	p_n	q_n
7	·37796	·77152	16	·25000	·90370
8	·35355	·80178	17	·24254	·90951
9	·33333	·82496	18	·23570	·91466
10	·31623	·84327	19	·22942	·91925
11	·30151	·85812	20	·22361	·92338
12	·28868	·87039	21	·21822	·92711
13	·27735	·88070	22	·21320	·93048
14	·26726	·88950	23	·20851	·93356
15	·25820	·89709	24	·20412	·93638

Illustration (i). Find the correlation between dullness and developmental defects as indicated in the following table for 26,287 children.

	Without Defects	With Defects	Totals
Not Dull ...	22,793	1,420	24,213
Dull ...	1,186	888	2,074
Totals ...	23,979	2,308	26,287

Here
$$\tau_0 = \tfrac{2074}{26287} = \cdot 078{,}898,$$
$$\tau_0' = \tfrac{2308}{26287} = \cdot 087{,}800.$$

Whence by interpolation from Table, p. 43:

$\tau_1 = \cdot 14712$, $\tau_1' = \cdot 15945$,
$\tau_2 = \cdot 14694$, $\tau_2' = \cdot 15268$,
$\tau_3 = \cdot 05977$, $\tau_3' = \cdot 05431$,
$\tau_4 = -\cdot 04262$, $\tau_4' = -\cdot 05137$,
$\tau_5 = -\cdot 06702$, $\tau_5' = -\cdot 06755$,
$\tau_6 = -\cdot 00752$, $\tau_6' = \cdot 00017$,
$h = 1{\cdot}41253$, $k = 1{\cdot}35442$.

Proceeding to apply the difference formula (xxxviii) for four further functions we have

$\tau_7 = \cdot 04770$, $\tau_7' = \cdot 05221$,
$\tau_8 = \cdot 02985$, $\tau_8' = \cdot 02486$,
$\tau_9 = -\cdot 02530$, $\tau_9' = -\cdot 03185$,
$\tau_{10} = -\cdot 03647$, $\tau_{10}' = -\cdot 03460$.

Hence the equation for r is

$$\begin{aligned}\cdot 026{,}854 = {} & \cdot 023{,}458r + \cdot 022{,}435r^2 + \cdot 003{,}246r^3 \\ & + \cdot 002{,}189r^4 + \cdot 004{,}527r^5 - \cdot 000{,}001r^6 \\ & + \cdot 002{,}490r^7 + \cdot 000{,}742r^8 + \cdot 000{,}806r^9 \\ & + \cdot 001{,}262r^{10}.\end{aligned}$$

Whence we find $r = \cdot 652 \pm \cdot 009$.

Illustration (ii). Find the tetrachoric correlation for the four-fold table given for *Houses in course of Erection* on p. xxxv. Here

$$\tfrac{1}{2}(1-\alpha_1) = \tau_0 = \tfrac{1761}{6771} = \cdot 260{,}080\,; \quad \tfrac{1}{2}(1-\alpha_2) = \tau_0' = \tfrac{62}{6771} = \cdot 009{,}157.$$

By simple linear interpolation,

$\tau_1 = \cdot 32442$, $\tau_1' = \cdot 02468$,
$\tau_2 = \cdot 14753$, $\tau_2' = \cdot 04116$,
$\tau_3 = -\cdot 07766$, $\tau_3' = \cdot 04599$,
$\tau_4 = -\cdot 11015$, $\tau_4' = \cdot 03048$.

Hence the equation for r:

$$-\cdot 000{,}6093 = \cdot 008{,}007r + \cdot 006{,}072r^2 - \cdot 003{,}572r^3 - \cdot 003{,}357r^4.$$

Whence $r = -\cdot 081 \pm \cdot 043$.

Or, the association is not definitely significant.

Illustration (iii). Find the tetrachoric r for the Table of Bradford Parents:

Mother's Habits.

Father's Habits.		Good	Bad	Totals
Good	...	994	67	1061
Bad	...	159	476	635
Totals	...	1153	543	1696

Here a brief experience will show the reader that to proceed by tetrachoric functions will require a very large amount of labour.

We have

$$\tfrac{1}{2}(1-\alpha_1) = \tau_0 = 543/1696 = \cdot 32017\,; \quad \tfrac{1}{2}(1-\alpha_2) = \tau_0' = 635/1696 = \cdot 37441,$$
$$d/N = 476/1696 = \cdot 28066.$$

We have accordingly the following series of tetrachoric functions—the first 6 from the table, the remaining 18 from the difference formula.

d/N	$\frac{1}{2}(1-\alpha)$	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6
·28066	·32017	·35769	·11817	−·11415	−·09489	·05674	·08012
—	·37441	·37901	·08581	−·13887	−·07178	·08288	·06325
τ_7	τ_8	τ_9	τ_{10}	τ_{11}	τ_{12}	τ_{13}	τ_{14}
−·02963	−·06913	·01368	·06032	−·00324	−·05294	−·00401	·04659
−·05629	−·05709	·04034	·05223	−·02957	−·04819	·02176	·04558
τ_{15}	τ_{16}	τ_{17}	τ_{18}	τ_{19}	τ_{20}	τ_{21}	τ_{22}
·00922	−·04103	−·01304	·03609	·01586	−·03167	−·01793	·02768
−·01575	−·04245	·01103	·03966	−·00723	−·03714	·00411	·03484
τ_{23}	τ_{24}	h	—	—	—	—	—
·01944	−·02407	·46732	—	—	—	—	—
−·00151	−·03272	·32020	—	—	—	—	—

Considering only the equation as far as Everitt's Tables extend, we have

$$\phi(r) = -\cdot16079 + \cdot13557r + \cdot01014r^2 + \cdot01585r^3 + \cdot00681r^4 + \cdot00470r^5 + \cdot00507r^6 = 0.$$

This leads to $r = \cdot9365$, but the series indicates that the terms are far from converging rapidly.

The first 12 tetrachoric functions were then used, the last six being found by the table of p_n and q_n above, and the value of r was found to be ·9152.

Then 18 functions were used and gave $r = \cdot9114$.

Lastly 24 tetrachoric functions were used, and the equation below obtained, which led to $r = \cdot9105$.

$$\begin{aligned}\phi(r) = &-\cdot16079 + \cdot13557r + \cdot01014r^2 + \cdot01585r^3 + \cdot00681r^4 + \cdot00470r^5 \\ &+ \cdot00507r^6 + \cdot00167r^7 + \cdot00395r^8 + \cdot00055r^9 + \cdot00315r^{10} \\ &+ 00010r^{11} + \cdot00255r^{12} - \cdot00009r^{13} + \cdot00212r^{14} - \cdot00015r^{15} \\ &+ 00174r^{16} - \cdot00014r^{17} + \cdot00143r^{18} - \cdot00011r^{19} + \cdot00118r^{20} \\ &- \cdot00057r^{21} + \cdot00096r^{22} - \cdot00003r^{23} + \cdot00079r^{24}.\end{aligned}$$

It will be seen that even with this very large amount of labour we cannot be sure of having reached a final result*. To obviate this the following table was constructed by Everitt, and there is no doubt that the extension of this table to the whole range of correlation would much simplify the discovery of tetrachoric r_t. At present the calculation of high values of r_t, for negative correlations is in hand.

Table XXX (pp. 52—27)

Supplementary Tables for determining High Correlations from Tetrachoric Groupings. (P. F. Everitt, *Biometrika*, Vol. VIII. pp. 385—395.)

Using the notation of p. l,

$$\frac{d}{N} = \frac{1}{2\pi\sqrt{1-r^2}} \int_h^\infty \int_k^\infty e^{-\frac{1}{2}\frac{1}{1-r^2}(x^2+y^2-2rxy)}\, dx\, dy \quad \text{.....(xl)}$$

in the case of a tetrachoric table, or

$$\left.\begin{aligned} \frac{d}{N} &= \frac{1}{\sqrt{2\pi}} \int_k^\infty e^{-\frac{1}{2}y^2}\, Y\, dy \\ \text{where } Y &= \frac{1}{\sqrt{2\pi}} \int_t^\infty e^{-\frac{1}{2}X\,dX}, \text{ if } t = \frac{h - yr}{\sqrt{1-r^2}} \end{aligned}\right\} \quad \text{.....(xli).}$$

* Mr H. E. Soper working out this example draws my attention to the fact that convergence is closely given by a form : $r_n = r_\infty (1 + a \cdot c^n)$, where n is the number of terms used and a and c are constants.

Hence
$$(r_n - r_\infty)(r_{n+2m} - r_\infty) = (r_{n+m} - r_\infty)^2,$$

or
$$r_\infty = \frac{r_n r_{n+2m} - r^2_{n+m}}{r_n + r_{n+2m} - 2r_{n+m}}.$$

In our case take $n = 6$, $m = 6$, and we find

$$r_\infty = \frac{r_6 r_{18} - r_{12}^2}{r_6 + r_{18} - 2r_{12}} = \cdot9106.$$

The value r_{24} is ·9105. In this case $a = \cdot1567$ and $c = \cdot7574$, but we cannot assert that these would be constants for all tables. If we use r_{12}, r_{18} and r_{24}, we find $r_\infty = \cdot9102$.

Hence r, h being known, Y is a tabled integral for each value of Y. Accordingly by aid of Table II we know $\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}y^2}$, and using a quadrature formula, d/N can be found for each value of h, k and r.

Table XXX gives, for $r = \cdot80$, $\cdot85$, $\cdot90$, $\cdot95$ and $1\cdot00$, and values of h and k proceeding by $\cdot1$, the values of d/N. For given values of h, k and d/N, we can then find r by interpolation from these tables. The process is far shorter than that required by Table XXIX when we have to proceed to many terms. Unfortunately opportunity has not yet arisen for fully completing similar tables for r negative and over $\cdot80$.

Illustration. Determine the correlation in habits between Mother and Father in Bradford. The data are

Mother.

Father.		Habits Good	Habits Bad	Totals
Habits Good ...		994	67	1061
Habits Bad ...		159	476	635
Totals		1153	543	1696

Here $(b+d)/N = \cdot32017$, $(c+d)/N = \cdot37441$, and therefore $h = \cdot46722$, $k = \cdot32020$ from Table II. Also $d/N = 476/1696 = \cdot28066$.

Inspection of Table XXX shows that r will be likely to lie between $\cdot90$ and $\cdot95$.

We extract from the Table for d/N:

$r = \cdot90$	$h = \cdot4$	$h = \cdot5$
$k = \cdot3$	$\cdot2943$	$\cdot2728$
$k = \cdot4$	$\cdot2784$	$\cdot2602$

$r = \cdot95$	$h = \cdot4$	$h = \cdot5$
$k = \cdot3$	$\cdot3135$	$\cdot2898$
$k = \cdot4$	$\cdot2980$	$\cdot2787$

Hence:

$r = \cdot90$	$h = \cdot4$	$h = \cdot5$
$k = \cdot32020$	$\cdot2911$	$\cdot2703$

$r = \cdot95$	$h = \cdot4$	$h = \cdot5$
$k = \cdot32020$	$\cdot3104$	$\cdot2876$

Thus:

$r = \cdot90$	$h = \cdot46722$
$k = \cdot32020$	$\cdot2771$

$r = \cdot95$	$h = \cdot46722$
$k = \cdot32020$	$\cdot2951$

We have now the desired h and k and have to interpolate $d/N = \cdot 28066$ between $\cdot 2771$ and $\cdot 2951$. There results $r = \cdot 9099$.

This is in excellent agreement with the value $\cdot 9105$ deduced from 24 terms, or from the final value $\cdot 9102$, which can be deduced from the 12, 18 and 24 term values on the logarithmic rate of decrease hypothesis: see footnote p. liii.

Table XXXI (pp. 58—61)

The Γ*-Function.* (J. H. Duffell: *Biometrika*, Vol. VII. pp. 43—47.)

It is well known that $\Gamma(x+1) = x\Gamma(x)$, and this property enables us to raise or lower the argument of the Γ-function at will. As a rule in most statistical investigations we require $\Gamma(x+1)/x^x e^{-x}$. The following formula due to Pearson will then be found to give $\Gamma(x+1)/x^x e^{-x}$ with great exactness:

$$\log\left(\frac{\Gamma(x+1)}{x^x e^{-x}}\right) = \cdot 399{,}0899 + \tfrac{1}{2}\log x + \cdot 080{,}929 \sin\frac{25^\circ\cdot 623}{x} \quad \ldots(\text{xlii}).$$

For values of $x+1$ less than 6 and often for values less than 10, we find $\log\Gamma(x+1)$ or $\log\Gamma(p)$ from Table XXXI by reduction to p between 1 and 2.

The reader's attention must be especially drawn as to the rules, given on the Table itself, as to (i) characteristic, (ii) change of third figure of mantissa at a bar, and (iii) the sign of the differences on the facing pages of the tables. The difference tabled under 1·144, say, is the drop from 1·144 to 1·145.

Illustration (i). Find $\Gamma(\cdot 2346)$.

By the reduction formula $\Gamma(\cdot 2346) = \Gamma(1\cdot 2346)/\cdot 2346$.

Hence
$$\log\Gamma(\cdot 2346) = \log\Gamma(1\cdot 2346) - \bar{1}\cdot 370{,}3280.$$
$$\log\Gamma(1\cdot 234) = \bar{1}\cdot 958{,}9685 \qquad \Delta = -1069,$$
$$\log\Gamma(1\cdot 235) = \bar{1}\cdot 958{,}8616 \qquad \cdot 6\Delta = -[641\cdot 4].$$
$$\therefore \log\Gamma(1\cdot 2346) = \bar{1}\cdot 958{,}9685 - [641]$$
$$= \bar{1}\cdot 958{,}9044.$$
$$\log\Gamma(\cdot 2346) = \bar{1}\cdot 958{,}9044$$
$$-\bar{1}\cdot 370{,}3280$$
$$\cdot 588{,}5764$$

Or
$$\Gamma(\cdot 2346) = 3\cdot 87772.$$

Illustration (ii). Find $\Gamma(8{\cdot}7614)$.

$$\Gamma(8{\cdot}7614) = 7{\cdot}7614 \times 6{\cdot}7614 \times 5{\cdot}7614 \times 4{\cdot}7614 \times 3{\cdot}7614 \times 2{\cdot}7614 \times 1{\cdot}7614\,\Gamma(1{\cdot}7614).$$

$$\begin{aligned}
\log\Gamma(8{\cdot}7614) = {} & {\cdot}889{,}9401 + \log\Gamma(1{\cdot}7614) \\
& {\cdot}830{,}0366 \\
& {\cdot}760{,}5280 \\
& {\cdot}677{,}7347 \\
& {\cdot}575{,}3495 \\
& {\cdot}441{,}1293 \\
& {\cdot}245{,}8580 \\
= {} & 4{\cdot}420{,}5762 + \log\Gamma(1{\cdot}7614).
\end{aligned}$$

$$\log\Gamma(1{\cdot}7614) = \bar{1}{\cdot}964{,}5473 + {\cdot}4\,[1113] = \bar{1}{\cdot}964{,}5918.$$

$$\therefore\ \log\Gamma(8{\cdot}7614) = 4{\cdot}385{,}1680.$$

Hence $\Gamma(8{\cdot}7614) = 24275{\cdot}49.$

TABLE XXXII (pp. 62—63)

TABLE XXXIII, A and B (p. 64).

Subtense from Arc and Chord in the case of the Common Catenary. (Julia Bell and H. E. Soper: see *Biometrika*, Vol. VIII. pp. 316, 338, and Vol. IX. pp. 401—2.)

If c be the parameter of the common catenary, then we know that

$$y = c\cosh u \quad \text{.....(xliii),}$$

where $u = x/c$ is its equation.

If the chord be $2x$, then

$$\left.\begin{aligned}\text{subtense/chord} &= (y-c)/(2x) \\ &= \frac{(\sinh \tfrac{1}{2}u)^2}{u}\end{aligned}\right\} \quad \text{.....(xliv),}$$

$$\text{arc/chord} = \frac{\sinh u}{u} \quad \text{.....(xlv),}$$

$$\frac{\text{arc} - \text{chord}}{\text{chord}} = \frac{\sinh u - u}{u} = \frac{\beta}{100} \quad \text{.....(xlvi),}$$

$$\frac{\text{subtense}}{\text{chord}} = \frac{(\sinh \tfrac{1}{2}u)^2}{u} = \frac{\alpha}{100} \quad \text{.....(xlvii).}$$

Corresponding values of α and β are given in the Tables XXXII and XXXIII.

Illustration (i). A cable of 132·5 is suspended over the gap between two towers of the same height, 115 feet apart. What will be the droop of the cable?

$$\beta = 100\,\frac{(132{\cdot}5 - 115)}{115} = 11{\cdot}52.$$

Table XXXIII A, gives us $\alpha = 21{\cdot}62 = 100$ subtense/chord.

$$\therefore \text{ subtense} = {\cdot}2162 \times 115$$
$$= 24{\cdot}86.$$

Thus the droop is 24·86 ft.

Illustration (ii). A catenary arch is to have a rise of 50 ft., centre line measurement, and a span of 200. What is the length of the centre line?

$$\alpha = 100 \times 50/200 = 25{\cdot}0,$$

but $\alpha = 25$ by Table XXXII gives $\beta = 15{\cdot}1$.

$$100\ (\text{arc} - \text{chord})/\text{chord} = 15{\cdot}1.$$

$$\therefore \text{ arc} = 230{\cdot}2 \text{ ft.}*$$

Illustration (iii). For some races the shape of the nasal bridge is very approximately a catenary. Thus if the nasal chord from dacryon to dacryon be measured and also the tape measure from dacryon to dacryon, we obtain the mesodacryal index β. The tables enable us to pass to the mesodacryal index α, and thus ascertain the nasal subtense, which is slightly harder of direct measurement than the arcual or tape measure.

In the skull of a male gorilla the mesodacryal chord was 22·6 mm., and the mesodacryal arc 30 mm. Determine the mesodacryal subtense

$$\beta = 100\,\frac{30 - 22{\cdot}6}{22{\cdot}6} = \frac{100 \times 7{\cdot}4}{22{\cdot}6} = 32{\cdot}74.$$

Hence, from Table XXXII:

$$\alpha = 38{\cdot}84 = 100 \text{ subtense}/22{\cdot}6.$$

$$\therefore \text{ subtense} = 22{\cdot}6 \times {\cdot}3884 = 8{\cdot}8 \text{ mm.}$$

The actual value of the mesodacryal subtense measured on the skull was 8·7 mm.

Abac XXXIV (p. 65)

Diagram to find the Correlation Coefficient r from Mean Contingency on the Hypothesis of a Normal Frequency Distribution. (Pearson: *Drapers' Company Research Memoirs,* No. 1, "On the Theory of Contingency.")

If n_{pq} be the frequency in the cell of the pth column and qth row of a correlation or contingency table, and m_p be the total frequency in the pth column, n_q the

* Should there be any use for this table for constructional purposes, which there ought to be when the value of the catenary arch is more fully recognised, I will in a later edition of this work give the value of u corresponding to each β, so that the parameter c can be at once read off and the form of the arch readily plotted. It might also be desirable to give the values of α and β to two decimal places. We have these data in our MS. copies.

total frequency in the qth row, and N the whole population, then if the two variates are independent, the frequency to be expected in the p, qth cell will be

$$N \times \frac{n_q}{N} \times \frac{m_p}{N} = \frac{n_q m_p}{N},$$

and the observed excess over this, i.e. $n_{pq} - \frac{n_q m_p}{N}$, is termed the 'contingency' in this cell. The total contingency must be of course zero, i.e. the sum of all the cell contingencies. If, however, we take only the positive excess contingencies and divide them by N, i.e. $\psi = \frac{1}{N} \Sigma_+ \left(n_{pq} - \frac{n_q m_p}{N} \right)$, we obtain the so-called 'mean contingency.' On the assumption of normal frequency distribution it is possible to deduce the actual correlation from ψ, provided that the cells are sufficiently small for summation to replace integration. As in practice our cells are hardly likely to exceed 8×8, and may be smaller and unequal in area, we shall generally find a value below that of the true correlation, even if the system be accurately normal. A corrective factor corresponding to the class-index correlation has not yet been theoretically deduced. But experience seems to show that to add *half* the correction due to class-index correlations gives good results. That is to say, that, if r_ψ be the correlation found from the Abac, p. 65, and r_{xC_x} and r_{xC_y} be the class-index correlations for x and y, we should take for the true correlation:

$$r = r_\psi + \tfrac{1}{2} \left[\frac{r_\psi}{r_{xC_x} r_{yC_y}} - r_\psi \right]$$

$$= \tfrac{1}{2} \left[r_\psi + \frac{r_\psi}{r_{xC_x} r_{yC_y}} \right] \quad \text{.....(xlviii).}$$

It is clear that this is the same thing as taking the mean of the crude mean contingency correlation and its value as corrected for the class-index correlations. The following illustrations may indicate the method of procedure.

Illustration (i). Find the correlation from the table on p. lix by mean contingency. The first number in each cell is the frequency reduced to 1000, the second number is that to be expected on the basis of independent probability, and the third is the mean contingency of the cell.

The sum of the positive contingencies is 94·136, hence the mean contingency is ·094. Entering the diagram with ·094 on the base scale, we pass up the vertical to the curve, and then along the horizontal to the left hand scale and find $r_\psi = ·285$.

The class-index correlation for the vertical marginal frequency is $r_{yC_y} = ·9645$, and that for the horizontal marginal frequency is ·9624*. Hence

$$r_\psi / (r_{xC_x} r_{yC_y}) = ·307,$$

and

$$r = \tfrac{1}{2} (·307 + ·285) = ·296.$$

The table is actually a true Gaussian distribution with correlation equal to ·300.

* *Biometrika*, Vol. IX. p. 218.

First Variate *A*.

Second Variate *B*.	*1*	*2*	*3*	*4*	*5+6*	*7*	*8*	Totals
1	4·04 (1·224) *2·816*	17·16 (10·948) *6·212*	7·55 (8·976) *−1·426*	3·30 (6·120) *−2·820*	0·91 (2·346) *−1·436*	0·92 (3·434) *−2·514*	0·12 (0·952) *−0·832*	34
2	17·41 (10·836) *6·574*	123·59 (96·922) *26·668*	79·76 (79·464) *0·296*	44·64 (54·180) *−9·540*	14·61 (20·769) *−6·159*	17·67 (30·401) *−12·731*	3·32 (8·428) *−5·108*	301
3	8·86 (10·224) *−1·364*	93·00 (91·448) *1·552*	78·31 (74·976) *3·334*	52·04 (51·120) *0·920*	19·20 (19·596) *−0·396*	26·40 (28·684) *−2·284*	6·19 (7·952) *−1·762*	284
4	2·83 (4·932) *−2·102*	37·73 (44·114) *−6·384*	37·24 (36·168) *1·072*	27·51 (24·660) *2·850*	10·95 (9·453) *1·497*	16·31 (13·837) *2·473*	4·43 (3·836) *0·594*	137
5+6	1·62 (3·780) *−2·160*	25·21 (33·810) *−8·600*	27·75 (27·720) *0·030*	22·09 (18·900) *3·190*	9·26 (7·245) *2·015*	14·64 (10·605) *4·035*	4·43 (2·940) *1·490*	105
7	1·02 (3·528) *−2·508*	19·50 (31·556) *−12·056*	24·47 (25·872) *−1·402*	21·39 (17·640) *3·750*	9·58 (6·762) *2·818*	16·36 (9·898) *6·462*	5·68 (2·744) *2·936*	98
8	0·22 (1·476) *−1·256*	5·81 (13·202) *−7·392*	8·92 (10·824) *−1·904*	9·03 (7·380) *1·650*	4·49 (2·829) *1·661*	8·70 (4·141) *4·559*	3·83 (1·148) *2·682*	41
Totals	36	322	264	180	69	101	28	1000

Illustration (ii). Find r_ψ by mean contingency for the table on p. lx:

The sum of the positive contingencies is 169·846, or we have mean contingency $\psi = \cdot 170$, whence the diagram leads us to $r_\psi = \cdot 480$. The marginal frequencies are the same as in *Illustration* (i). Thus we have

$$r_\psi / (r_{x\,C_x} r_{y\,C_y}) = \cdot 517,$$

$$r = \tfrac{1}{2}(\cdot 517 + \cdot 480) = \cdot 499.$$

The table gives actually a true Gaussian distribution with correlation ·500. It will be seen from *Illustrations* (i) and (ii), that if the distribution be Gaussian, even if the marginal frequencies are in fairly irregular groupings, r_ψ will be reasonably close to the true contingency, and corrected as suggested above will give excellent results.

First Variate *A*.

Second Variate *B*.	*1*	*2*	*3*	*4*	*5+6*	*7*	*8*	Totals
1	7·38 (1·224) *6·156*	19·85 (10·948) *8·902*	4·94 (8·976) *−4·036*	1·38 (6·120) *−4·740*	0·26 (2·346) *−2·086*	0·18 (3·434) *−3·254*	0·01 (0·952) *−0·951*	34
2	20·58 (10·836) *9·744*	145·47 (96·922) *48·548*	78·94 (79·464) *−0·524*	35·98 (54·180) *−18·200*	9·72 (20·769) *−11·049*	9·27 (30·401) *−21·131*	1·04 (8·428) −7·388	301
3	6·01 (10·224) *−4·214*	93·63 (91·182) *2·182*	85·41 (74·976) *10·434*	54·34 (51·120) *3·220*	18·59 (19·596) *−1·006*	22·33 (28·684) *−6·354*	3·69 (7·952) *−4·262*	284
4	1·26 (4·932) *−3·672*	31·81 (44·114) *−12·304*	39·49 (36·168) *3·322*	31·03 (24·660) *6·370*	12·29 (9·453) *2·837*	17·36 (13·837) *3·523*	3·76 (3·836) *−·076*	137
5+6	0·53 (3·780) *−3·250*	18·11 (33·810) *−15·700*	27·79 (27·720) *0·070*	25·14 (18·90) *6·240*	11·09 (7·245) *3·845*	17·62 (10·605) *7·015*	4·72 (2·940) *1·780*	105
7	0·22 (3·528) *−3·308*	11·02 (31·556) *−20·536*	21·59 (25·872) *−4·282*	23·66 (17·640) *6·020*	11·86 (6·762) *5·098*	21·89 (9·898) *11·992*	7·76 (2·744) *5·016*	98
8	0·02 (1·476) *−1·456*	2·11 (13·202) *−11·092*	5·84 (10·824) *−4·984*	8·47 (7·380) *1·090*	5·19 (2·829) *2·361*	12·35 (4·141) *8·209*	7·02 (1·148) *5·872*	41
Totals	36	322	264	180	69	101	28	1000

TABLES XXXV—XLVI (pp. 66—87)

Criteria for Frequency Types and Probable Errors of Frequency Constants. (A. J. Rhind: *Biometrika,* Vol. VII. pp. 127—147 and pp. 386—397.)

It is desirable to consider all these tables under one heading, namely the general investigation of frequency type and of the probable errors of frequency constants.

The main lines of Pearson's theory of frequency are involved in the following statements:

If the differential equation to the uni-modal frequency distribution be

$$\frac{1}{y}\frac{dy}{dx}=\frac{x-a}{f(x)} \quad \text{.....(xlix),}$$

we may suppose $f(x)$ expanded in a series of powers of x, and so

$$\frac{1}{y}\frac{dy}{dx}=\frac{x-a}{c_0+c_1x+c_2x^2+\ldots+c_nx^n+\ldots} \quad \text{.....(l),}$$

then a, c_0, c_1, c_2, ... c_n ... can be uniquely determined from the 'moment coefficients' of the frequency distribution. These constants are functions of certain other constants β_1, β_2-3, β_3, β_4-15, ... which vanish for the Gaussian curve, and are small for any distribution not widely divergent from the Gaussian. Further c_0, c_1, $c_2 \ldots c_n \ldots$ converge, if, as usual, these constants are less than unity, the factors of convergence being of the order $\sqrt{\beta\text{-constant}}$. As a matter of fact c_n involves the $(n+2)$th moment coefficient, and thus we obtain values of the c-constants subject to very large errors, if we retain terms beyond c_2. If we stop at c_2 then our differential equation is of the form

$$\frac{1}{y}\frac{dy}{dx}=\frac{x-a}{c_0+c_1x+c_2x^2} \quad \text{.....(li),}$$

and we need only $\beta_1=\mu_3^2/\mu_2^3$ and $\beta_2=\mu_4/\mu_2^2$, where μ_2, μ_3, μ_4 are the second, third and fourth moment coefficients about the mean.

If we take the form $\frac{1}{y}\frac{dy}{dx}=\frac{x-a}{c_0}$, we reach the Gaussian, in which each contributory cause-group is independent, and if the number of groups be not very large, each cause-group is of equal valency and contributes with equal frequency results in excess and defect of its mean contribution. If we take $\frac{1}{y}\frac{dy}{dx}=\frac{x-a}{c_0+c_1x}$, then each contributory cause-group is still of equal valency and independent, but does not give contributions in excess and defect of equal frequency.

Finally if we take $\frac{1}{y}\frac{dy}{dx}=\frac{x-a}{c_0+c_1x+c_2x^2}$, then contributory cause-groups are not of equal valency, they are not independent, but their results correlated, and further contributions in excess and defect are not equally probable. The use of this form $\frac{1}{y}\frac{dy}{dx}=\frac{x-a}{c_0+c_1x+c_2x^2}$ was adopted to allow of this wide generalisation of the Gaussian hypothesis.

If we adopt it, every β-constant is expressible by means of the formulae:

$$\beta_n(\text{even})=(n+1)\{\tfrac{1}{2}\beta_{n-1}+(1+\tfrac{1}{2}\alpha)\beta_{n-2}\}/(1-\tfrac{1}{2}(n-1)\alpha) \quad \text{.....(lii),}$$

$$\beta_n(\text{odd})\ =(n+1)\{\tfrac{1}{2}\beta_1\beta_{n-1}+(1+\tfrac{1}{2}\alpha)\beta_{n-2}\}/(1-\tfrac{1}{2}(n-1)\alpha) \quad \text{.....(liii),}$$

where

$$\alpha=(2\beta_2-3\beta_1-6)/(\beta_2+3) \quad \text{.....(liv),}$$

in terms of lower β-constants.

Table XLII, (*a*)—(*d*) gives the values of β_3, β_4, β_5 and β_6 in terms of β_1 and β_2. Hence as soon as β_1 and β_2 are calculated we can find the numerical values of

$$\beta_3 = \mu_3\mu_5/\mu_2^4, \quad \beta_4 = \mu_6/\mu_2^3, \quad \beta_5 = \mu_7\mu_3/\mu_2^5, \quad \beta_6 = \mu_8/\mu_2^4 \quad \ldots\ldots\ldots(\text{lv}),$$

theoretically. Although these values will not be those which would be absolutely deduced from the data themselves, they will, considering the large probable errors of μ_5, μ_6, μ_7 and μ_8 be reasonable approximations to them. The values of the probable errors of β_1 and β_2 are determinable by formulae involving β_1, $\beta_2 \ldots \beta_8$.

From these formulae, Tables XXXVII and XXXVIII, giving the values of $\sqrt{N}\Sigma_{\beta_1}$ and $\sqrt{N}\Sigma_{\beta_2}$ have been constructed. Hence multiplying by χ_1 from Table V, we obtain

$$\cdot 67449\,\Sigma_{\beta_1} \text{ and } \cdot 67449\,\Sigma_{\beta_2}$$

the probable errors of β_1 and β_2.

If we add to the standard deviations of β_1 and β_2, the correlation between deviations in β_1 and β_2, namely $R_{\beta_1\beta_2}$, which correlation is given in Table XXXIX, we can find the probable errors of any functions of β_1 and β_2. Two such important functions are the distance d from mean to mode and the skewness sk of the distribution. The probable errors of d and sk can be found from Tables XL and XLI respectively, the former by multiplying the tabulated value $\sqrt{N}\Sigma_d/\sigma$ by $\sigma \times \chi_1$ (from Table V), and the latter by multiplying the tabulated value $\sqrt{N}\Sigma_{sk}$ by χ_1 (from Table V).

Thus far we have only been concerned with the constants which describe certain physical characters of the frequency distribution without regard to the type of curve suited to the distribution. We now turn to the latter subject.

It is known that the type of frequency depends upon a certain criterion κ_2. Hence near the critical values of κ_2 more than one type of curve may describe the frequency within the limit of the probable error of κ_2. Table XLIII gives the probable error of κ_2, if the entries in that table be multiplied by the χ_1 of Table V.

The following are the series of Type curves which arise according to the value of the criteria

$$\kappa_1 = 2\beta_2 - 3\beta_1 - 6 \quad \ldots\ldots\ldots(\text{lvi}),$$

$$\kappa_2 = \frac{\beta_1(\beta_2+3)^2}{4(4\beta_2 - 3\beta_1)(2\beta_2 - 3\beta_1 - 6)} \quad \ldots\ldots\ldots(\text{lvii}).$$

β_2 is by necessity $> \frac{3}{4}\beta_1$. Hence for our curves all possible values of β_1, β_2 lie in the positive quadrant between the lines $\beta_2 = \frac{3}{4}\beta_1$ and $\beta_2 = \frac{15}{8}\beta_1 + \frac{9}{2}$, the latter being if we go to β_8 the limit of failure of Type IV, for its μ_6 becomes infinite. Beyond the latter line distributions are heterotypic.

Criterion	*Type*	*Equation to Curve*
$\kappa_2 = 0 \quad \beta_1 = 0, \quad \beta_2 > 3$	VII	$y = y_0 \dfrac{1}{\left(1 + \dfrac{x^2}{a^2}\right)^m}$ .....(lviii).
$\kappa_2 = 0 \quad \beta_1 = 0, \quad \beta_2 = 3$	Normal	$y = y_0 e^{-\frac{1}{2}\frac{x^2}{\sigma^2}}$ .....(lix).
$\kappa_2 = 0 \quad \beta_1 = 0, \quad \beta_2 < 3$	$\mathrm{II_a}$	$y = y_0 \left(1 - \dfrac{x^2}{a^2}\right)^m$ .....(lx).
$\kappa_2 = 0 \quad \beta_1 = 0, \quad \beta_2 < 1{\cdot}8$	$\mathrm{II_b}$	$y = y_0 \dfrac{1}{\left(1 - \dfrac{x^2}{a^2}\right)^m}$ .....(lxi).
$\kappa_2 > 0 < 1$	IV	$y = y_0 \dfrac{e^{-\nu \tan^{-1}\frac{x}{a}}}{\left(1 + \dfrac{x^2}{a^2}\right)^m}$ .....(lxii).
$\kappa_2 = 1$*	V	$y = y_0 e^{-\gamma/x} x^{-p}$ .....(lxiii).
$\kappa_2 > 1 < \infty$	VI	$y = y_0 (x-a)^{m_1}/x^{m_2}$ .....(lxiv).
$\kappa_2 = \infty$, i.e. $2\beta_2 - 3\beta_1 - 6 = 0$	III	$y = y_0 e^{-p\frac{x}{a}} \left(1 + \dfrac{x}{a}\right)^p$ .....(lxv).
$\kappa_2 < 0$, i.e. negative. Below $f = 0$	$\mathrm{I_I}$	$y = y_0 \left(1 + \dfrac{x}{a_1}\right)^{m_1} \left(1 - \dfrac{x}{a_2}\right)^{m_2}$ .....(lxvi).
$\kappa_2 < 0$. Inside $f = 0$	$\mathrm{J_I}$	$y = y_0 \left(1 + \dfrac{x}{a_1}\right)^{m_1} \Big/ \left(1 - \dfrac{x}{a_2}\right)^{m_2}$ ...(lxvii).
$\kappa_2 < 0$. Above $f = 0$	$\mathrm{U_I}$	$y = y_0 \dfrac{1}{\left(1 + \dfrac{x}{a_1}\right)^{m_1} \left(1 - \dfrac{x}{a_2}\right)^{m_2}}$ ...(lxviii).

For $\kappa_2 < 0$, $f = 0$ represents the biquadratic

$$\beta_1 (8\beta_2 - 9\beta_1 - 12)/(4\beta_2 - 3\beta_1) = (10\beta_2 - 12\beta_1 - 18)^2/(\beta_2 + 3)^2 \ldots(\text{lxix}).$$

Type I is thus divided into three subclasses, limited range curves, J-shaped curves and U-shaped curves.

Diagram XXXV enables the reader at once to find the type appropriate to his distribution, and Diagram XXXVI gives the same figure on a much larger scale to indicate the changes that occur with large values of β_1 and β_2.

Knowing the values of β_1 and β_2 the computer can fix his point on the Diagram XXXV, but he may come so near a critical point or line, that one curve may appear as reasonable as another. It is clear, for example, that in the neighbourhood of the Gaussian point G, he might possibly use II, $\mathrm{I_I}$, III, VI, V,

* The branch of the cubic $\kappa_2 = 1$ with which we are concerned passes through the Gaussian point, at which $p = \infty$, and along this branch p is always > 5.

IV or VIII, and as all these types at that point transform into each other, the forms actually deduced will be almost identical, however different their equations. But there will be other occasions when doubt as to the use of the simpler of two curves may arise; for example if $\beta_1 = \cdot 8$, $\beta_2 = 4 \cdot 15$, are we justified in using Type III as simpler than Type I?

Now we have to remember that the variates β_1, β_2 form a frequency surface, of which the equation is

$$z = \frac{M}{2\pi \Sigma_{\beta_1} \Sigma_{\beta_2} \sqrt{1 - R^2_{\beta_1 \beta_2}}} e^{-\frac{1}{2} \frac{1}{(1 - R^2_{\beta_1 \beta_2})} \left(\frac{\beta_1^2}{\Sigma_{\beta_1}^2} + \frac{\beta_2^2}{\Sigma_{\beta_2}^2} - \frac{2 R_{\beta_1 \beta_2} \beta_1 \beta_2}{\Sigma_{\beta_1} \Sigma_{\beta_2}} \right)} \quad \ldots\ldots\ldots\ldots\text{(lxx)},$$

and that the contours of this surface projected onto the β_1, β_2 plane of Diagram XXXV form a series of similar and similarly placed ellipses. Within any one of these ellipses a certain amount of the volume of the β_1, β_2-frequency lies, and therefore if this system of contours were properly placed round the β_1, β_2 point on Diagram XXXV we could tell at once the probability that the given point, owing to random sampling, should fall outside a given elliptic contour.

The ellipse which has for principal semi-axes $1 \cdot 177 \Sigma_1$ and $1 \cdot 177 \Sigma_2$, where Σ_1 and Σ_2 are the principal axes of the ellipse:

$$1 = \frac{1}{1 - R^2_{\beta_1 \beta_2}} \left(\frac{\beta_1^2}{\Sigma_{\beta_1}^2} + \frac{\beta_2^2}{\Sigma_{\beta_2}^2} - \frac{2 R_{\beta_1 \beta_2} \beta_1 \beta_2}{\Sigma_{\beta_1} \Sigma_{\beta_2}} \right) \quad \ldots\ldots\ldots\ldots\text{(lxxi)},$$

covers an area on which stands just one half the frequency, i.e. it is the ellipse determined by the generalised probable error of two variates (see Table X, p. 24).

The semi-minor axis $1 \cdot 177 \Sigma_1$ and the semi-major axis $1 \cdot 177 \Sigma_2$ of this "Probability Ellipse" multiplied by $\sqrt{N}$ are given in Tables XLIV and XLV respectively, and Table XLVI gives the angle in degrees between the major axis of this ellipse and the axis of β_2. It is thus possible to construct from Tables XLIV—XLVI the "probability ellipse" round a given point β_1, β_2, and to test the area within which half the frequency lies. If the probability required be not $\frac{1}{2}$, but much less, then we note that the probability, that a point will lie outside the ellipse with semi-axes $\lambda \Sigma_1$ and $\lambda \Sigma_2$ is $P = e^{-\frac{1}{2}\lambda^2}$.

Let $$\lambda \Sigma_2 = 1 \cdot 177 \sqrt{N} \Sigma_2 \times \frac{\cdot 67449}{\sqrt{N/q}} \quad \ldots\ldots\ldots\ldots\text{(lxxii)},$$

or $$\lambda^2 = q \times \cdot 630{,}672,$$

and $$P = e^{-q \times \cdot 315{,}336}.$$

Hence $$\log P \doteq -q \times \cdot 136{,}949.$$

Accordingly

$q = 10$: $P = \cdot 0427$,

$q = 12$: $P = \cdot 0227$,

$q = 15$: $P = \cdot 0088$,

$q = 20$: $P = \cdot 0018$.

Hence we select the grade of working probability we require, roughly 1 in 23, 1 in 44, 1 in 114 or 1 in 555, and this determines q. Divide N the total frequency by q and look up in Table V, χ_1 for N/q, multiply this by the $1{\cdot}177\sqrt{N}\Sigma_2$ of Table XLV, p. 84, and we obtain the semi-major axis of the required ellipse. Multiply the same χ_1 by $1{\cdot}177\sqrt{N}\Sigma_1$ of Table XLIV and we have the semi-minor axis. We can then construct round the point β_1, β_2 this ellipse and ascertain if it cuts critical boundaries on Diagram XXXV, p. 66, the orientation being given by Table XLVI, p. 86. Less accurately, but for practical purposes effectively, we may work on Diagram XLVII, p. 88. We proceed just as before, to select our q and so determine our $\lambda\Sigma_2$ and $\lambda\Sigma_1$. Then we take the ratio of Σ_1/Σ_2. We now pick out of the ellipses on p. 88 the set having the nearest Σ_1/Σ_2 value and out of this set the ellipse with the nearest $\lambda\Sigma_2$ value of its semi-major axis. This ellipse or if necessary an interpolated one is transferred to tracing paper and placed with its centre at the given point (β_1, β_2), and its major axis touching the dotted curve. If this ellipse does not cut a critical line, we can be certain that to the given degree of probability the curve is of the type into the area of which its β_1, β_2 point falls.

It would be impossible in an Introduction to these tables to give the whole theory of frequency curves*. But one or two formulae may be usefully placed here for reference.

$$\text{Distance } d \text{ from mode to mean} = \frac{\sigma\sqrt{\beta_1}(\beta_2+3)}{2(5\beta_2-6\beta_1-9)} \quad \text{.....(lxxiii),}$$

$$\text{Skewness } sk = \frac{\sqrt{\beta_1}(\beta_2+3)}{2(5\beta_2-6\beta_1-9)} \quad \text{.....(lxxiv),}$$

$$N\Sigma_{\beta_1}{}^2 = \beta_1(4\beta_4-24\beta_2+36+9\beta_1\beta_2-12\beta_3+35\beta_1) \quad \text{.....(lxxv),}$$

$$N\Sigma_{\beta_2}{}^2 = \beta_6-4\beta_2\beta_4+4\beta_2{}^3-\beta_2{}^2+16\beta_2\beta_1-8\beta_3+16\beta_1 \quad \text{.....(lxxv } bis\text{),}$$

$$N\Sigma_{\beta_1}\Sigma_{\beta_2}R_{\beta_1\beta_2} = 2\beta_5-3\beta_4\beta_1-4\beta_3\beta_2+6\beta_2{}^2\beta_1+3\beta_1\beta_2-6\beta_3+12\beta_1{}^2+24\beta_1 \quad \text{(lxxvi).}$$

It is from the above formulae that the Tables now under discussion have been calculated.

Illustration. The following percentages of black measured with a colour top are stated to occur with the recorded frequencies in the skin colour of white and negro crosses†.

Discuss the type of frequency curve suited to the data and determine the chief physical constants of the distribution and their probable errors.

* The general theory is given in "Skew Variation in Homogeneous Material," *Phil. Trans.* Vol. 186 (1895), A, pp. 343—414: Supplement, Vol. 197 (1901), A, pp. 443—459; "On the Mathematical Theory of Errors of Judgment," *Phil. Trans.* Vol. 198 (1902), pp. 274—279; "Das Fehlergesetz und seine Verallgemeinerungen durch Fechner und Pearson," A Rejoinder, *Biometrika*, Vol. IV. pp. 169—212. "Skew Frequency Curves," A Rejoinder to Professor Kapteyn, *Ibid.* Vol. v. pp. 168—171, and "On the curves which are most suitable for describing the frequency of Random Samples of a Population," *Ibid.* Vol. v. pp. 172—175.

† Extracted from C. B. Davenport, *Heredity of Skin Color in Negro-White Crosses*, Carnegie Institution of Washington, 1913.

The working origin was taken at 20, the centre of the group 18—22. The centre of the first group at 1·47 % is $\frac{1}{5}(20 - 1{\cdot}47) = 3{\cdot}706$ on the negative side of the working origin and may be taken to contribute -56, $+206$, -763, $+2830$

Percentage	Frequency	Percentage	Frequency
*0— 2**	15	*43—47*	45
3— 7	120	*48—52*	24
8—12	139	*53—57*	14
13—17	157	*58—62*	6
18—22	158	*63—67*	3
23—27	139	*68—72*	3
28—32	117	*73—77*	2
33—37	92	*78—82*	2
38—42	50	*83—87*	—

to the first, second, third and fourth moments respectively. The working unit being 5 %, the raw moment coefficients are:

$$\nu_1' = \cdot 567{,}2191, \qquad \nu_2' = 7{\cdot}703{,}4990,$$
$$\nu_3' = 28{\cdot}982{,}5042, \qquad \nu_4' = 253{\cdot}268{,}8730.$$

Whence transferring to mean and correcting, we have

$$\text{Mean} = 22{\cdot}8361, \qquad \sigma = 2{\cdot}70156,$$
$$\mu_2 = 7{\cdot}298{,}428, \qquad \mu_3 = 16{\cdot}238{,}780, \qquad \mu_4 = 198{\cdot}909{,}921.$$

These lead to

$$\beta_1 = \cdot 678{,}295, \qquad \beta_2 = 3{\cdot}734{,}202,$$
$$\kappa_1 = 2\beta_2 - 3\beta_1 - 6 = -\cdot 566{,}483, \qquad \kappa_2 = -1{\cdot}052{,}180,$$
$$sk = \cdot 495{,}087, \qquad \text{Distance from Mean to Mode} = d = 1{\cdot}337{,}508.$$

These values, except the mean, are all in working units. Therefore in percentages of black:

$$\sigma = 13{\cdot}5078 \text{ and } d = 6{\cdot}6875.$$

We can now find the probable errors of these constants. We first want χ_1 from Table V, but 1086 is outside the limit of n. We therefore take χ_2 for 543 and have $\chi_1 = \cdot 02047$, and we find $\chi_2 = \cdot 014{,}47$. We can repeat our constants with their probable errors

$$\text{Mean} = 22{\cdot}8361 \pm \cdot 2765, \qquad \sigma = 13{\cdot}5078 \pm \cdot 1955.$$

Then from Table XXXVII,

$$\beta_2 = 3{\cdot}7: \quad \sqrt{N}\Sigma_{\beta_1} = 4{\cdot}70 + \tfrac{283}{500}[2] = 4{\cdot}71,$$
$$\beta_2 = 3{\cdot}8: \quad \sqrt{N}\Sigma_{\beta_1} = 5{\cdot}05 + \tfrac{283}{500}[2] = 5{\cdot}06.$$

Hence for $\beta_2 = 3{\cdot}7342$: $\sqrt{N}\Sigma_{\beta_1} = 4{\cdot}71 + \tfrac{342}{1000}[35]$,

$$\sqrt{N}\Sigma_{\beta_1} = 4{\cdot}83.$$

* 4 at 0 and 11 at 2, giving a mean at 1·47 %.

Similarly from Table XXXVIII:

$$\beta_2 = 3{\cdot}7: \quad \sqrt{N}\Sigma_{\beta_2} = 12{\cdot}02 - \tfrac{283}{500}[66] = 11{\cdot}65.$$
$$\beta_2 = 3{\cdot}8: \quad \sqrt{N}\Sigma_{\beta_2} = 13{\cdot}60 - \tfrac{283}{500}[72] = 13{\cdot}19.$$

Hence for $\beta_2 = 3{\cdot}7342$: $\sqrt{N}\Sigma_{\beta_2} = 11{\cdot}65 + \tfrac{342}{1000}[1{\cdot}54]$,

$$\sqrt{N}\Sigma_{\beta_2} = 12{\cdot}18.$$

Thus we find, multiplying by χ_1.

$$\beta_1 = \ {\cdot}6783 \pm {\cdot}0989,$$
$$\beta_2 = 3{\cdot}7342 \pm {\cdot}2493.$$

It is clear that the β_1 and β_2 are significantly different from the Gaussian $\beta_1 = 0$ and $\beta_2 = 3$.

We next turn to the skewness, using Table XLI:

$$\beta_2 = 3{\cdot}7: \quad \sqrt{N}\Sigma_{sk} = 1{\cdot}98 + \tfrac{283}{500}[21] = 2{\cdot}10,$$
$$\beta_2 = 3{\cdot}8: \quad \sqrt{N}\Sigma_{sk} = 1{\cdot}88 + \tfrac{283}{500}[16] = 1{\cdot}97.$$

Hence for $\beta_2 = 3{\cdot}7342$: $\sqrt{N}\Sigma_{sk} = 2{\cdot}10 - \tfrac{342}{1000}[13]$

$$= 2{\cdot}06.$$

Thus the skewness $= {\cdot}4951 \pm {\cdot}0422$, or the distribution is significantly skew.

Passing to Table XL for the probable error of d, we have

$$\beta_2 = 3{\cdot}7: \quad \sqrt{N}\Sigma_{d/\sigma} = 2{\cdot}14 + \tfrac{283}{500}[20] = 2{\cdot}25,$$
$$\beta_2 = 3{\cdot}8 \quad \sqrt{N}\Sigma_{d/\sigma} = 2{\cdot}03 + \tfrac{283}{500}[17] = 2{\cdot}13.$$

Hence for $\beta_2 = 3{\cdot}7342$: $\sqrt{N}\Sigma_{d/\sigma} = 2{\cdot}25 - \tfrac{342}{1000}[12]$

$$= 2{\cdot}21.$$

Thus Probable Error of $d = \chi_1 \times \sigma \times 2{\cdot}21 = {\cdot}6111,$

and $$d = 6{\cdot}6875 \pm {\cdot}6111.$$

The probable error of κ_1 is to be found from the relation:

$$(\sqrt{N}\Sigma_{\kappa_1})^2 = 4\,(\sqrt{N}\Sigma_{\beta_2})^2 + 9\,(\sqrt{N}\Sigma_{\beta_1})^2 - 12\,(\sqrt{N}\Sigma_{\beta_2})\,(\sqrt{N}\Sigma_{\beta_1}) \times R_{\beta_1\beta_2}. \quad \text{(lxxvii)}$$

Thus we require $R_{\beta_1\beta}$ Table XXXIX, p. 72, will provide this:

$$\beta_2 = 3{\cdot}7: \ R_{\beta_1\beta_2} = {\cdot}892 + \tfrac{283}{500}[5] = {\cdot}895,$$
$$\beta_2 = 3{\cdot}8: \ R_{\beta_1\beta_2} = {\cdot}893 + \tfrac{283}{500}[5] = {\cdot}896.$$

Hence for $\beta_2 = 3{\cdot}7342$ we may take $R_{\beta_1\beta_2} = {\cdot}895$. Accordingly

$$(\sqrt{N}\Sigma_{\kappa_1})^2 = 593{\cdot}4096 + 209{\cdot}9601 - 631{\cdot}8278$$
$$= 171{\cdot}5419.$$

Or, $$\sqrt{N}\Sigma_{\kappa_1} = 13{\cdot}0974.$$

Hence p.e. of $\kappa_1 = \chi_1 \times \sqrt{N}\Sigma_{\kappa_1} = {\cdot}2681,$

or, $$\kappa_1 = -{\cdot}566{,}483 \pm {\cdot}2681.$$

It would look therefore as if κ_1 were significantly negative, but it is just possible that κ_1 might be zero. Such a big probable error for κ_1 suggests our being in the neighbourhood of a critical limit. This is verified on examining Table XLIII to find the value of $\sqrt{N}\Sigma_{\kappa_2}$. We see that it is over 80, and we thus conclude that the probable error of κ_2 may lie between 1 and 2. Thus we cannot be definitely certain of the sign or magnitude of κ_2, when we are even relatively in the neighbourhood of $\kappa_1 = 0$.

If we turn to Diagram XXXV (p. 66) we see that the point $\beta_1 = \cdot 678$ and $\beta_2 = 3\cdot 734$ is not very close but is approaching the line along which Type III is applicable, and this is the source of the disturbance noted.

We accordingly try to measure the probability that Type III would be as satisfactory as Type I within the area of which our β_1, β_2 values actually lie. We must take $1\cdot 177\sqrt{N}\Sigma_1$ from Table XLIV and $1\cdot 177\sqrt{N}\Sigma_2$ from Table XLV. We have

$$\beta_1 = \cdot 6789,\ \beta_2 = 3\cdot 7 \text{ then } 1\cdot 177\sqrt{N}\Sigma_1 = 2\cdot 3$$

$$\text{„} \quad \text{„} \quad \beta_2 = 3\cdot 8 \quad \text{„} \quad \text{„} \quad = 2\cdot 5.$$

Hence for $\beta_2 = 3\cdot 7342$, $1\cdot 177\sqrt{N}\Sigma_1 = 2\cdot 3 + \frac{342}{1000}[2] = 2\cdot 37$.

Again: $\beta_2 = 3\cdot 7 \quad 1\cdot 177\sqrt{N}\Sigma_2 = 16 - \frac{283}{500}[1] = 15\cdot 43,$

$\beta_2 = 3\cdot 8 \quad 1\cdot 177\sqrt{N}\Sigma_2 = 18 - \frac{283}{500}[1] = 17\cdot 43,$

$\beta_2 = 3\cdot 7342 \quad 1\cdot 177\sqrt{N}\Sigma_2 = 15\cdot 43 + \frac{342}{1000}[2] = 16\cdot 11.$

Thus: $\Sigma_1/\Sigma_2 = 2\cdot 37/16\cdot 11 = \cdot 147 = \cdot 15$, say. Or, if we turn to Diagram XLVII (p. 88), our system of ellipses is half-way between the 3rd ($\Sigma_1/\Sigma_2 = \cdot 14$) and the 4th ($\Sigma_1/\Sigma_2 = \cdot 16$). Now if such a system of ellipses be traced off and centred at the point $\beta_1 = \cdot 678$, $\beta_2 = 3\cdot 734$ on Diagram XLVII to the right and then the major-axis be brought into parallelism with the dotted lines, we find that the biggest of these ellipses $\lambda\Sigma_2 = \cdot 5$ fails to reach the critical line III. But the semi-major axis of the probability-ellipse is $1\cdot 177\Sigma_2 = 16\cdot 11/\sqrt{N} = \cdot 493$. Hence we must conclude that it is more probable that the curve is of Type I than of Type III. This is readily determined and is usually sufficient guide. Actually the value of $\lambda\Sigma_2$ must be about $\cdot 6$ before we get an ellipse to approximately touch the Type III line. But $\Sigma_2 = \cdot 493/1\cdot 177 = \cdot 419$, and accordingly $\lambda = \cdot 6/\cdot 419 = 1\cdot 432$, which gives $P = e^{-\frac{1}{2}\lambda^2} = \cdot 36$ nearly, or the odds are 16 to 9 that the point would not lie outside this contour. But if it did lie outside this contour, the chance of its being on or over the Type III line corresponds to only a very small section of the total frequency outside this contour. If we invert the problem and put the system of ellipses on the nearest point of the Type III line we find that the odds are very much in favour of the point $\beta_1 = \cdot 678$, $\beta_2 = 3\cdot 734$ lying outside such a system. On the whole it is reasonable to conclude that Type I is properly used although we should probably not get bad results from a Type III curve. In some respects a suitable fit would be obtained by using Type I, and fixing its

start at zero*, but the vagueness of what is meant by 'percentage of black' as a factor, when the entire pigmentation of the skin probably arises from a single melanin pigment, only varying in concentration in the pigment granules and in the density of granules themselves. We have therefore contented ourselves by fitting a Type I curve, as further illustration of the use of the tables in the present work. The theory of fitting is given in the paper cited below†. Following the usual notation we find:

$$r = 6(\beta_2 - \beta_1 - 1)/(3\beta_1 - 2\beta_2 + 6) = 21{\cdot}7755,$$
$$e = r^2/\{4 + \tfrac{1}{4}\beta_1 (r+2)^2/(r+1)\} = 57{\cdot}764{,}468,$$
$$b^2 = \mu_2 r^2 (r+1)/\epsilon = (36{\cdot}9391)^2.$$

Hence:
$$m_1 = 2{\cdot}0917, \quad m_2 = 17{\cdot}6838,$$
$$a_1 = 3{\cdot}9071, \quad a_2 = 33{\cdot}0320,$$

and:
$$y = y_0 \left(1 + \frac{x}{3{\cdot}9071}\right)^{2{\cdot}0917} \left(1 - \frac{x}{33{\cdot}0320}\right)^{17{\cdot}6838}.$$

To find y_0 since m_2 is large, we use the approximation to the formula:

$$y_0 = \frac{N}{b} \frac{(m_1 + m_2 + 1)}{\left\{\dfrac{\Gamma(m_1+1)}{e^{-m_1} m_1^{m_1}}\right\}} \times \frac{\left\{\dfrac{\Gamma(m_1+m_2+1)}{e^{-(m_1+m_2)}(m_1+m_2)^{m_1+m_2}}\right\}}{\left\{\dfrac{\Gamma(m_2+1)}{e^{-m_2} m_2^{m_2}}\right\}} \quad \text{.....(lxxviii),}$$

namely,
$$y_0 = \frac{N}{b} \frac{(m_1+m_2+1)}{\Gamma(m_1+1)/(e^{-m_1} m_1^{m_1})} \sqrt{\frac{m_1+m_2}{m_2}}\, e^{\frac{1}{12}\left(\frac{1}{m_1+m_2} - \frac{1}{m_2}\right)} \quad \text{...(lxxix)}$$

the evaluation of the two Γ-functions for $m_1 + m_2 + 1$ and $m_2 + 1$ following easily by Stirling's Theorem. If we write $Z = \Gamma(3{\cdot}0917)/\{e^{-2{\cdot}0917}(2{\cdot}0917)^{2{\cdot}0917}\}$ we have

$$\begin{aligned} \log Z = &\ \log 2{\cdot}0917 \ - 2{\cdot}0917 \log 2{\cdot}0917, \\ &+ \log 1{\cdot}0917, \\ &+ \log \Gamma(1{\cdot}0917), \\ &+ 2{\cdot}0917 \log e. \end{aligned}$$

From Table XXXI (p. 58) we find $\log \Gamma(1{\cdot}0917) = \bar{1}{\cdot}979{,}8897$ and $\log e$ is given by Table LV (p. 143). Hence we determine, $\log Z = {\cdot}576{,}5176$. Evaluating the rest of the expression for $\log y_0$ we have:

$$\log y_0 = 2{\cdot}233{,}3936,$$
$$y_0 = 171{\cdot}157.$$

Thus our curve is

$$y = 171{\cdot}157 \left(1 + \frac{x}{3{\cdot}9071}\right)^{2{\cdot}0917} \left(1 - \frac{x}{33{\cdot}0320}\right)^{17{\cdot}6838}$$

* For method, see *Phil. Trans.* Vol. 186, A, pp. 370, 371.

† *Phil. Trans.* Vol. 186, A, pp. 367—370. See also Palin Elderton: *Frequency Curves and Correlation*, Layton Brothers.

We must interpolate between these two series for $n = 79$, that is we must take 0·42 times the first series and 0·58 times the second series. The results are given below, and set against the direct calculation from formula (lxxx), using Table XLIX.

	By Interpolation.	*Direct Calculation.*
	$n=79,\ p=4,\ m=40$	$n=79,\ p=4,\ m=40$
0	14·9130	12·6143
1	21·1943	21·9379
2	19·3642	22·5152
3	14·7828	17·6667
4	10·3562	11·6727
5	6·9472	6·8143
6	4·5578	3·6137
7	2·9529	1·7713
8	1·8940	·8118
9	1·2004	·3507
10	·7489	·1436
11	·4582	·0560
12	·2740	·0208
13	·1598	·0074
14	·0908	·0025
15	·0503	·0008
16	·0271	·0003
17	·0142	
18 and over	·0140	

The interpolation does not give a result very close to the actual series. For example, not more than three syphilitics might be anticipated in 70 °/₀ of samples of 40 by the interpolated series; actually not more than 3 are to be expected in 75 °/₀ of samples. At the same time the result is much better than the normal curve theory provides. In the latter case we have

$$\text{Mean} = 40 \times \tfrac{4}{79} = 2{\cdot}025,$$

$$\text{Standard-Deviation} = \sqrt{40 \times \tfrac{4}{79} \times \tfrac{75}{79}} = 1{\cdot}387.$$

Hence
$$(3{\cdot}5 - 2{\cdot}025)/1{\cdot}387 = 1{\cdot}064$$

and by Table II this value of x corresponds to $\frac{1}{2}(1+\alpha) = {\cdot}86$, i.e. in 86 °/₀ per cent. of samples of 40, we should have not more than 3 cases. It will be seen therefore that (i) the values at the latter end of the Table are not close enough to obtain very accurate results by interpolation, but (ii) that the Gaussian gives a still poorer approximation.

Illustration (ii). Of 10 patients subjected by a first surgeon to a given operation only one dies. A second surgeon in performing the same operation on 7 patients, presumably equally affected, loses 4 cases. Would it be reasonable to assume the second surgeon had inferior operative skill?

On p. 91, we have the series for $p = 2$ when $n = 10$ for the values $m = 5$ and $m = 10$. Taking ·6 of the first series and ·4 of the second we have:

	Interpolation from Table.	*Actual Value from formula.*
	$n=10,\ m=7,\ p=1$	$n=10,\ m=7,\ p=1$
0	37·9762	35·9477
1	30·6704	31·4542
2	17·3366	18·8725
3	8·2114	8·9869
4	3·5248	3·4565
5	1·4446	1·0370
6	·5676	·2200
7	·1996	·0251
8	[·0561]	—
9	[·0114]	—
10	[·0012]	—

The chance that if the two surgeons are of equal skill 4 or more patients will die out of the second surgeon's 7 operations is ·058 by interpolation and ·047 actually. Hence the odds against the occurrence are 16 to 1 by the table and 20 to 1 actually. It will be observed that interpolation gives small values at impossible numbers of deaths, but these have to be reckoned in to obtain the total number 100. That all seven patients should die under the second surgeon, if of equal skill, involves odds of 500 to 1 about in the interpolation result, but 4000 to 1 about actually. On the Gaussian hypothesis in the original problem the mean $= 7 \times \frac{1}{10} = ·7$ and the S. D. $= \sqrt{7 \times \frac{1}{10} \times \frac{9}{10}} = ·7937$, and $(3·5 - ·7)/·7937 = 3·52$ roughly, or this corresponds to odds of about 4545 to 1—which are wholly unreasonable. Thus the Table gives by interpolation odds of approximately the right value, which may serve many useful purposes, for those who are unable to work out the values required from formula (lxxx). At the same time it is clear that a much larger Table with closer values of the quantities involved is desirable.

Table XLIX (pp. 98—101)

The Logarithms of Factorials. (Calculated by Julia Bell, published here for the first time.)

This table was obtained by adding up in succession consecutive logarithms in a table of logarithms to 12 figures. Not until the work was completed did we realise the existence of the splendid table of C. F. Degen*, which was then used to confirm our own results. De Morgan in his *Treatise on the Theory of Probabilities* of 1837 published an abridgement to six decimals of Degen's Table of Factorials. His values cannot, however, be trusted to the sixth figure of the mantissa. The

* *Tabularum ad faciliorem et breviorem probabilitatis computationem utilium Enneas.* Havniae, MDCCCXXIV. This gives the logarithms of the factorials up to 1200 with 18 figures in the mantissa.

use of a factorial table is extremely varied, especially in problems in probability involving high numbers.

Illustration. In a certain district the number of children born per month is 662 and the chance of a birth being male is ·51 and of its being female ·49. Evaluate the chance that in a given month there should be an equal number of boys and girls born, and compare it with the chance of the most probable numbers (338 boys and 324 girls) being born.

The chance of equal numbers of boys and girls being born is:

$$C_e = (\cdot 51)^{331}(\cdot 49)^{331}\frac{\lfloor 662}{\lfloor 331 \lfloor 331}.$$

Therefore

$$\log C_e = 331\left\{\begin{matrix}\bar{1}\cdot 707,5702\\ +\bar{1}\cdot 690,1961\end{matrix}\right\}\begin{matrix}+1581\cdot 714,6156\\ -1383\cdot 941,4114\end{matrix}$$

where the logs of the factorials are found from Table XLIX. Hence

$$\log C_e = \left\{\begin{matrix}\overline{200}\cdot 660,6453\\ +197\cdot 773,2042\end{matrix}\right\} = \bar{2}\cdot 433,8495$$

or $C_e = \cdot 027155$, or once in about 36·8 months, say once in three years the records may be expected to show equal numbers of boys and girls born in the month*.

The chance of the most probable number of boys and girls is given by

$$C_m = (\cdot 51)^{338}(\cdot 49)^{324}\frac{\lfloor 662}{\lfloor 338 \lfloor 324},$$

$$\begin{aligned}\log C_m = {} & 338\times\bar{1}\cdot 707,5702 + 1581\cdot 714,6156\\ & + 324\times\bar{1}\cdot 690,1961 - 709\cdot 645,9652\\ & - 674\cdot 359,6453\\ = {} & \left\{\begin{matrix}\overline{200}\cdot 782,2640\\ +197\cdot 709,0051\end{matrix}\right\} = \bar{2}\cdot 491,2691.\end{aligned}$$

Or $C_m = \cdot 030993$, or the most probable numbers will only be born once in 32·3 months, or say once in two years and eight months.

We have $C_e/C_m = \cdot 876$, or the chance of equal boys and girls is 88% of the chance of the most probable numbers of boys and girls.

Table L (pp. 102—112)

Tables of Fourth-Moments of Subgroup Frequencies. (Calculated by Alice Lee and P. F. Everitt; published here for the first time.)

In the usual method of determining the raw moments of a frequency, we take moments about an arbitrary origin, which is towards the apparent mode and

* Actually of course the problem is more complex, because the number of children born per month is not constant.

multiply by plus and minus abscissae increasing by units—the 'working unit.' Thus an error made in an early moment may be carried on to the later moments. To control the results Table L was calculated a number of years ago, and from it the fourth moments for such frequencies as most usually occur can be read off at sight, and the raw fourth moment column thus tested before proceeding further.

(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	
Head Length	Frequency	Abscissa	$N\nu_1'$	$N\nu_2'$	$N\nu_3'$	$N\nu_4'$	Table L
171	1	−20	− 20	+ 400	− 8,000	+160,000	160,000
2	1	19	19	361	6,859	130,321	130,321
3	2	18	36	648	11,664	209,952	209,952
4	0	17	—	—	—	—	—
5	3	16	48	768	12,288	196,608	196,608
6	3	15	45	675	10,125	151,875	151,875
7	5	14	70	980	13,720	192,080	192,080
8	7	13	91	1,183	15,379	199,927	199,927
9	12	12	144	1,728	20,736	248,832	248,832
180	13	11	143	1,573	17,303	190,333	190,333
1	17	10	170	1,700	17,000	170,000	170,000
2	28	9	252	2,268	20,412	183,708	183,708
3	24	8	192	1,536	12,288	98,304	98,304
4	43	7	301	2,107	14,749	103,243	103,243
5	53	6	318	1,908	11,448	68,688	68,688
6	57	5	285	1,425	7,125	35,625	35,625
7	55	4	220	880	3,520	14,080	14,080
8	68	3	204	612	1,836	5,508	5,508
9	83	2	166	332	664	1,328	1,328
190	85	− 1	− 85	+ 85	− 85	+ 85	85
1	96	0	—	—		—	—
2	102	+ 1	+102	+ 102	+ 102	+ 102	102
3	79	2	158	316	632	1,264	1,264
4	83	3	249	747	2,241	6,723	6,723
5	66	4	264	1,056	4,224	16,896	16,896
6	66	5	330	1,650	8,250	42,250	41,250
7	56	6	336	2,016	12,096	72,576	72,576
8	43	7	301	2,107	14,749	103,243	103,243
9	35	8	280	2,240	17,920	143,360	143,360
200	30	9	270	2,430	21,870	196,830	196,830
1	20	10	200	2,000	20,000	200,000	200,000
2	24	11	264	2,904	31,944	351,384	351,384
3	14	12	168	2,016	24,192	290,304	290,304
4	13	13	169	2,197	28,561	371,293	371,293
5	8	14	112	1,568	21,952	307,328	307,328
6	3	15	45	675	10,125	151,875	151,875
7	6	16	96	1,536	24,576	393,216	393,216
8	0	17	—	—	—	—	—
9	1	18	18	324	5,832	104,976	104,976
210	1	+19	+ 19	+ 361	+ 6,859	+130,321	130,321
Totals	1306	—	+	+	+	+	—

The mean will be mn_s/N and the standard deviation $\sqrt{m\frac{n_s}{N}\left(1-\frac{n_s}{N}\right)}$. If we only have a single sample of m and do not know the distribution in the actual population we are compelled to give n_s/N the value m_s/m, where m_s is the number found in the sth cell of the sample. If n_s/N or m_s/m be very small and m large, the binomial will approach Poisson's Exponential Limit, and in such cases the deviations in the samples for the sth cell will be distributed very differently from those following a Gaussian law, and the usual rule for deducing the probability of deviations of a given size by means of the probability integral fails markedly. It is not till we get something like 30 out of 1000 in a cell that we can trust the Gaussian to give us at all a reasonable approach. The present table endeavours to provide material in the case of cell frequencies 1 to 30, which will supply the place of the probability integral.

Illustration (i). Suppose the actual number to be expected in a cell is 17, what is the probability that the observed number will deviate by more than 5 from this result? Looking at p. 123 we see that in 8·467 °/₀ of cases there will be a deviation in defect of 6 or more and in 9·526 °/₀ of cases a deviation in excess of 6 or more. Hence in 17·993 °/₀ say 18 °/₀ of cases we should get values less than 12 or greater than 22. Thus once in every 5 or 6 trials we should get values which differ as widely as 6 or more from the true value.

Now look at the matter from the Gaussian standpoint. The standard deviation is

$$\sqrt{m\frac{17}{m}\left(1-\frac{17}{m}\right)}=\sqrt{17\left(1-\frac{17}{m}\right)}.$$

Here m is supposed large compared with 17, so that the S. D. $=\sqrt{17}=4{\cdot}123$ nearly. But suppose $m=800$, we should have

$$\text{S. D.}=\sqrt{17\,(1-{\cdot}02125)}=\sqrt{17\times{\cdot}97875}=4{\cdot}079.$$

Now we want deviations in excess of 5, i.e. we must take $5{\cdot}5/4{\cdot}079=1{\cdot}348$. If we turn to Table II we find for this argument

$$\tfrac{1}{2}(1+\alpha)={\cdot}9102 \text{ or } \tfrac{1}{2}(1-\alpha)={\cdot}0898.$$

Hence we should conclude that in not more than 17·96 °/₀ of cases would deviations exceed ± 5. Actually such occur in 17·99 °/₀ of cases. Thus the actual percentages are very close, but the Poisson series tells us that 8·47 °/₀ of cases will be in defect and 9·53 °/₀ in excess, while the Gaussian gives 8·98 °/₀ in both excess and defect. We may further ask the percentage of times that 17 itself would occur; according to the Gaussian it will occur in 9·76 °/₀ of trials, actually it will occur in 9·63 °/₀. With values of cell-frequency less than 17, say in the single digits, far greater divergences will be encountered.

Illustration (ii). Consider the fourfold Table below and discuss the relative probabilities that it has arisen from a population which shews 0, 1, 2, 3, etc. indi-

	A	Not-*A*	Totals
B ...	127·5	0	127·5
Not-*B* ...	863·5	87	950·5
	991	87	1078

viduals for this size of sample in the cell *B*, not-*A*. On the assumption that 0 is really the population of this cell, the probability is unity. Hence we have the following result.

Population of cell	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13* & over
Probability of 0 occurring	1	·36788	·13534	·04979	·01832	·00674	·00248	·00091	·00034	·00012	·00005	·00002	·00001	·00000

Sum = 1·58200.

Whence taking the *à priori* probabilities proportional to the probability of 0 occurring on the separate possibilities we have:

Probabilities that the Table arose from a population with x in the B, not-A cell.

x	Probability	*x*	Probability
0	·632,110	*7*	·000,575
1	·232,541	*8*	·000,215
2	·085,550	*9*	·000,076
3	·031,473	*10*	·000,032
4	·011,580	*11*	·000,013
5	·004,260	*12*	·000,006
6	·001,568	*13* and over	·000,000

The "association" of such a Table cannot therefore be considered "perfect," for in 37 °/₀ of cases it would arise from a Table with a unit or more in the *B*, not-*A* cell. The above is actually a Table of the correlation of stature in father and son. Grave caution is therefore needful in discussing such "perfect association" tables.

TABLE LIII (p. 125)

Angles, Arcs and Decimals of Degrees. (Based on Hutton's *Mathematical Tables.*)

This Table gives degrees in radians for the first two quadrants; it then gives minutes and seconds from 1 to 60 in fractions of a degree and in radians. The

need of such a table is very obvious, and arises in too great a variety of circumstances to be specified.

Illustration. It is required to plot the curve*:

$$x = 14{\cdot}9917 \tan\theta,$$

$$y = 235{\cdot}323 \cos^{32{\cdot}8023}\theta e^{-4{\cdot}5696\theta}.$$

Here $\log y = \log 235{\cdot}323 + 32{\cdot}8023 \log\cos\theta - 4{\cdot}5696 \log e \times \theta.$

To cover the whole range of observations we must proceed from $\theta = -45°$ to $\theta = +45°$ roughly. It will be found sufficient to take θ by steps of 3° and ultimately perhaps of 4°. Hence 14·9917 is put on the arithmometer and multiplied in succession by the natural tangents of 3°, 6°, 9° ... etc. Plus and minus signs are given to these values of x. The corresponding values of y are found in three columns. The first is obtained by putting 32·8023 on the arithmometer and multiplying by the logarithmic cosines of 3°, 6°, 9°, etc. The second is obtained by multiplying (taking the third factor from Table LIII)

$$4{\cdot}5696 \times \log e \times {\cdot}017{,}4533 = {\cdot}216{,}7955$$

on the machine and multiplying the result in succession by 3, 6, 9, etc.

The first column is added to $\log 235{\cdot}323 = 2{\cdot}371{,}6644$ and the second column first subtracted and then added to the result to obtain the value of $\log y$ for positive and negative abscissae respectively. The antilogarithms give the ordinates.

Another problem sometimes arises given x to find y. For example: In the above curve the mode is at 117·9998 cms. of stature and the origin at 113·8228, thus the distance between them = 4·1770 cms. or since the working unit of $x = 2$ cms. and the positive direction of x is towards dwarfs, the mode is at $x = -2{\cdot}0885$. Required to find the maximum ordinate y_{mo}. We have

$$\tan\theta = -2{\cdot}0885/14{\cdot}9917 = -{\cdot}139{,}3104,$$

whence by a table of natural tangents

$$\theta = -7° \; 55' \; {\cdot}851265,$$
$$= -7° \; 55' \; 51''.$$

The log cosine of this value of θ is

$$\bar{1}{\cdot}995{,}8962.$$

Table LIII gives us:

$7° =$	·122,1730	in arc
$55' =$	·015,9989	„ „
$·851{,}265' = ·851{,}265 \times ·000{,}2909 =$	·000,2476	„ „
Hence $\theta =$	$-$ ·138,4195	„ „

* See *Phil. Trans.* Vol. 186, A, p. 387. Pearson's Type IV frequency curve fitted to the stature of 2192 St Louis School Girls aged 8.

Hence $\log y_{mo} = 2{\cdot}371{,}6644$

$+ 32{\cdot}8023\ (-\ {\cdot}004{,}1038)$

$+\ 1{\cdot}984{,}5521 \times {\cdot}138{,}4195$

$= 2{\cdot}511{,}7510.$

Hence $y_{mo} = 324{\cdot}901.$

Table LIV (pp. 126—142)

Tables of the $G(r, \nu)$ Integrals. (Calculated by Alice Lee, D.Sc. *Transactions British Association Report,* Dover, 1899, pp. 65—120.)

The purpose of this table is to obtain the value of the integral

$$G(r, \nu) = \int_0^\pi \sin^r \theta e^{\nu\theta} d\theta \quad \text{.....(lxxxiii).}$$

In order to obtain small differences in tabulated values two additional functions $F(r, \nu)$ and $H(r, \nu)$ are introduced.

The relations between the three functions are then expressed by the following series of equations:

$$F(r, \nu) = e^{-\frac{1}{2}\nu\pi} G(r, \nu) \quad \text{.....(lxxxiv),}$$

$$F(r, \nu) = \frac{e^{\nu\phi} (\cos\phi)^{r+1}}{\sqrt{r-1}} H(r, \nu) \quad \text{.....(lxxxv),}$$

$$G(r, \nu) = e^{\frac{1}{2}\nu\pi} F(r, \nu) \quad \text{.....(lxxxvi),}$$

$$G(r, \nu) = \frac{e^{\nu\phi + \frac{1}{2}\nu\pi} (\cos\phi)^{r+1}}{\sqrt{r-1}} H(r, \nu) \quad \text{.....(lxxxvii),}$$

$$H(r, \nu) = \frac{\sqrt{r-1}\, e^{-\nu\phi}}{(\cos\phi)^{r+1}} F(r, \nu) \quad \text{.....(lxxxviii),}$$

$$H(r, \nu) = \frac{\sqrt{r-1}\, e^{-\nu\phi - \frac{1}{2}\nu\pi}}{(\cos\phi)^{r+1}} G(r, \nu) \quad \text{.....(lxxxix),}$$

where $\tan\phi = \nu/r$.

Pearson's Type IV Skew Frequency Curve is of the form

$$y = y_0 \frac{e^{-\nu \tan^{-1}\frac{x}{a}}}{\left\{1 + \left(\frac{x}{a}\right)^2\right\}^{\frac{1}{2}(r+2)}} \quad \text{.....(xc).}$$

Hence if N be its total area, i.e. the entire population under discussion,

$$N = y_0 a e^{-\frac{1}{2}\nu\pi} \int_0^\pi \sin^r \theta e^{\nu\theta} d\theta,$$

or

$$y_0 = \frac{N}{a} \frac{1}{F(r, \nu)} \quad \text{.....(xci).}$$

The function $H(r, \nu)$ is introduced because, as a rule, its logarithms have far smaller differences and it is thus capable of more exact determination from a table of double entry. Its physical relation to the curve may be expressed as follows; let the origin be transferred to the mean, then if y_1 be the ordinate at the mean,

$$y_1 = \frac{N}{\sigma} \frac{1}{H(r, \nu)} \quad \text{.....(xci)}^{\text{bis}},$$

where σ is the standard-deviation of the curve

$$= \frac{a}{\sqrt{r-1}\cos\phi} \quad \text{.....(xcii).}$$

The distance of the mean from the origin is given by

$$\mu_1' = -a\tan\phi \quad \text{.....(xciii).}$$

When r is fairly large:

$$\frac{1}{F(r, \nu)} = \sqrt{\frac{r}{2\pi}} \frac{e^{\frac{\cos^2\phi}{3r} - \frac{1}{12r} - \phi r\tan\phi}}{(\cos\phi)^{r+1}} \quad \text{.....(xciv).}$$

Hence

$$\frac{1}{H(r, \nu)} = \sqrt{\frac{r}{r-1}} \times \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}q^2} \quad \text{.....(xcv),}$$

where

$$q = \sqrt{\frac{1 - 4\cos^2\phi}{6r}},$$

and thus the evaluation if ϕ be $> 60°$ may be made by aid of Table II*.

Illustration. In the curve fitted to the statures of St Louis School Girls, aged 8 (p. lxxx), we have

$$N = 2192, \qquad a = 14{\cdot}9917,$$
$$r = 30{\cdot}8023, \qquad \nu = 4{\cdot}56967.$$

Find y_0.

We have $\tan\phi = \nu/r = {\cdot}148{,}3548.$

Hence $\phi = 8°\ 26'{\cdot}31315 = 8°{\cdot}43855.$

Turning to the Tables, p. 136, we see the large differences of $\log F(r, \nu)$ at this value of ϕ, and accordingly settle to work with $\log H(r, \nu)$.

We have for $\log H(r, \nu)$,

	$r = 30$	$r = 31$
$\phi = 8°$	·388,2032	·388,5583,
$\phi = 9°$	·388,2278	·388,5822,

$\phi = 8°{\cdot}4386$, $r = 30$:

$$\log H(r, \nu) = {\cdot}388{,}2032 + ({\cdot}4386)[246] - \tfrac{1}{2}({\cdot}4386) \times ({\cdot}5614)[28]$$
$$= {\cdot}388{,}2137.$$

* For a fuller discussion of these integrals see *Phil. Trans.* Vol. 186, A, pp. 376—381, *B. A. Trans. Report*, Liverpool, 1896, Preliminary Report of Committee..., and the *B.A. Trans. Report*, Dover, 1899, already cited.

$\phi = 8°{\cdot}4386,\ r = 31$:

$$\log H(r, \nu) = {\cdot}388{,}5583 + ({\cdot}4386)[238] - \tfrac{1}{2}({\cdot}4386)({\cdot}5614)[27]$$
$$= {\cdot}388{,}5684.$$

$\phi = 8°{\cdot}4386,\ r = 32$:

$$\log H(r, \nu) = {\cdot}388{,}8910 + ({\cdot}4386)[231] - \tfrac{1}{2}({\cdot}4386)({\cdot}5614)[26]$$
$$= {\cdot}388{,}9008.$$

Hence $\phi = 8°{\cdot}4386,\ r = 30{\cdot}8023$:

$$\log H(r, \nu) = {\cdot}388{,}2137 + {\cdot}8023\,[3547] - \tfrac{1}{2}({\cdot}8023)({\cdot}1977)[-223]$$
$$= {\cdot}388{\cdot}5001.$$

Hence by formula (lxxxv):

$$\log F(r, \nu) = \nu\phi \log e + \overline{r+1} \log \cos\phi - \tfrac{1}{2}\log(r-1) + \log H(r, \nu).$$

Or, using Tables LIII and LV, we have

$$\begin{aligned} \log F(r, \nu) = {} & {\cdot}292{,}2901 - {\cdot}737{,}1249 \\ & + \bar{1}{\cdot}849{,}6578 \\ & + {\cdot}388{,}5001 \\ & \quad {\cdot}530{,}4480 \\ & - {\cdot}737{,}1249 \\ \log F(r, \nu) = {} & \bar{1}{\cdot}793{,}3231 \end{aligned}$$

Finally from formula (xci):

$$\begin{aligned} \log y_0 &= \log N - \log a - \bar{1}{\cdot}793{,}3231 \\ &= 3{\cdot}340{,}8405 \quad - 1{\cdot}175{,}8509 \\ &\qquad\qquad - \bar{1}{\cdot}793{,}3231 \\ &\qquad\qquad - {\cdot}969{,}1740 \\ &= 2{\cdot}371{,}6665. \end{aligned}$$

Or $y_0 = 235{\cdot}324$*.

Table LV.

This table contains some miscellaneous constants in frequent statistical or biometric use and requires no illustration. It has already been used in the illustrations to previous tables.

I have had the generous assistance of my colleagues Miss E. M. Elderton and Mr H. E. Soper in the preparation of the Illustrations to these Tables. I can hardly hope that arithmetical slips have wholly escaped us in a first edition, and I shall be grateful for the communication of any corrections that my readers may discover are necessary.

* The value 235·323 obtained in *Phil. Trans.* Vol. 186, A, p. 387, was found by the approximate formula (xciv) before tables were calculated.

Every reader may now see in what way the higher branches of mathematics are concerned in our present subject. They are the abbreviators of long and tedious operations, and it would be perfectly possible, with sufficient time and industry, to do without their use.....When both the ordinary and the mathematical result are derived from the same hypothesis, the latter must be the more correct: and in those numerous cases in which the difficulty lies in reducing the original circumstances to a mathematical form, there is nothing to show that we are less liable to error in deducing a common sense result from principles too indefinite for calculation, than we should be in attempting to define more closely, and to apply numerical reasoning.—De Morgan.

TABLE I.

Table of Deviates of the Normal Curve for each Permille of Frequency.

Permille	·000	·001	·002	·003	·004	·005	·006	·007	·008	·009	·010	
·00	∞	3·0902	2·8782	2·7478	2·6521	2·5758	2·5121	2·4573	2·4089	2·3656	2·3263	·99
·01	2·3263	2·2904	2·2571	2·2262	2·1973	2·1701	2·1444	2·1201	2·0969	2·0749	2·0537	·98
·02	2·0537	2·0335	2·0141	1·9954	1·9774	1·9600	1·9431	1·9268	1·9110	1·8957	1·8808	·97
·03	1·8808	1·8663	1·8522	1·8384	1·8250	1·8119	1·7991	1·7866	1·7744	1·7624	1·7507	·96
·04	1·7507	1·7392	1·7279	1·7169	1·7060	1·6954	1·6849	1·6747	1·6646	1·6546	1·6449	·95
·05	1·6449	1·6352	1·6258	1·6164	1·6072	1·5982	1·5893	1·5805	1·5718	1·5632	1·5548	·94
·06	1·5548	1·5464	1·5382	1·5301	1·5220	1·5141	1·5063	1·4985	1·4909	1·4833	1·4758	·93
·07	1·4758	1·4684	1·4611	1·4538	1·4466	1·4395	1·4325	1·4255	1·4187	1·4118	1·4051	·92
·08	1·4051	1·3984	1·3917	1·3852	1·3787	1·3722	1·3658	1·3595	1·3532	1·3469	1·3408	·91
·09	1·3408	1·3346	1·3285	1·3225	1·3165	1·3106	1·3047	1·2988	1·2930	1·2873	1·2816	·90
·10	1·2816	1·2759	1·2702	1·2646	1·2591	1·2536	1·2481	1·2426	1·2372	1·2319	1·2265	·89
·11	1·2265	1·2212	1·2160	1·2107	1·2055	1·2004	1·1952	1·1901	1·1850	1·1800	1·1750	·88
·12	1·1750	1·1700	1·1650	1·1601	1·1552	1·1503	1·1455	1·1407	1·1359	1·1311	1·1264	·87
·13	1·1264	1·1217	1·1170	1·1123	1·1077	1·1031	1·0985	1·0939	1·0893	1·0848	1·0803	·86
·14	1·0803	1·0758	1·0714	1·0669	1·0625	1·0581	1·0537	1·0494	1·0450	1·0407	1·0364	·85
·15	1·0364	1·0322	1·0279	1·0237	1·0194	1·0152	1·0110	1·0069	1·0027	0·9986	0·9945	·84
·16	0·9945	0·9904	0·9863	0·9822	0·9782	0·9741	0·9701	0·9661	0·9621	0·9581	0·9542	·83
·17	0·9542	0·9502	0·9463	0·9424	0·9385	0·9346	0·9307	0·9269	0·9230	0·9192	0·9154	·82
·18	0·9154	0·9116	0·9078	0·9040	0·9002	0·8965	0·8927	0·8890	0·8853	0·8816	0·8779	·81
·19	0·8779	0·8742	0·8705	0·8669	0·8633	0·8596	0·8560	0·8524	0·8488	0·8452	0·8416	·80
·20	0·8416	0·8381	0·8345	0·8310	0·8274	0·8239	0·8204	0·8169	0·8134	0·8099	0·8064	·79
·21	0·8064	0·8030	0·7995	0·7961	0·7926	0·7892	0·7858	0·7824	0·7790	0·7756	0·7722	·78
·22	0·7722	0·7688	0·7655	0·7621	0·7588	0·7554	0·7521	0·7488	0·7454	0·7421	0·7388	·77
·23	0·7388	0·7356	0·7323	0·7290	0·7257	0·7225	0·7192	0·7160	0·7128	0·7095	0·7063	·76
·24	0·7063	0·7031	0·6999	0·6967	0·6935	0·6903	0·6871	0·6840	0·6808	0·6776	0·6745	·75
·25	0·6745	0·6713	0·6682	0·6651	0·6620	0·6588	0·6557	0·6526	0·6495	0·6464	0·6433	·74
·26	0·6433	0·6403	0·6372	0·6341	0·6311	0·6280	0·6250	0·6219	0·6189	0·6158	0·6128	·73
·27	0·6128	0·6098	0·6068	0·6038	0·6008	0·5978	0·5948	0·5918	0·5888	0·5858	0·5828	·72
·28	0·5828	0·5799	0·5769	0·5740	0·5710	0·5681	0·5651	0·5622	0·5592	0·5563	0·5534	·71
·29	0·5534	0·5505	0·5476	0·5446	0·5417	0·5388	0·5359	0·5330	0·5302	0·5273	0·5244	·70
·30	0·5244	0·5215	0·5187	0·5158	0·5129	0·5101	0·5072	0·5044	0·5015	0·4987	0·4959	·69
·31	0·4959	0·4930	0·4902	0·4874	0·4845	0·4817	0·4789	0·4761	0·4733	0·4705	0·4677	·68
·32	0·4677	0·4649	0·4621	0·4593	0·4565	0·4538	0·4510	0·4482	0·4454	0·4427	0·4399	·67
·33	0·4399	0·4372	0·4344	0·4316	0·4289	0·4261	0·4234	0·4207	0·4179	0·4152	0·4125	·66
·34	0·4125	0·4097	0·4070	0·4043	0·4016	0·3989	0·3961	0·3934	0·3907	0·3880	0·3853	·65
·35	0·3853	0·3826	0·3799	0·3772	0·3745	0·3719	0·3692	0·3665	0·3638	0·3611	0·3585	·64
·36	0·3585	0·3558	0·3531	0·3505	0·3478	0·3451	0·3425	0·3398	0·3372	0·3345	0·3319	·63
·37	0·3319	0·3292	0·3266	0·3239	0·3213	0·3186	0·3160	0·3134	0·3107	0·3081	0·3055	·62
·38	0·3055	0·3029	0·3002	0·2976	0·2950	0·2924	0·2898	0·2871	0·2845	0·2819	0·2793	·61
·39	0·2793	0·2767	0·2741	0·2715	0·2689	0·2663	0·2637	0·2611	0·2585	0·2559	0·2533	·60
·40	0·2533	0·2508	0·2482	0·2456	0·2430	0·2404	0·2378	0·2353	0·2327	0·2301	0·2275	·59
·41	0·2275	0·2250	0·2224	0·2198	0·2173	0·2147	0·2121	0·2096	0·2070	0·2045	0·2019	·58
·42	0·2019	0·1993	0·1968	0·1942	0·1917	0·1891	0·1866	0·1840	0·1815	0·1789	0·1764	·57
·43	0·1764	0·1738	0·1713	0·1687	0·1662	0·1637	0·1611	0·1586	0·1560	0·1535	0·1510	·56
·44	0·1510	0·1484	0·1459	0·1434	0·1408	0·1383	0·1358	0·1332	0·1307	0·1282	0·1257	·55
·45	0·1257	0·1231	0·1206	0·1181	0·1156	0·1130	0·1105	0·1080	0·1055	0·1030	0·1004	·54
·46	0·1004	0·0979	0·0954	0·0929	0·0904	0·0878	0·0853	0·0828	0·0803	0·0778	0·0753	·53
·47	0·0753	0·0728	0·0702	0·0677	0·0652	0·0627	0·0602	0·0577	0·0552	0·0527	0·0502	·52
·48	0·0502	0·0476	0·0451	0·0426	0·0401	0·0376	0·0351	0·0326	0·0301	0·0276	0·0251	·51
·49	0·0251	0·0226	0·0201	0·0175	0·0150	0·0125	0·0100	0·0075	0·0050	0·0025	0·0000	·50
	·010	·009	·008	·007	·006	·005	·004	·003	·002	·001	·000	Permille

TABLE II. *Area and Ordinate in terms of Abscissa.*

x	$\frac{1}{2}(1+a)$	Δ +	Δ^2 −	z	Δ −	Δ^2 −
·00	·5000000			·3989423		
		39894	0		199	399
·01	·5039894		4	·3989223		399
		39890			598	
·02	·5079783		8	·3988625		399
		39882			997	
·03	·5119665		12	·3987628		398
		39870			1395	
·04	·5159534		16	·3986233		398
		39854			1793	
·05	·5199388		20	·3984439		397
		39834			2191	
·06	·5239222		24	·3982248		397
		39810			2588	
·07	·5279032		28	·3979661		396
		39782			2984	
·08	·5318814		32	·3976677		395
		39750			3379	
·09	·5358564		36	·3973298		394
		39714			3773	
·10	·5398278		40	·3969525		393
		39675			4166	
·11	·5437953		44	·3965360		392
		39631			4558	
·12	·5477584		48	·3960802		390
		39584			4948	
·13	·5517168		51	·3955854		389
		39532			5337	
·14	·5556700		55	·3950517		387
		39477			5724	
·15	·5596177		59	·3944793		386
		39418			6110	
·16	·5635595		63	·3938684		384
		39355			6493	
·17	·5674949		67	·3932190		382
		39288			6875	
·18	·5714237		71	·3925315		380
		39217			7255	
·19	·5753454		74	·3918060		378
		39143			7633	
·20	·5792597		78	·3910427		375
		39065			8008	
·21	·5831662		82	·3902419		373
		38983			8381	
·22	·5870644		86	·3894038		371
		38897			8752	
·23	·5909541		89	·3885286		368
		38808			9120	
·24	·5948349		93	·3876166		365
		38715			9485	
·25	·5987063		97	·3866681		362
		38618			9847	
·26	·6025681		100	·3856834		360
		38518			10207	
·27	·6064199		104	·3846627		357
		38414			10564	
·28	·6102612		107	·3836063		354
		38306			10917	
·29	·6140919		111	·3825146		350
		38195			11268	
·30	·6179114		114	·3813878		347
		38081			11615	
·31	·6217195		118	·3802264		344
		37963			11958	
·32	·6255158		121	·3790305		340
		37842			12298	
·33	·6293000		125	·3778007		337
		37717			12635	
·34	·6330717		128	·3765372		333
		37589			12968	
·35	·6368307		131	·3752403		329
		37458			13297	
·36	·6405764		135	·3739106		325
		37323			13623	
·37	·6443088		138	·3725483		322
		37185			13944	
·38	·6480273		141	·3711539		318
		37044			14262	
·39	·6517317		144	·3697277		313
		36900			14575	
·40	·6554217		147	·3682701		309
		36753			14885	
·41	·6590970		150	·3667817		305
		36602			15190	
·42	·6627573		153	·3652627		301
		36449			15491	
·43	·6664022		156	·3637136		296
		36293			15787	
·44	·6700314		159	·3621349		292
		36133			16079	
·45	·6736448		162	·3605270		288
		35971			16367	
·46	·6772419		165	·3588903		283
		35806			16650	
·47	·6808225		168	·3572253		278
		35638			16928	
·48	·6843863		171	·3555325		274
		35467			17202	
·49	·6879331		173	·3538124		269
		35294			17470	
·50	·6914625		176	·3520653		264

x	$\frac{1}{2}(1+a)$	Δ +	Δ^2 −
·50	·6914625		
		35118	176
·51	·6949743		179
		34939	
·52	·6984682		181
		34758	
·53	·7019440		184
		34574	
·54	·7054015		186
		34388	
·55	·7088403		189
		34200	
·56	·7122603		191
		34009	
·57	·7156612		193
		33815	
·58	·7190427		196
		33620	
·59	·7224047		198
		33422	
·60	·7257469		200
		33222	
·61	·7290691		202
		33020	
·62	·7323711		204
		32816	
·63	·7356527		206
		32610	
·64	·7389137		208
		32402	
·65	·7421539		210
		32192	
·66	·7453731		212
		31980	
·67	·7485711		214
		31767	
·68	·7517478		215
		31551	
·69	·7549029		217
		31334	
·70	·7580363		219
		31116	
·71	·7611479		220
		30896	
·72	·7642375		222
		30674	
·73	·7673049		223
		30451	
·74	·7703500		225
		30226	
·75	·7733726		226
		30001	
·76	·7763727		227
		29773	
·77	·7793501		228
		29545	
·78	·7823046		230
		29316	
·79	·7852361		231
		29085	
·80	·7881446		232
		28853	
·81	·7910299		233
		28620	
·82	·7938919		234
		28387	
·83	·7967306		235
		28152	
·84	·7995458		235
		27917	
·85	·8023375		236
		27680	
·86	·8051055		237
		27443	
·87	·8078498		238
		27205	
·88	·8105703		238
		26967	
·89	·8132671		239
		26728	
·90	·8159399		239
		26489	
·91	·8185887		240
		26249	
·92	·8212136		240
		26008	
·93	·8238145		241
		25768	
·94	·8263912		241
		25527	
·95	·8289439		241
		25285	
·96	·8314724		242
		25044	
·97	·8339768		242
		24802	
·98	·8364569		242
		24560	
·99	·8389129		242
		24318	
1·00	·8413447		242

TABLE II.—(*continued*).

z	Δ −	Δ^2 −
·3520653		264
	17734	
·3502919		259
	17994	
·3484925		254
	18248	
·3466677		249
	18497	
·3448180		244
	18741	
·3429439		239
	18981	
·3410458		234
	19215	
·3391243		229
	19444	
·3371799		224
	19667	
·3352132		219
	19886	
·3332246		213
	20099	
·3312147		208
	20307	
·3291840		203
	20510	
·3271330		197
	20707	
·3250623		192
	20899	
·3229724		187
	21086	
·3208638		181
	21267	
·3187371		176
	21442	
·3165929		170
	21613	
·3144317		165
	21777	
·3122539		159
	21936	
·3100603		154
	22090	
·3078513		148
	22239	
·3056274		143
	22381	
·3033893		137
	22519	
·3011374		132
	22650	
·2988724		126
	22777	
·2965948		121
	22897	
·2943050		115
	23013	
·2920038		110
	23122	
·2896916		104
	23227	
·2873689		99
	23325	
·2850364		93
	23419	
·2826945		88
	23507	
·2803438		83
	23589	
·2779849		77
	23666	
·2756182		72
	23738	
·2732444		66
	23805	
·2708640		61
	23866	
·2684774		56
	23922	
·2660852		51
	23972	
·2636880		45
	24017	
·2612863		40
	24058	
·2588805		35
	24093	
·2564713		30
	24122	
·2540591		25
	24147	
·2516443		20
	24167	
·2492277		15
	24182	
·2468095		10
	24191	
·2443904		5
	24196	
·2419707		0

x	$\frac{1}{2}(1+a)$	Δ +	Δ^2 −
1·00	·8413447		242
		24076	
1·01	·8437524		242
		23834	
1·02	·8461358		242
		23592	
1·03	·8484950		242
		23351	
1·04	·8508300		242
		23109	
1·05	·8531409		241
		22868	
1·06	·8554277		241
		22626	
1·07	·8576903		241
		22386	
1·08	·8599289		240
		22145	
1·09	·8621434		240
		21905	
1·10	·8643339		240
		21665	
1·11	·8665005		239
		21426	
1·12	·8686431		239
		21188	
1·13	·8707619		238
		20950	
1·14	·8728568		237
		20712	
1·15	·8749281		237
		20475	
1·16	·8769756		236
		20239	
1·17	·8789995		235
		20004	
1·18	·8809999		235
		19769	
1·19	·8829768		234
		19535	
1·20	·8849303		233
		19302	
1·21	·8868606		232
		19070	
1·22	·8887676		231
		18839	
1·23	·8906514		230
		18609	
1·24	·8925123		229
		18379	
1·25	·8943502		228
		18151	
1·26	·8961653		227
		17924	
1·27	·8979577		226
		17697	
1·28	·8997274		225
		17472	
1·29	·9014747		224
		17248	
1·30	·9031995		223
		17026	
1·31	·9049021		222
		16804	
1·32	·9065825		220
		16584	
1·33	·9082409		219
		16365	
1·34	·9098773		218
		16147	
1·35	·9114920		217
		15930	
1·36	·9130850		215
		15715	
1·37	·9146565		214
		15501	
1·38	·9162067		212
		15289	
1·39	·9177356		211
		15078	
1·40	·9192433		210
		14868	
1·41	·9207302		208
		14660	
1·42	·9221962		207
		14453	
1·43	·9236415		205
		14248	
1·44	·9250663		204
		14044	
1·45	·9264707		202
		13842	
1·46	·9278550		201
		13642	
1·47	·9292191		199
		13443	
1·48	·9305634		197
		13245	
1·49	·9318879		196
		13049	
1·50	·9331928		194

z	Δ −	Δ^2 +
·2419707		0
	24196	
·2395511		5
	24191	
·2371320		10
	24182	
·2347138		14
	24168	
·2322970		19
	24149	
·2298821		24
	24125	
·2274696		28
	24097	
·2250599		33
	24064	
·2226535		37
	24027	
·2202508		41
	23986	
·2178522		46
	23940	
·2154582		50
	23890	
·2130691		54
	23836	
·2106856		58
	23778	
·2083078		62
	23715	
·2059363		66
	23649	
·2035714		70
	23578	
·2012135		74
	23504	
·1988631		78
	23426	
·1965205		82
	23344	
·1941861		85
	23259	
·1918602		89
	23170	
·1895432		93
	23077	
·1872354		96
	22981	
·1849373		99
	22882	
·1826491		103
	22779	
·1803712		106
	22673	
·1781038		109
	22564	
·1758474		112
	22452	
·1736022		115
	22337	
·1713686		118
	22218	
·1691468		121
	22097	
·1669370		124
	21973	
·1647397		127
	21847	
·1625551		129
	21717	
·1603833		132
	21585	
·1582248		134
	21451	
·1560797		137
	21314	
·1539483		139
	21175	
·1518308		142
	21033	
·1497275		144
	20890	
·1476385		146
	20744	
·1455641		148
	20596	
·1435046		150
	20446	
·1414600		152
	20294	
·1394306		154
	20140	
·1374165		155
	19985	
·1354181		157
	19828	
·1334353		159
	19669	
·1314684		160
	19508	
·1295176		162

TABLE II. *Area and Ordinate in terms of Abscissa.*

x	$\frac{1}{2}(1+\alpha)$	Δ +	Δ^2 −	z	Δ −	Δ^2 +
1·50	·9331928	12855	194	·1295176	19346	162
1·51	·9344783	12662	193	·1275830	19183	163
1·52	·9357445	12471	191	·1256646	19018	165
1·53	·9369916	12282	189	·1237628	18853	166
1·54	·9382198	12094	188	·1218775	18685	167
1·55	·9394292	11908	186	·1200090	18517	168
1·56	·9406201	11724	184	·1181573	18348	169
1·57	·9417924	11541	183	·1163225	18177	170
1·58	·9429466	11360	181	·1145048	18006	171
1·59	·9440826	11181	179	·1127042	17834	172
1·60	·9452007	11004	177	·1109208	17661	173
1·61	·9463011	10828	176	·1091548	17487	174
1·62	·9473839	10654	174	·1074061	17312	174
1·63	·9484493	10482	172	·1056748	17137	175
1·64	·9494974	10311	170	·1039611	16962	176
1·65	·9505285	10142	169	·1022649	16786	176
1·66	·9515428	9975	167	·1005864	16609	177
1·67	·9525403	9810	165	·0989255	16432	177
1·68	·9535213	9647	163	·0972823	16255	177
1·69	·9544860	9485	162	·0956568	16077	178
1·70	·9554345	9325	160	·0940491	15899	178
1·71	·9563671	9167	158	·0924591	15722	178
1·72	·9572838	9011	156	·0908870	15544	178
1·73	·9581849	8856	155	·0893326	15366	178
1·74	·9590705	8704	153	·0877961	15188	178
1·75	·9599408	8553	151	·0862773	15010	178
1·76	·9607961	8403	149	·0847764	14832	178
1·77	·9616364	8256	147	·0832932	14654	178
1·78	·9624620	8110	146	·0818278	14477	177
1·79	·9632730	7966	144	·0803801	14300	177
1·80	·9640697	7824	142	·0789502	14123	177
1·81	·9648521	7684	140	·0775379	13946	176
1·82	·9656205	7545	139	·0761433	13770	176
1·83	·9663750	7409	137	·0747663	13594	176
1·84	·9671159	7273	135	·0734068	13419	175
1·85	·9678432	7140	133	·0720649	13245	175
1·86	·9685572	7009	132	·0707404	13071	174
1·87	·9692581	6879	130	·0694333	12897	173
1·88	·9699460	6751	128	·0681436	12725	173
1·89	·9706210	6624	126	·0668711	12553	172
1·90	·9712834	6500	125	·0656158	12382	171
1·91	·9719334	6377	123	·0643777	12211	170
1·92	·9725711	6255	121	·0631566	12041	170
1·93	·9731966	6136	120	·0619524	11873	169
1·94	·9738102	6018	118	·0607652	11705	168
1·95	·9744119	5902	116	·0595947	11538	167
1·96	·9750021	5787	115	·0584409	11372	166
1·97	·9755808	5674	113	·0573038	11206	165
1·98	·9761482	5563	111	·0561831	11042	164
1·99	·9767045	5453	110	·0550789	10879	163
2·00	·9772499		108	·0539910		162

x	$\frac{1}{2}(1+\alpha)$	Δ +	Δ^2 −
2·00	·9772499	5345	108
2·01	·9777844	5239	106
2·02	·9783083	5134	105
2·03	·9788217	5031	103
2·04	·9793248	4929	102
2·05	·9798178	4829	100
2·06	·9803007	4731	98
2·07	·9807738	4634	97
2·08	·9812372	4539	95
2·09	·9816911	4445	94
2·10	·9821356	4352	92
2·11	·9825708	4262	91
2·12	·9829970	4172	89
2·13	·9834142	4084	88
2·14	·9838226	3998	86
2·15	·9842224	3913	85
2·16	·9846137	3829	84
2·17	·9849966	3747	82
2·18	·9853713	3666	81
2·19	·9857379	3587	79
2·20	·9860966	3509	78
2·21	·9864474	3432	77
2·22	·9867906	3357	75
2·23	·9871263	3283	74
2·24	·9874545	3210	73
2·25	·9877755	3138	71
2·26	·9880894	3068	70
2·27	·9883962	2999	69
2·28	·9886962	2932	68
2·29	·9889893	2865	66
2·30	·9892759	2800	65
2·31	·9895559	2736	64
2·32	·9898296	2674	63
2·33	·9900969	2612	62
2·34	·9903581	2552	60
2·35	·9906133	2492	59
2·36	·9908625	2434	58
2·37	·9911060	2377	57
2·38	·9913437	2321	56
2·39	·9915758	2267	55
2·40	·9918025	2213	54
2·41	·9920237	2160	53
2·42	·9922397	2108	52
2·43	·9924506	2058	51
2·44	·9926564	2008	50
2·45	·9928572	1960	49
2·46	·9930531	1912	48
2·47	·9932443	1865	47
2·48	·9934309	1820	46
2·49	·9936128	1775	45
2·50	·9937903		44

TABLE II.—(*continued*).

z	Δ −	Δ^2 +
·0539910	10717	162
·0529192	10557	161
·0518636	10397	160
·0508239	10238	159
·0498001	10081	157
·0487920	9924	156
·0477996	9769	155
·0468226	9616	154
·0458611	9463	153
·0449148	9312	151
·0439836	9162	150
·0430674	9013	149
·0421661	8866	147
·0412795	8720	146
·0404076	8575	145
·0395500	8432	143
·0387069	8290	142
·0378779	8149	140
·0370629	8010	139
·0362619	7873	138
·0354746	7737	136
·0347009	7602	135
·0339408	7468	133
·0331939	7337	132
·0324603	7206	130
·0317397	7077	129
·0310319	6950	127
·0303370	6824	126
·0296546	6699	125
·0289847	6576	123
·0283270	6455	122
·0276816	6335	120
·0270481	6216	119
·0264265	6099	117
·0258166	5984	116
·0252182	5870	114
·0246313	5757	113
·0240556	5646	111
·0234910	5536	110
·0229374	5428	108
·0223945	5322	107
·0218624	5217	105
·0213407	5113	104
·0208294	5011	102
·0203284	4910	101
·0198374	4811	99
·0193563	4713	98
·0188850	4617	96
·0184233	4522	95
·0179711	4428	93
·0175283		92

x	$\frac{1}{2}(1+a)$	Δ +	Δ^2 −
2·50	·9937903	1731	44
2·51	·9939634	1688	43
2·52	·9941323	1646	42
2·53	·9942969	1605	41
2·54	·9944574	1565	40
2·55	·9946139	1525	39
2·56	·9947664	1487	39
2·57	·9949151	1449	38
2·58	·9950600	1412	37
2·59	·9952012	1376	36
2·60	·9953388	1341	35
2·61	·9954729	1306	35
2·62	·9956035	1272	34
2·63	·9957308	1239	33
2·64	·9958547	1207	32
2·65	·9959754	1176	32
2·66	·9960930	1145	31
2·67	·9962074	1115	30
2·68	·9963189	1085	29
2·69	·9964274	1056	29
2·70	·9965330	1028	28
2·71	·9966358	1001	27
2·72	·9967359	974	27
2·73	·9968333	948	26
2·74	·9969280	922	26
2·75	·9970202	897	25
2·76	·9971099	873	24
2·77	·9971972	849	24
2·78	·9972821	825	23
2·79	·9973646	803	23
2·80	·9974449	781	22
2·81	·9975229	759	22
2·82	·9975988	738	21
2·83	·9976726	717	21
2·84	·9977443	697	20
2·85	·9978140	678	20
2·86	·9978818	658	19
2·87	·9979476	640	19
2·88	·9980116	622	18
2·89	·9980738	604	18
2·90	·9981342	587	17
2·91	·9981929	570	17
2·92	·9982498	553	16
2·93	·9983052	537	16
2·94	·9983589	522	16
2·95	·9984111	507	15
2·96	·9984618	492	15
2·97	·9985110	478	14
2·98	·9985588	464	14
2·99	·9986051	450	14
3·00	·9986501		13

z	Δ −	Δ^2 +
·0175283	4336	92
·0170947	4246	91
·0166701	4157	89
·0162545	4069	88
·0158476	3982	86
·0154493	3897	85
·0150596	3814	84
·0146782	3731	82
·0143051	3650	81
·0139401	3571	80
·0135830	3493	78
·0132337	3416	77
·0128921	3340	76
·0125581	3266	74
·0122315	3193	73
·0119122	3121	72
·0116001	3051	70
·0112951	2981	69
·0109969	2913	68
·0107056	2847	67
·0104209	2781	66
·0101428	2717	64
·0098712	2654	63
·0096058	2592	62
·0093466	2531	61
·0090936	2471	60
·0088465	2413	59
·0086052	2355	57
·0083697	2299	56
·0081398	2244	55
·0079155	2189	54
·0076965	2136	53
·0074829	2084	52
·0072744	2033	51
·0070711	1983	50
·0068728	1934	49
·0066793	1886	48
·0064907	1839	47
·0063067	1793	46
·0061274	1748	45
·0059525	1704	44
·0057821	1661	43
·0056160	1619	42
·0054541	1578	41
·0052963	1537	40
·0051426	1497	40
·0049929	1459	39
·0048470	1421	38
·0047050	1384	37
·0045666	1347	36
·0044318		35

TABLE II. *Area and Ordinate in terms of Abscissa.*

x	$\frac{1}{2}(1+a)$	Δ +	Δ^2 −	z	Δ −	Δ^2 +
3·00	·9986501			·0044318		
		437	13		1312	35
3·01	·9986938		13	·0043007		35
		424			1277	
3·02	·9987361		13	·0041729		34
		411			1243	
3·03	·9987772		12	·0040486		33
		399			1210	
3·04	·9988171		12	·0039276		32
		387			1178	
3·05	·9988558		12	·0038098		32
		375			1146	
3·06	·9988933		11	·0036951		31
		364			1115	
3·07	·9989297		11	·0035836		30
		353			1085	
3·08	·9989650		11	·0034751		29
		342			1056	
3·09	·9989992		10	·0033695		29
		332			1027	
3·10	·9990324		10	·0032668		28
		322			999	
3·11	·9990646		10	·0031669		27
		312			971	
3·12	·9990957		10	·0030698		27
		302			944	
3·13	·9991260		9	·0029754		26
		293			918	
3·14	·9991553		9	·0028835		26
		284			893	
3·15	·9991836		9	·0027943		25
		275			868	
3·16	·9992112		9	·0027075		24
		267			843	
3·17	·9992378		8	·0026231		24
		258			820	
3·18	·9992636		8	·0025412		23
		250			797	
3·19	·9992886		8	·0024615		23
		242			774	
3·20	·9993129		8	·0023841		22
		235			752	
3·21	·9993363		7	·0023089		21
		227			731	
3·22	·9993590		7	·0022358		21
		220			710	
3·23	·9993810		7	·0021649		20
		213			689	
3·24	·9994024		7	·0020960		20
		206			669	
3·25	·9994230		7	·0020290		19
		200			650	
3·26	·9994429		6	·0019641		19
		193			631	
3·27	·9994623		6	·0019010		18
		187			612	
3·28	·9994810		6	·0018397		18
		181			595	
3·29	·9994991		6	·0017803		17
		175			577	
3·30	·9995166		6	·0017226		17
		169			560	
3·31	·9995335		6	·0016666		17
		164			543	
3·32	·9995499		5	·0016122		16
		159			527	
3·33	·9995658		5	·0015595		16
		153			512	
3·34	·9995811		5	·0015084		15
		148			496	
3·35	·9995959		5	·0014587		15
		143			481	
3·36	·9996103		5	·0014106		15
		139			467	
3·37	·9996242		5	·0013639		14
		134			453	
3·38	·9996376		4	·0013187		14
		130			439	
3·39	·9996505		4	·0012748		13
		125			426	
3·40	·9996631		4	·0012322		13
		121			413	
3·41	·9996752		4	·0011910		13
		117			400	
3·42	·9996869		4	·0011510		12
		113			388	
3·43	·9996982		4	·0011122		12
		109			376	
3·44	·9997091		4	·0010747		12
		106			364	
3·45	·9997197		4	·0010383		11
		102			353	
3·46	·9997299		3	·0010030		11
		99			342	
3·47	·9997398		3	·0009689		11
		95			331	
3·48	·9997493		3	·0009358		10
		92			320	
3·49	·9997585		3	·0009037		10
		89			310	
3·50	·9997674		3	·0008727		10

x	$\frac{1}{2}(1+a)$	Δ +	Δ^2 −
3·50	·9997674		
		86	3
3·51	·9997759		3
		83	
3·52	·9997842		3
		80	
3·53	·9997922		3
		77	
3·54	·9997999		3
		74	
3·55	·9998074		3
		72	
3·56	·9998146		3
		69	
3·57	·9998215		2
		67	
3·58	·9998282		2
		65	
3·59	·9998347		2
		62	
3·60	9998409		2
		60	
3·61	9998469		2
		58	
3·62	·9998527		2
		56	
3·63	·9998583		2
		54	
3·64	·9998637		2
		52	
3·65	·9998689		2
		50	
3·66	·9998739		2
		48	
3·67	·9998787		2
		47	
3·68	·9998834		2
		45	
3·69	·9998879		2
		43	
3·70	·9998922		2
		42	
3·71	·9998964		2
		40	
3·72	·9999004		1
		39	
3·73	·9999043		1
		37	
3·74	·9999080		1
		36	
3·75	·9999116		1
		35	
3·76	·9999150		1
		33	
3·77	·9999184		1
		32	
3·78	·9999216		1
		31	
3·79	·9999247		1
		30	
3·80	·9999277		1
		29	
3·81	·9999305		1
		28	
3·82	·9999333		1
		27	
3·83	·9999359		1
		26	
3·84	·9999385		1
		25	
3·85	·9999409		1
		24	
3·86	·9999433		1
		23	
3·87	·9999456		1
		22	
3·88	·9999478		1
		21	
3·89	·9999499		1
		20	
3·90	·9999519		1
		19	
3·91	·9999539		1
		19	
3·92	·9999557		1
		18	
3·93	·9999575		1
		17	
3·94	·9999593		1
		17	
3·95	·9999609		1
		16	
3·96	·9999625		1
		15	
3·97	·9999641		1
		15	
3·98	·9999655		1
		14	
3·99	·9999670		1
		14	
4·00	·9999683		1

TABLE II.—(*continued*).

z	Δ −	Δ^2 +
·0008727		
	301	10
·0008426		10
	291	
·0008135		9
	282	
·0007853		9
	273	
·0007581		9
	264	
·0007317		8
	256	
·0007061		8
	247	
·0006814		8
	239	
·0006575		8
	232	
·0006343		8
	224	
·0006119		7
	217	
·0005902		7
	210	
·0005693		7
	203	
·0005490		7
	196	
·0005294		6
	189	
·0005105		6
	183	
·0004921		6
	177	
·0004744		6
	171	
·0004573		6
	165	
·0004408		6
	160	
·0004248		5
	155	
·0004093		5
	149	
·0003944		5
	144	
·0003800		5
	139	
·0003661		5
	135	
·0003526		5
	130	
·0003396		4
	125	
·0003271		4
	121	
·0003149		4
	117	
·0003032		4
	113	
·0002919		4
	109	
·0002810		4
	105	
·0002705		4
	102	
·0002604		4
	98	
·0002506		3
	95	
·0002411		3
	91	
·0002320		3
	88	
·0002232		3
	85	
·0002147		3
	82	
·0002065		3
	79	
·0001987		3
	76	
·0001910		3
	73	
·0001837		3
	71	
·0001766		3
	68	
·0001698		2
	66	
·0001633		2
	63	
·0001569		2
	61	
·0001508		2
	59	
·0001449		2
	57	
·0001393		2
	55	
·0001338		2

x	$\frac{1}{2}(1+a)$	Δ +	Δ^2 −
4·00	·9999683		
		13	1
4·01	·9999696		1
		13	
4·02	·9999709		0
		12	
4·03	·9999721		
		12	
4·04	·9999733		
		11	
4·05	·9999744		
		11	
4·06	·9999755		
		10	
4·07	·9999765		
		10	
4·08	·9999775		
		9	
4·09	·9999784		
		9	
4·10	·9999793		
		9	
4·11	·9999802		
		8	
4·12	·9999811		
		8	
4·13	·9999819		
		8	
4·14	·9999826		
		7	
4·15	·9999834		
		7	
4·16	·9999841		
		7	
4·17	·9999848		
		7	
4·18	·9999854		
		6	
4·19	·9999861		
		6	
4·20	·9999867		
		6	
4·21	·9999872		
		6	
4·22	·9999878		
		5	
4·23	·9999883		
		5	
4·24	·9999888		
		5	
4·25	·9999893		
		5	
4·26	·9999898		
		4	
4·27	·9999902		
		4	
4·28	·9999907		
		4	
4·29	·9999911		
		4	
4·30	·9999915		
		4	
4·31	·9999918		
		4	
4·32	·9999922		
		3	
4·33	·9999925		
		3	
4·34	·9999929		
		3	
4·35	·9999932		
		3	
4·36	·9999935		
		3	
4·37	·9999938		
		3	
4·38	·9999941		
		3	
4·39	·9999943		
		3	
4·40	·9999946		
		2	
4·41	·9999948		
		2	
4·42	·9999951		
		2	
4·43	·9999953		
		2	
4·44	·9999955		
		2	
4·45	·9999957		
		2	
4·46	·9999959		
		2	
4·47	·9999961		
		2	
4·48	·9999963		
		2	
4·49	·9999964		
		2	
4·50	·9999966		

z	Δ −	Δ^2 +
·0001338		
	53	2
·0001286		2
	51	
·0001235		2
	49	
·0001186		2
	47	
·0001140		2
	45	
·0001094		2
	43	
·0001051		2
	42	
·0001009		2
	40	
·0000969		2
	39	
·0000930		1
	37	
·0000893		1
	36	
·0000857		1
	35	
·0000822		1
	33	
·0000789		1
	32	
·0000757		1
	31	
·0000726		1
	30	
·0000697		1
	28	
·0000668		1
	27	
·0000641		1
	26	
·0000615		1
	25	
·0000589		1
	24	
·0000565		1
	23	
·0000542		1
	22	
·0000519		1
	22	
·0000498		1
	21	
·0000477		1
	20	
·0000457		1
	19	
·0000438		1
	18	
·0000420		1
	18	
·0000402		1
	17	
·0000385		1
	16	
·0000369		1
	16	
·0000354		1
	15	
·0000339		1
	14	
·0000324		1
	14	
·0000310		1
	13	
·0000297		1
	13	
·0000284		1
	12	
·0000272		0
	12	
·0000261		
	11	
·0000249		
	11	
·0000239		
	10	
·0000228		
	10	
·0000218		
	9	
·0000209		
	9	
·0000200		
	9	
·0000191		
	8	
·0000183		
	8	
·0000175		
	8	
·0000167		
	7	
·0000160		

TABLE II. *Area and Ordinate in terms of Abscissa**.

x	$\frac{1}{2}(1+\alpha)$	z
4·50	66023	159837
4·51	67586	152797
4·52	69080	146051
4·53	70508	139590
4·54	71873	133401
4·55	73177	127473
4·56	74423	121797
4·57	75614	116362
4·58	76751	111159
4·59	77838	106177
4·60	78875	101409
4·61	79867	96845
4·62	80813	92477
4·63	81717	88297
4·64	82580	84298
4·65	83403	80472
4·66	84190	76812
4·67	84940	73311
4·68	85656	69962
4·69	86340	66760
4·70	86992	63698
4·71	87614	60771
4·72	88208	57972
4·73	88774	55296
7·74	89314	52739
4·75	89829	50295
4·76	90320	47960
4·77	90789	45728
4·78	91235	43596
4·79	91661	41559
4·80	92067	39613
4·81	92453	37755
4·82	92822	35980
4·83	93173	34285
4·84	93508	32667
4·85	93827	31122
4·86	94131	29647
4·87	94420	28239
4·88	94696	26895
4·89	94958	25613
4·90	95208	24390
4·91	95446	23222
4·92	95673	22108
4·93	95889	21046
4·94	96094	20033
4·95	96289	19066
4·96	96475	18144
4·97	96652	17265
4·98	96821	16428
4·99	96981	15629
5·00	97133	14867
5·01	97278	14141
5·02	97416	13450
5·03	97548	12791
5·04	97672	12162
5·05	97791	11564
5·06	97904	10994
5·07	98011	10451
5·08	98113	9934
5·09	98210	9441
5·10	98302	8972
5·11	98389	8526
5·12	98472	8101
5·13	98551	7696
5·14	98626	7311
5·15	98698	6944
5·16	98765	6595
5·17	98830	6263
5·18	98891	5947
5·19	98949	5647
5·20	99004	5361
5·21	99056	5089
5·22	99105	4831
5·23	99152	4585
5·24	99197	4351
5·25	99240	4128
5·26	99280	3917
5·27	99318	3716
5·28	99354	3525
5·29	99388	3344
5·30	99421	3171
5·31	99452	3007
5·32	99481	2852
5·33	99509	2704
5·34	99535	2563
5·35	99560	2430
5·36	99584	2303
5·37	99606	2183
5·38	99628	2069
5·39	99648	1960
5·40	99667	1857
5·41	99685	1760
5·42	99702	1667
5·43	99718	1579
5·44	99734	1495
5·45	99748	1416
5·46	99762	1341
5·47	99775	1270
5·48	99787	1202
5·49	99799	1138
5·50	99810	1077
5·51	99821	1019
5·52	99831	965
5·53	99840	913
5·54	99849	864
5·55	99857	817
5·56	99865	773
5·57	99873	731
5·58	99880	691
5·59	99886	654
5·60	99893	618
5·61	99899	585
5·62	99905	553
5·63	99910	522
5·64	99915	494
5·65	99920	467
5·66	99924	441
5·67	99929	417
5·68	99933	394
5·69	99936	372
5·70	99940	351
5·71	99944	332
5·72	99947	313
5·73	99950	296
5·74	99953	280
5·75	99955	264
5·76	99958	249
5·77	99960	235
5·78	99963	222
5·79	99965	210
5·80	99967	198
5·81	99969	187
5·82	99971	176
5·83	99972	166
5·84	99974	157
5·85	99975	148
5·86	99977	139
5·87	99978	131
5·88	99979	124
5·89	99981	117
5·90	99982	110
5·91	99983	104
5·92	99984	98
5·93	99985	92
5·94	99986	87
5·95	99987	82
5·96	99987	77
5·97	99988	73
5·98	99989	68
5·99	99990	65
6·00	99990	61

* Prefix ·99999 to each entry.

TABLE III. *Abscissa and Ordinate in terms of difference of Areas.*

a	x	Δ +	Δ^2 +	Δ^3 +	z	Δ −	Δ^2 −	Δ^3 −
·00	·0000000				·3989423			
		125335	0	20		313	627	
·01	·0125335				·3989109			0
		125354	20	20		940	627	0
·02	·0250689				·3988169			
		125394	39	20		1567	627	0
·03	·0376083				·3986603			
		125453	59	20		2194	627	0
·04	·0501536				·3984408			
		125532	79	20		2821	627	0
·05	·0627068				·3981587			
		125631	99	20		3449	628	1
·06	·0752699				·3978138			
		125750	119	20		4078	628	1
·07	·0878448				·3974060			
		125889	139	20		4707	629	1
·08	·1004337				·3969353			
		126048	159	20		5337	630	1
·09	·1130385				·3964016			
		126228	180	21		5967	631	1
·10	·1256613				·3958049			
		126429	201	21		6599	632	1
·11	·1383042				·3951450			
		126650	221	21		7232	633	1
·12	·1509692				·3944218			
		126893	243	21		7866	634	1
·13	·1636585				·3936352			
		127157	264	22		8501	635	1
·14	·1763742				·3927852			
		127443	286	22		9137	636	1
·15	·1891184				·3918715			
		127751	308	22		9775	638	2
·16	·2018935				·3908939			
		128081	330	23		10415	640	2
·17	·2147016				·3898525			
		128434	353	23		11056	641	2
·18	·2275450				·3887469			
		128811	376	24		11699	643	2
·19	·2404260				·3875769			
		129211	400	24		12344	645	2
·20	·2533471				·3863425			
		129635	424	25		12991	647	2
·21	·2663106				·3850434			
		130084	449	25		13641	649	2
·22	·2793190				·3836794			
		130559	474	26		14292	652	2
·23	·2923749				·3822501			
		131059	500	27		14946	654	3
·24	·3054808				·3807555			
		131586	527	27		15603	657	3
·25	·3186394				·3791952			
		132140	554	28		16262	659	3
·26	·3318533				·3775690			
		132722	582	29		16924	662	3
·27	·3451255				·3758766			
		133333	611	30		17589	665	3
·28	·3584588				·3741177			
		133973	640	30		18258	668	3
·29	·3718561				·3722919			
		134644	671	31		18929	672	3
·30	·3853205				·3703990			
		135346	702	32		19604	675	4
·31	·3988551				·3684386			
		136081	735	34		20283	679	4
·32	·4124631				·3664103			
		136849	768	35		20965	682	4
·33	·4261480				·3643138			
		137652	803	36		21651	686	4
·34	·4399132				·3621487			
		138490	839	37		22342	690	4
·35	·4537622				·3599146			
		139366	876	39		23036	695	4
·36	·4676988				·3576109			
		140281	914	40		23735	699	5
·37	·4817268				·3552374			
		141235	954	42		24439	704	5
·38	·4958503				·3527935			
		142231	996	43		25148	709	5
·39	·5100735				·3502788			
		143271	1039	45		25861	714	5
·40	·5244005				·3476926			
		144355	1085	47		26580	719	6
·41	·5388360				·3450346			
		145487	1132	49		27305	725	6
·42	·5533847				·3423041			
		146668	1181	51		28035	730	6
·43	·5680515				·3395005			
		147900	1232	54		28772	736	6
·44	·5828415				·3366233			
		149186	1286	56		29514	743	7
·45	·5977601				·3336719			
		150529	1342	59		30264	749	7
·46	·6128130				·3306455			
		151930	1402	62		31020	756	7
·47	·6280060				·3275435			
		153394	1464	65		31783	763	7
·48	·6433454				·3243652			
		154923	1529	69		32554	771	8
·49	·6588377				·3211098			
		156521	1598	72		33333	779	8
·50	·6744898		1670		·3177766		787	

TABLE III. *Abscissa and Ordinate in terms of difference of Areas.*

a	x	Δ +	Δ^2 +	Δ^3 +	z	Δ −	Δ^2 −	Δ^3 −
·50	·6744898				·3177766			
		158191	1670	76		34119	787	9
·51	·6903088	159937	1747	81	·3143646	34915	795	9
·52	·7063026	161765	1828	86	·3108732	35719	804	9
·53	·7224791	163678	1913	91	·3073013	36532	814	10
·54	·7388468	165682	2004	96	·3036481	37356	823	10
·55	·7554150	167782	2100	102	·2999125	38189	834	11
·56	·7721932		2203		·2960936		844	
		169984		109		39034		11
·57	·7891917	172296	2312	116	·2921902	39889	856	12
·58	·8064212	174724	2428	124	·2882013	40757	867	12
·59	·8238936	177276	2552	133	·2841256	41637	880	13
·60	·8416212	179961	2685	143	·2799619	42530	893	14
·61	·8596174		2828		·2757089		907	
		182789		153		43437		15
·62	·8778963	185771	2981	165	·2713653	44358	921	15
·63	·8964734	188917	3147	178	·2669295	45295	937	16
·64	·9153651	192242	3325	193	·2624000	46247	953	17
·65	·9345893	195760	3518	209	·2577753	47217	970	18
·66	·9541653		3727		·2530535		988	
		199486		227		48205		19
·67	·9741139	203440	3954	248	·2482330	49213	1007	20
·68	·9944579	207641	4201	271	·2433117	50240	1028	22
·69	1·0152220	212114	4472	297	·2382877	51289	1049	23
·70	1·0364334	216882	4769	326	·2331588	52362	1072	25
·71	1·0581216		5095		·2279226		1097	
		221977		360		53459		26
·72	1·0803193	227432	5455	399	·2225767	54582	1123	28
·73	1·1030626	233286	5854	443	·2171185	55734	1152	30
·74	1·1263911	239583	6297	495	·2115451	56916	1182	33
·75	1·1503494	246374	6792	555	·2058535	58130	1215	35
·76	1·1749868		7347		·2000405		1250	
		253721		625		59380		38
·77	1·2003589	261693	7972	709	·1941024	60669	1288	42
·78	1·2265281	270373	8681	808	·1880356	61999	1330	45
·79	1·2535654	279861	9488	926	·1818357	63374	1375	50
·80	1·2815516		10414		·1754983		1425	

TABLE IV.

Extension of Table of the Probability Integral $F = \frac{1}{2}(1 - \alpha)$.

$$F = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{1}{2}x^2} dx.$$ *The table gives* $(-\log F)$ *for* x.

x	$-\log F$	x	$-\log F$	x	$-\log F$
5	6·54265	*30*	197·30921	*50*	544·96634
6	9·00586	*31*	210·56940	*60*	783·90743
7	11·89285	*32*	224·26344	*70*	1066·26576
8	15·20614	*33*	238·39135	*80*	1392·04459
9	18·94746	*34*	252·95315	*90*	1761·24604
10	23·11805	*35*	267·94888	*100*	2173·87154
11	27·71882	*36*	283·37855	*150*	4888·38812
12	32·75044	*37*	299·24218	*200*	8688·58977
13	38·21345	*38*	315·53979	*250*	13574·49960
14	44·10827	*39*	332·27139	*300*	19546·12790
15	50·43522	*40*	349·43701	*350*	26603·48018
16	57·19458	*41*	367·03664	*400*	34746·55970
17	64·38658	*42*	385·07032	*450*	43975·36860
18	72·01140	*43*	403·53804	*500*	54289·40830
19	80·06919	*44*	422·43983		
20	88·56010	*45*	441·77568		N.B. To obtain anything but a rough appreciation after $x = 50$, the table would require much extension, but for many practical problems it suffices to take after $x = 50$:
21	97·48422	*46*	461·54561		
22	106·84167	*47*	481·74964		
23	116·63253	*48*	502·38776		
24	126·85686	*49*	523·45999		
25	137·51475	*50*	544·96634		
26	148·60624				
27	160·13139				
28	172·09024				
29	184·48283				
30	197·30921				

$$F = \frac{1}{\sqrt{2\pi}} \frac{1}{x} e^{-\frac{1}{2}x^2}.$$

From each of the values in this table ·30103 must be subtracted, if we wish to obtain the probability $2F$, then given by $(-\log 2F)$, that the value is greater than x, without regard to sign.

TABLE V. *Probable Errors of Means and Standard Deviations.*

n	χ_1	χ_2
1	·67449	·47694
2	·47694	·33724
3	·38942	·27536
4	·33724	·23847
5	·30164	·21329
6	·27536	·19471
7	·25493	·18026
8	·23847	·16862
9	·22483	·15898
10	·21329	·15082
11	·20337	·14380
12	·19471	·13768
13	·18707	·13228
14	·18026	·12747
15	·17415	·12314
16	·16862	·11923
17	·16359	·11567
18	·15898	·11241
19	·15474	·10942
20	·15082	·10665
21	·14719	·10408
22	·14380	·10168
23	·14064	·09945
24	·13768	·09735
25	·13490	·09539
26	·13228	·09353
27	·12981	·09179
28	·12747	·09013
29	·12525	·08856
30	·12314	·08708
31	·12114	·08566
32	·11923	·08431
33	·11741	·08302
34	·11567	·08179
35	·11401	·08062
36	·11241	·07949
37	·11088	·07841
38	·10942	·07737
39	·10800	·07637
40	·10665	·07541
41	·10534	·07448
42	·10408	·07359
43	·10286	·07273
44	·10168	·07190
45	·10055	·07110
46	·09945	·07032
47	·09838	·06957
48	·09735	·06884
49	·09636	·06813
50	·09539	·06745
51	·09445	·06678
52	·09353	·06614
53	·09265	·06551
54	·09179	·06490
55	·09095	·06431
56	·09013	·06373
57	·08934	·06317
58	·08856	·06262
59	·08781	·06209
60	·08708	·06157
61	·08636	·06107
62	·08566	·06057
63	·08498	·06009
64	·08431	·05962
65	·08366	·05916
66	·08302	·05871
67	·08240	·05827
68	·08179	·05784
69	·08120	·05742
70	·08062	·05700
71	·08005	·05660
72	·07949	·05621
73	·07894	·05582
74	·07841	·05544
75	·07788	·05507
76	·07737	·05471
77	·07687	·05435
78	·07637	·05400
79	·07589	·05366
80	·07541	·05332
81	·07494	·05299
82	·07448	·05267
83	·07403	·05235
84	·07359	·05204
85	·07316	·05173
86	·07273	·05143
87	·07231	·05113
88	·07190	·05084
89	·07150	·05056
90	·07110	·05027
91	·07071	·05000
92	·07032	·04972
93	·06994	·04946
94	·06957	·04919
95	·06920	·04893
96	·06884	·04868
97	·06848	·04843
98	·06813	·04818
99	·06779	·04793
100	·06745	·04769
101	·06711	·04746
102	·06678	·04722
103	·06646	·04699
104	·06614	·04677
105	·06582	·04654
106	·06551	·04632
107	·06521	·04611
108	·06490	·04589
109	·06460	·04568
110	·06431	·04547
111	·06402	·04527
112	·06373	·04507
113	·06345	·04487
114	·06317	·04467
115	·06290	·04447
116	·06262	·04428
117	·06236	·04409
118	·06209	·04391
119	·06183	·04372
120	·06157	·04354
121	·06132	·04336
122	·06107	·04318
123	·06082	·04300
124	·06057	·04283
125	·06033	·04266
126	·06009	·04249
127	·05985	·04232
128	·05962	·04216
129	·05939	·04199
130	·05916	·04183
131	·05893	·04167
132	·05871	·04151
133	·05849	·04136
134	·05827	·04120
135	·05805	·04105
136	·05784	·04090
137	·05763	·04075
138	·05742	·04060
139	·05721	·04045
140	·05700	·04031
141	·05680	·04017
142	·05660	·04002
143	·05640	·03988
144	·05621	·03974
145	·05601	·03961
146	·05582	·03947
147	·05563	·03934
148	·05544	·03920
149	·05526	·03907
150	·05507	·03894

TABLE V. *Probable Errors of Means and Standard Deviations.*

n	χ_1	χ_2
151	·05489	·03881
152	·05471	·03868
153	·05453	·03856
154	·05435	·03843
155	·05418	·03831
156	·05400	·03819
157	·05383	·03806
158	·05366	·03794
159	·05349	·03782
160	·05332	·03771
161	·05316	·03759
162	·05299	·03747
163	·05283	·03736
164	·05267	·03724
165	·05251	·03713
166	·05235	·03702
167	·05219	·03691
168	·05204	·03680
169	·05188	·03669
170	·05173	·03658
171	·05158	·03647
172	·05143	·03637
173	·05128	·03626
174	·05113	·03616
175	·05099	·03605
176	·05084	·03595
177	·05070	·03585
178	·05056	·03575
179	·05041	·03565
180	·05027	·03555
181	·05013	·03545
182	·05000	·03535
183	·04986	·03526
184	·04972	·03516
185	·04959	·03507
186	·04946	·03497
187	·04932	·03488
188	·04919	·03478
189	·04906	·03469
190	·04893	·03460
191	·04880	·03451
192	·04868	·03442
193	·04855	·03433
194	·04843	·03424
195	·04830	·03415
196	·04818	·03407
197	·04806	·03398
198	·04793	·03389
199	·04781	·03381
200	·04769	·03372

n	χ_1	χ_2
201	·04757	·03364
202	·04746	·03356
203	·04734	·03347
204	·04722	·03339
205	·04711	·03331
206	·04699	·03323
207	·04688	·03315
208	·04677	·03307
209	·04666	·03299
210	·04654	·03291
211	·04643	·03283
212	·04632	·03276
213	·04622	·03268
214	·04611	·03260
215	·04600	·03253
216	·04589	·03245
217	·04579	·03238
218	·04568	·03230
219	·04558	·03223
220	·04547	·03216
221	·04537	·03208
222	·04527	·03201
223	·04517	·03194
224	·04507	·03187
225	·04497	·03180
226	·04487	·03173
227	·04477	·03166
228	·04467	·03159
229	·04457	·03152
230	·04447	·03145
231	·04438	·03138
232	·04428	·03131
233	·04419	·03125
234	·04409	·03118
235	·04400	·03111
236	·04391	·03105
237	·04381	·03098
238	·04372	·03092
239	·04363	·03085
240	·04354	·03079
241	·04345	·03072
242	·04336	·03066
243	·04327	·03060
244	·04318	·03053
245	·04309	·03047
246	·04300	·03041
247	·04292	·03035
248	·04283	·03029
249	·04274	·03022
250	·04266	·03016

n	χ_1	χ_2
251	·04257	·03010
252	·04249	·03004
253	·04240	·02998
254	·04232	·02993
255	·04224	·02987
256	·04216	·02981
257	·04207	·02975
258	·04199	·02969
259	·04191	·02964
260	·04183	·02958
261	·04175	·02952
262	·04167	·02947
263	·04159	·02941
264	·04151	·02935
265	·04143	·02930
266	·04136	·02924
267	·04128	·02919
268	·04120	·02913
269	·04112	·02908
270	·04105	·02903
271	·04097	·02897
272	·04090	·02892
273	·04082	·02887
274	·04075	·02881
275	·04067	·02876
276	·04060	·02871
277	·04053	·02866
278	·04045	·02860
279	·04038	·02855
280	·04031	·02850
281	·04024	·02845
282	·04017	·02840
283	·04009	·02835
284	·04002	·02830
285	·03995	·02825
286	·03988	·02820
287	·03981	·02815
288	·03974	·02810
289	·03968	·02806
290	·03961	·02801
291	·03954	·02796
292	·03947	·02791
293	·03940	·02786
294	·03934	·02782
295	·03927	·02777
296	·03920	·02772
297	·03913	·02767
298	·03907	·02763
299	·03901	·02758
300	·03894	·02754

TABLE V. *Probable Errors of Means and Standard Deviations.*

n	χ_1	χ_2
301	·03888	·02749
302	·03881	·02744
303	·03875	·02740
304	·03868	·02735
305	·03862	·02731
306	·03856	·02726
307	·03850	·02722
308	·03843	·02718
309	·03837	·02713
310	·03831	·02709
311	·03825	·02704
312	·03819	·02700
313	·03812	·02696
314	·03806	·02692
315	·03800	·02687
316	·03794	·02683
317	·03788	·02679
318	·03782	·02675
319	·03776	·02670
320	·03771	·02666
321	·03765	·02662
322	·03759	·02658
323	·03753	·02654
324	·03747	·02650
325	·03741	·02646
326	·03736	·02642
327	·03730	·02637
328	·03724	·02633
329	·03719	·02629
330	·03713	·02625
331	·03707	·02621
332	·03702	·02618
333	·03696	·02614
334	·03691	·02610
335	·03685	·02606
336	·03680	·02602
337	·03674	·02598
338	·03669	·02594
339	·03663	·02590
340	·03658	·02587
341	·03653	·02583
342	·03647	·02579
343	·03642	·02575
344	·03637	·02571
345	·03631	·02568
346	·03626	·02564
347	·03621	·02560
348	·03616	·02557
349	·03610	·02553
350	·03605	·02549

n	χ_1	χ_2
351	·03600	·02546
352	·03595	·02542
353	·03590	·02538
354	·03585	·02535
355	·03580	·02531
356	·03575	·02528
357	·03570	·02524
358	·03565	·02521
359	·03560	·02517
360	·03555	·02514
361	·03550	·02510
362	·03545	·02507
363	·03540	·02503
364	·03535	·02500
365	·03530	·02496
366	·03526	·02493
367	·03521	·02490
368	·03516	·02486
369	·03511	·02483
370	·03507	·02479
371	·03502	·02476
372	·03497	·02473
373	·03492	·02469
374	·03488	·02466
375	·03483	·02463
376	·03478	·02460
377	·03474	·02456
378	·03469	·02453
379	·03465	·02450
380	·03460	·02447
381	·03456	·02443
382	·03451	·02440
383	·03446	·02437
384	·03442	·02434
385	·03438	·02431
386	·03433	·02428
387	·03429	·02424
388	·03424	·02421
389	·03420	·02418
390	·03415	·02415
391	·03411	·02412
392	·03407	·02409
393	·03402	·02406
394	·03398	·02403
395	·03394	·02400
396	·03389	·02397
397	·03385	·02394
398	·03381	·02391
399	·03377	·02388
400	·03372	·02385

n	χ_1	χ_2
401	·03368	·02382
402	·03364	·02379
403	·03360	·02376
404	·03356	·02373
405	·03352	·02370
406	·03347	·02367
407	·03343	·02364
408	·03339	·02361
409	·03335	·02358
410	·03331	·02355
411	·03327	·02353
412	·03323	·02350
413	·03319	·02347
414	·03315	·02344
415	·03311	·02341
416	·03307	·02338
417	·03303	·02336
418	·03299	·02333
419	·03295	·02330
420	·03291	·02327
421	·03287	·02324
422	·03283	·02322
423	·03279	·02319
424	·03276	·02316
425	·03272	·02313
426	·03268	·02311
427	·03264	·02308
428	·03260	·02305
429	·03256	·02303
430	·03253	·02300
431	·03249	·02297
432	·03245	·02295
433	·03241	·02292
434	·03238	·02289
435	·03234	·02287
436	·03230	·02284
437	·03227	·02281
438	·03223	·02279
439	·03219	·02276
440	·03216	·02274
441	·03212	·02271
442	·03208	·02269
443	·03205	·02266
444	·03201	·02263
445	·03197	·02261
446	·03194	·02258
447	·03190	·02256
448	·03187	·02253
449	·03183	·02251
450	·03180	·02248

TABLE V. *Probable Errors of Means and Standard Deviations.*

n	χ_1	χ_2
451	·03176	·02246
452	·03173	·02243
453	·03169	·02241
454	·03166	·02238
455	·03162	·02236
456	·03159	·02233
457	·03155	·02231
458	·03152	·02229
459	·03148	·02226
460	·03145	·02224
461	·03141	·02221
462	·03138	·02219
463	·03135	·02217
464	·03131	·02214
465	·03128	·02212
466	·03125	·02209
467	·03121	·02207
468	·03118	·02205
469	·03115	·02202
470	·03111	·02200
471	·03108	·02198
472	·03105	·02195
473	·03101	·02193
474	·03098	·02191
475	·03095	·02188
476	·03092	·02186
477	·03088	·02184
478	·03085	·02181
479	·03082	·02179
480	·03079	·02177
481	·03075	·02175
482	·03072	·02172
483	·03069	·02170
484	·03066	·02168
485	·03063	·02166
486	·03060	·02163
487	·03056	·02161
488	·03053	·02159
489	·03050	·02157
490	·03047	·02155
491	·03044	·02152
492	·03041	·02150
493	·03038	·02148
494	·03035	·02146
495	·03032	·02144
496	·03029	·02142
497	·03026	·02139
498	·03022	·02137
499	·03019	·02135
500	·03016	·02133

n	χ_1	χ_2
501	·03013	·02131
502	·03010	·02129
503	·03007	·02127
504	·03004	·02124
505	·03001	·02122
506	·02998	·02120
507	·02996	·02118
508	·02993	·02116
509	·02990	·02114
510	·02987	·02112
511	·02984	·02110
512	·02981	·02108
513	·02978	·02106
514	·02975	·02104
515	·02972	·02102
516	·02969	·02100
517	·02966	·02098
518	·02964	·02096
519	·02961	·02094
520	·02958	·02092
521	·02955	·02089
522	·02952	·02087
523	·02949	·02085
524	·02947	·02084
525	·02944	·02082
526	·02941	·02080
527	·02938	·02078
528	·02935	·02076
529	·02933	·02074
530	·02930	·02072
531	·02927	·02070
532	·02924	·02068
533	·02922	·02066
534	·02919	·02064
535	·02916	·02062
536	·02913	·02060
537	·02911	·02058
538	·02908	·02056
539	·02905	·02054
540	·02903	·02052
541	·02900	·02051
542	·02897	·02049
543	·02895	·02047
544	·02892	·02045
545	·02889	·02043
546	·02887	·02041
547	·02884	·02039
548	·02881	·02037
549	·02879	·02036
550	·02876	·02034

n	χ_1	χ_2
551	·02873	·02032
552	·02871	·02030
553	·02868	·02028
554	·02866	·02026
555	·02863	·02024
556	·02860	·02023
557	·02858	·02021
558	·02855	·02019
559	·02853	·02017
560	·02850	·02015
561	·02848	·02014
562	·02845	·02012
563	·02843	·02010
564	·02840	·02008
565	·02838	·02006
566	·02835	·02005
567	·02833	·02003
568	·02830	·02001
569	·02828	·01999
570	·02825	·01998
571	·02823	·01996
572	·02820	·01994
573	·02818	·01992
574	·02815	·01991
575	·02813	·01990
576	·02810	·01987
577	·02808	·01986
578	·02806	·01984
579	·02803	·01982
580	·02801	·01980
581	·02798	·01978
582	·02796	·01977
583	·02793	·01975
584	·02791	·01974
585	·02789	·01972
586	·02786	·01970
587	·02784	·01969
588	·02782	·01967
589	·02779	·01965
590	·02777	·01964
591	·02774	·01962
592	·02772	·01960
593	·02770	·01959
594	·02767	·01957
595	·02765	·01955
596	·02763	·01954
597	·02761	·01952
598	·02758	·01950
599	·02756	·01949
600	·02754	·01947

TABLE V. *Probable Errors of Means and Standard Deviations.*

n	χ_1	χ_2
601	·02751	·01945
602	·02749	·01944
603	·02747	·01942
604	·02744	·01941
605	·02742	·01939
606	·02740	·01937
607	·02738	·01936
608	·02735	·01934
609	·02733	·01933
610	·02731	·01931
611	·02729	·01929
612	·02726	·01928
613	·02724	·01926
614	·02722	·01925
615	·02720	·01923
616	·02718	·01922
617	·02715	·01920
618	·02713	·01919
619	·02711	·01917
620	·02709	·01915
621	·02707	·01914
622	·02704	·01912
623	·02702	·01911
624	·02700	·01909
625	·02698	·01908
626	·02696	·01906
627	·02694	·01905
628	·02692	·01903
629	·02689	·01902
630	·02687	·01900
631	·02685	·01899
632	·02683	·01897
633	·02681	·01896
634	·02679	·01894
635	·02677	·01893
636	·02675	·01891
637	·02672	·01890
638	·02670	·01888
639	·02668	·01887
640	·02666	·01885
641	·02664	·01884
642	·02662	·01822
643	·02660	·01881
644	·02658	·01879
645	·02656	·01878
646	·02654	·01876
647	·02652	·01875
648	·02650	·01874
649	·02648	·01872
650	·02646	·01871

n	χ_1	χ_2
651	·02644	·01869
652	·02642	·01868
653	·02639	·01866
654	·02637	·01865
655	·02635	·01864
656	·02633	·01862
657	·02631	·01861
658	·02629	·01859
659	·02627	·01858
660	·02625	·01856
661	·02623	·01855
662	·02621	·01854
663	·02620	·01852
664	·02618	·01851
665	·02616	·01849
666	·02614	·01848
667	·02612	·01847
668	·02610	·01845
669	·02608	·01844
670	·02606	·01843
671	·02604	·01841
672	·02602	·01840
673	·02600	·01838
674	·02598	·01837
675	·02596	·01836
676	·02594	·01834
677	·02592	·01833
678	·02590	·01832
679	·02588	·01830
680	·02587	·01829
681	·02585	·01828
682	·02583	·01826
683	·02581	·01825
684	·02579	·01824
685	·02577	·01822
686	·02575	·01821
687	·02573	·01820
688	·02571	·01818
689	·02570	·01817
690	·02568	·01816
691	·02566	·01814
692	·02564	·01813
693	·02562	·01812
694	·02560	·01810
695	·02558	·01809
696	·02557	·01808
697	·02555	·01807
698	·02553	·01805
699	·02551	·01804
700	·02549	·01803

n	χ_1	χ_2
701	·02548	·01801
702	·02546	·01800
703	·02544	·01799
704	·02542	·01798
705	·02540	·01796
706	·02538	·01795
707	·02537	·01794
708	·02535	·01792
709	·02533	·01791
710	·02531	·01790
711	·02530	·01789
712	·02528	·01787
713	·02526	·01786
714	·02524	·01785
715	·02522	·01784
716	·02521	·01782
717	·02519	·01781
718	·02517	·01780
719	·02515	·01779
720	·02514	·01777
721	·02512	·01776
722	·02510	·01775
723	·02508	·01774
724	·02507	·01773
725	·02505	·01771
726	·02503	·01770
727	·02502	·01769
728	·02500	·01768
729	·02498	·01766
730	·02496	·01765
731	·02495	·01764
732	·02493	·01763
733	·02491	·01762
734	·02490	·01760
735	·02488	·01759
736	·02486	·01758
737	·02485	·01757
738	·02483	·01756
739	·02481	·01754
740	·02479	·01753
741	·02478	·01752
742	·02476	·01751
743	·02474	·01750
744	·02473	·01749
745	·02471	·01747
746	·02469	·01746
747	·02468	·01745
748	·02466	·01744
749	·02465	·01743
750	·02463	·01742

TABLE V. *Probable Errors of Means and Standard Deviations.*

n	χ_1	χ_2
751	·02461	·01740
752	·02460	·01739
753	·02458	·01738
754	·02456	·01737
755	·02455	·01736
756	·02453	·01735
757	·02451	·01733
758	·02450	·01732
759	·02448	·01731
760	·02447	·01730
761	·02445	·01729
762	·02443	·01728
763	·02442	·01727
764	·02440	·01725
765	·02439	·01724
766	·02437	·01723
767	·02435	·01722
768	·02434	·01721
769	·02432	·01720
770	·02431	·01719
771	·02429	·01718
772	·02428	·01717
773	·02426	·01715
774	·02424	·01714
775	·02423	·01713
776	·02421	·01712
777	·02420	·01711
778	·02418	·01710
779	·02417	·01709
780	·02415	·01708
781	·02414	·01707
782	·02412	·01706
783	·02410	·01704
784	·02409	·01703
785	·02407	·01702
786	·02406	·01701
787	·02404	·01700
788	·02403	·01699
789	·02401	·01698
790	·02400	·01697
791	·02398	·01696
792	·02397	·01695
793	·02395	·01694
794	·02394	·01693
795	·02392	·01692
796	·02391	·01690
797	·02389	·01689
798	·02388	·01688
799	·02386	·01687
800	·02385	·01686

n	χ_1	χ_2
801	·02383	·01685
802	·02382	·01684
803	·02380	·01683
804	·02379	·01682
805	·02377	·01681
806	·02376	·01680
807	·02374	·01679
808	·02373	·01678
809	·02371	·01677
810	·02370	·01676
811	·02368	·01675
812	·02367	·01674
813	·02366	·01673
814	·02364	·01672
815	·02363	·01671
816	·02361	·01670
817	·02360	·01669
818	·02358	·01668
819	·02357	·01667
820	·02355	·01666
821	·02354	·01665
822	·02353	·01664
823	·02351	·01662
824	·02350	·01661
825	·02348	·01660
826	·02347	·01659
827	·02345	·01658
828	·02344	·01657
829	·02343	·01656
830	·02341	·01655
831	·02340	·01654
832	·02338	·01653
833	·02337	·01652
834	·02336	·01651
835	·02334	·01651
836	·02333	·01650
837	·02331	·01649
838	·02330	·01648
839	·02329	·01647
840	·02327	·01646
841	·02326	·01645
852	·02324	·01644
843	·02323	·01643
844	·02322	·01642
845	·02320	·01641
846	·02319	·01640
847	·02318	·01639
848	·02316	·01638
849	·02315	·01637
850	·02313	·01636

n	χ_1	χ_2
851	·02312	·01635
852	·02311	·01634
853	·02309	·01633
854	·02308	·01632
855	·02307	·01631
856	·02305	·01630
857	·02304	·01629
858	·02303	·01628
859	·02301	·01627
860	·02300	·01626
861	·02299	·01625
862	·02297	·01624
863	·02296	·01624
864	·02295	·01623
865	·02293	·01622
866	·02292	·01621
867	·02291	·01620
868	·02289	·01619
869	·02288	·01618
870	·02287	·01617
871	·02285	·01616
872	·02284	·01615
873	·02283	·01614
874	·02281	·01613
875	·02280	·01612
876	·02279	·01611
877	·02278	·01610
878	·02276	·01610
879	·02275	·01609
880	·02274	·01608
881	·02272	·01607
882	·02271	·01606
883	·02270	·01605
884	·02269	·01604
885	·02267	·01603
886	·02266	·01602
887	·02265	·01601
888	·02263	·01600
889	·02262	·01600
890	·02261	·01599
891	·02260	·01598
892	·02258	·01597
893	·02257	·01596
894	·02256	·01595
895	·02255	·01594
896	·02253	·01593
897	·02252	·01592
898	·02251	·01592
899	·02250	·01591
900	·02248	·01590

TABLE V.

Probable Errors of Means and Standard Deviations.

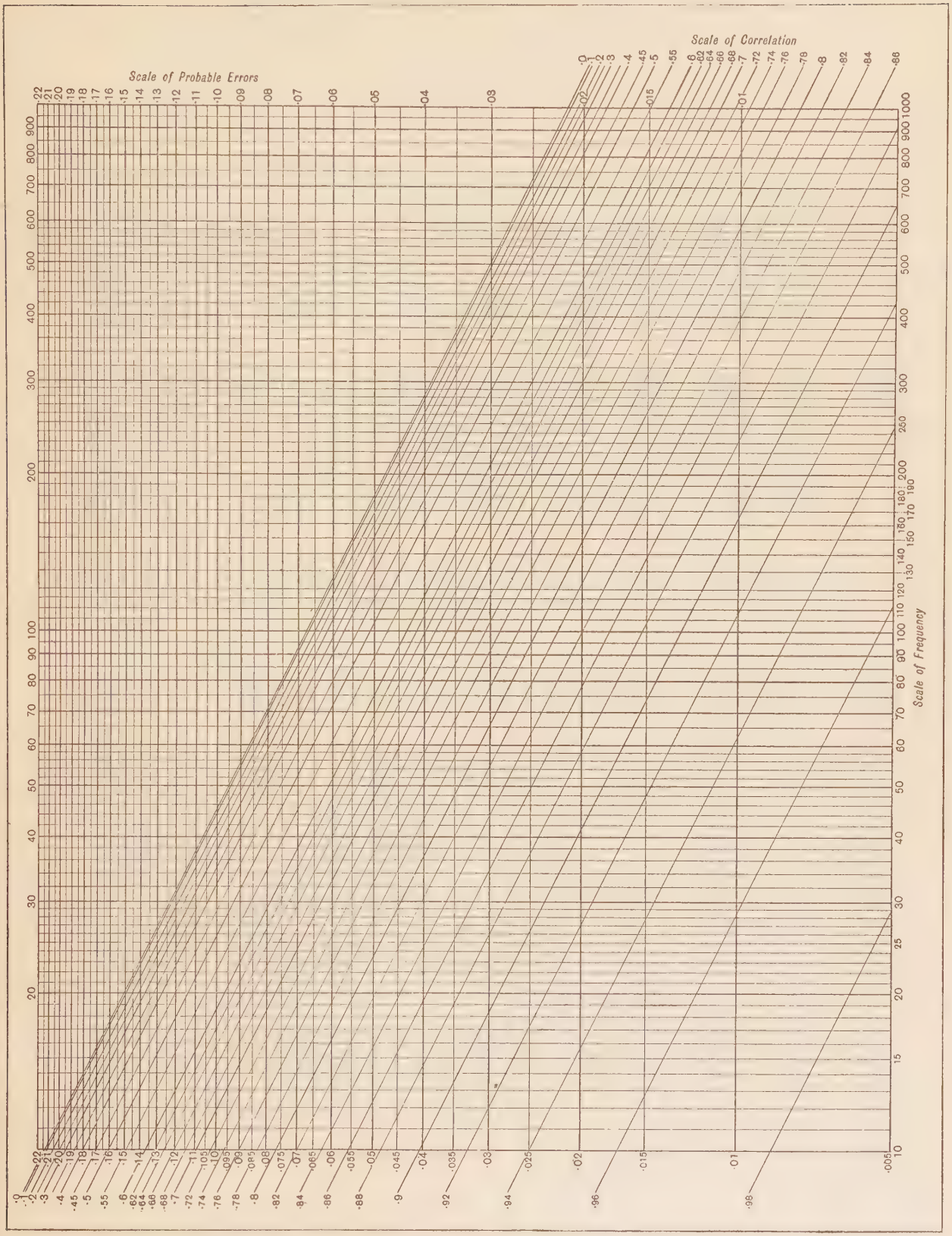
n	χ_1	χ_2	n	χ_1	χ_2
901	·02247	·01589	951	·02187	·01547
902	·02246	·01588	952	·02186	·01546
903	·02245	·01587	953	·02185	·01545
904	·02243	·01586	954	·02184	·01544
905	·02242	·01585	955	·02183	·01543
906	·02241	·01585	956	·02181	·01543
907	·02240	·01584	957	·02180	·01542
908	·02238	·01583	958	·02179	·01541
909	·02237	·01582	959	·02178	·01540
910	·02236	·01581	960	·02177	·01539
911	·02235	·01580	961	·02176	·01539
912	·02233	·01579	962	·02175	·01538
913	·02232	·01578	963	·02174	·01537
914	·02231	·01578	964	·02172	·01536
915	·02230	·01577	965	·02171	·01535
916	·02229	·01576	966	·02170	·01535
917	·02227	·01575	967	·02169	·01534
918	·02226	·01574	968	·02168	·01533
919	·02225	·01573	969	·02167	·01532
920	·02224	·01572	970	·02166	·01531
921	·02223	·01572	971	·02165	·01531
922	·02221	·01571	972	·02163	·01530
923	·02220	·01570	973	·02162	·01529
924	·02219	·01569	974	·02161	·01528
925	·02218	·01568	975	·02160	·01527
926	·02217	·01567	976	·02159	·01527
927	·02215	·01566	977	·02158	·01526
928	·02214	·01566	978	·02157	·01525
929	·02213	·01565	979	·02156	·01524
930	·02212	·01564	980	·02155	·01524
931	·02211	·01563	981	·02153	·01523
932	·02209	·01562	982	·02152	·01522
933	·02208	·01561	983	·02151	·01521
934	·02207	·01561	984	·02150	·01520
935	·02206	·01560	985	·02149	·01520
936	·02205	·01559	986	·02148	·01519
937	·02203	·01558	987	·02147	·01518
938	·02202	·01557	988	·02146	·01517
939	·02201	·01556	989	·02145	·01517
940	·02200	·01556	990	·02144	·01516
941	·02199	·01555	991	·02143	·01515
942	·02198	·01554	992	·02142	·01514
943	·02196	·01553	993	·02140	·01514
944	·02195	·01552	994	·02139	·01513
945	·02194	·01551	995	·02138	·01512
946	·02193	·01551	996	·02137	·01511
947	·02192	·01550	997	·02136	·01510
948	·02191	·01549	998	·02135	·01510
949	·02189	·01548	999	·02134	·01509
950	·02188	·01547	1000	·02133	·01508

TABLE VI.

Probable Errors of Coefficient of Variation.

V	ψ	Δ +	Δ^2 +	Δ^3 +
0	0·00000		0	
		1·00010		60
1	1·00010		60	
		1·00070		60
2	2·00080		120	
		1·00190		60
3	3·00270		180	
		1·00370		60
4	4·00639		239	
		1·00609		59
5	5·01248		299	
		1·00908		59
6	6·02156		358	
		1·01266		59
7	7·03422		417	
		1·01682		58
8	8·05104		475	
		1·02157		58
9	9·07261		533	
		1·02690		57
10	10·09950		590	
		1·03280		57
11	11·13230		647	
		1·03927		56
12	12·17157		703	
		1·04630		56
13	13·21787		759	
		1·05389		55
14	14·27176		814	
		1·06202		54
15	15·33379		868	
		1·07070		53
16	16·40449		921	
		1·07991		53
17	17·48440		974	
		1·08965		52
18	18·57405		1025	
		1·09990		51
19	19·67395		1076	
		1·11066		50
20	20·78461		1126	
		1·12192		49
21	21·90653		1175	
		1·13368		48
22	23·04021		1223	
		1·14591		47
23	24·18612		1270	
		1·15861		46
24	25·34473		1316	
		1·17177		45
25	26·51650		1362	
		1·18539		44
26	27·70190		1406	
		1·19945		43
27	28·90135		1449	
		1·21395		42
28	30·11530		1491	
		1·22886		41
29	31·34416		1533	
		1·24418		40
30	32·58834		1573	
		1·25991		39
31	33·84825		1612	
		1·27603		38
32	35·12428		1650	
		1·29253		37
33	36·41681		1687	
		1·30940		36
34	37·72621		1723	
		1·32664		35
35	39·05285		1758	
		1·34422		34
36	40·39707		1793	
		1·36215		33
37	41·75922		1826	
		1·38041		32
38	43·13962		1858	
		1·39899		31
39	44·53861		1890	
		1·41789		30
40	45·95650		1920	
		1·43709		30
41	47·39359		1950	
		1·45658		29
42	48·85017		1978	
		1·47636		28
43	50·32654		2006	
		1·49642		27
44	51·82296		2033	
		1·51675		26
45	53·33971		2059	
		1·53734		25
46	54·87706		2084	
		1·55818		24
47	56·43524		2109	
		1·57927		24
48	58·01451		2132	
		1·60059		23
49	59·61510		2155	
		1·62214		22
50	61·23724		2177	

TABLE VII. *Abac for Probable Errors of r.*



Abac for determining the Probable Errors of Correlation Coefficients.

TABLE VIII. *Values of* $1-r^2$ *for* $r = \cdot001$ *to* $\cdot999$.

Values of $1-r^2$.

r	·000	·001	·002	·003	·004	·005	·006	·007	·008	·009
·000	1·000 000	·999 999	·999 996	·999 991	·999 984	·999 975	·999 964	·999 951	·999 936	·999 919
·010	·999 900	·999 879	·999 856	·999 831	·999 804	·999 775	·999 744	·999 711	·999 676	·999 639
·020	·999 600	·999 559	·999 516	·999 471	·999 424	·999 375	·999 324	·999 271	·999 216	·999 159
·030	·999 100	·999 039	·998 976	·998 911	·998 844	·998 775	·998 704	·998 631	·998 556	·998 479
·040	·998 400	·998 319	·998 236	·998 151	·998 064	·997 975	·997 884	·997 791	·997 696	·997 599
·050	·997 500	·997 399	·997 296	·997 191	·997 084	·996 975	·996 864	·996 751	·996 636	·996 519
·060	·996 400	·996 279	·996 156	·996 031	·995 904	·995 775	·995 644	·995 511	·995 376	·995 239
·070	·995 100	·994 959	·994 816	·994 671	·994 524	·994 375	·994 224	·994 071	·993 916	·993 759
·080	·993 600	·993 439	·993 276	·993 111	·992 944	·992 775	·992 604	·992 431	·992 256	·992 079
·090	·991 900	·991 719	·991 536	·991 351	·991 164	·990 975	·990 784	·990 591	·990 396	·990 199
·100	·990 000	·989 799	·989 596	·989 391	·989 184	·988 975	·988 764	·988 551	·988 336	·988 119
·110	·987 900	·987 679	·987 456	·987 231	·987 004	·986 775	·986 544	·986 311	·986 076	·985 839
·120	·985 600	·985 359	·985 116	·984 871	·984 624	·984 375	·984 124	·983 871	·983 616	·983 359
·130	·983 100	·982 839	·982 576	·982 311	·982 044	·981 775	·981 504	·981 231	·980 956	·980 679
·140	·980 400	·980 119	·979 836	·979 551	·979 264	·978 975	·978 684	·978 391	·978 096	·977 799
·150	·977 500	·977 199	·976 896	·976 591	·976 284	·975 975	·975 664	·975 351	·975 036	·974 719
·160	·974 400	·974 079	·973 756	·973 431	·973 104	·972 775	·972 444	·972 111	·971 776	·971 439
·170	·971 100	·970 759	·970 416	·970 071	·969 724	·969 375	·969 024	·968 671	·968 316	·967 959
·180	·967 600	·967 239	·966 876	·966 511	·966 144	·965 775	·965 404	·965 031	·964 656	·964 279
·190	·963 900	·963 519	·963 136	·962 751	·962 364	·961 975	·961 584	·961 191	·960 796	·960 399
·200	·960 000	·959 599	·959 196	·958 791	·958 384	·957 975	·957 564	·957 151	·956 736	·956 319
·210	·955 900	·955 479	·955 056	·954 631	·954 204	·953 775	·953 344	·952 911	·952 476	·952 039
·220	·951 600	·951 159	·950 716	·950 271	·949 824	·949 375	·948 924	·948 471	·948 016	·947 559
·230	·947 100	·946 639	·946 176	·945 711	·945 244	·944 775	·944 304	·943 831	·943 356	·942 879
·240	·942 400	·941 919	·941 436	·940 951	·940 464	·939 975	·939 484	·938 991	·938 496	·937 999
·250	·937 500	·936 999	·936 496	·935 991	·935 484	·934 975	·934 464	·933 951	·933 436	·932 919
·260	·932 400	·931 879	·931 356	·930 831	·930 304	·929 775	·929 244	·928 711	·928 176	·927 639
·270	·927 100	·926 559	·926 016	·925 471	·924 924	·924 375	·923 824	·923 271	·922 716	·922 159
·280	·921 600	·921 039	·920 476	·919 911	·919 344	·918 775	·918 204	·917 631	·917 056	·916 479
·290	·915 900	·915 319	·914 736	·914 151	·913 564	·912 975	·912 384	·911 791	·911 196	·910 599
·300	·910 000	·909 399	·908 796	·908 191	·907 584	·906 975	·906 364	·905 751	·905 136	·904 519
·310	·903 900	·903 279	·902 656	·902 031	·901 404	·900 775	·900 144	·899 511	·898 876	·898 239
·320	·897 600	·896 959	·896 316	·895 671	·895 024	·894 375	·893 724	·893 071	·892 416	·891 759
·330	·891 100	·890 439	·889 776	·889 111	·888 444	·887 775	·887 104	·886 431	·885 756	·885 079
·340	·884 400	·883 719	·883 036	·882 351	·881 664	·880 975	·880 284	·879 591	·878 896	·878 199
·350	·877 500	·876 799	·876 096	·875 391	·874 684	·873 975	·873 264	·872 551	·871 836	·871 119
·360	·870 400	·869 679	·868 956	·868 231	·867 504	·866 775	·866 044	·865 311	·864 576	·863 839
·370	·863 100	·862 359	·861 616	·860 871	·860 124	·859 375	·858 624	·857 871	·857 116	·856 359
·380	·855 600	·854 839	·854 076	·853 311	·852 544	·851 775	·851 004	·850 231	·849 456	·848 679
·390	·847 900	·847 119	·846 336	·845 551	·844 764	·843 975	·843 184	·842 391	·841 596	·840 799
·400	·840 000	·839 199	·838 396	·837 591	·836 784	·835 975	·835 164	·834 351	·833 536	·832 719
·410	·831 900	·831 079	·830 256	·829 431	·828 604	·827 775	·826 944	·826 111	·825 276	·824 439
·420	·823 600	·822 759	·821 916	·821 071	·820 224	·819 375	·818 524	·817 671	·816 816	·815 959
·430	·815 100	·814 239	·813 376	·812 511	·811 644	·810 775	·809 904	·809 031	·808 156	·807 279
·440	·806 400	·805 519	·804 636	·803 751	·802 864	·801 975	·801 084	·800 191	·799 296	·798 399
·450	·797 500	·796 599	·795 696	·794 791	·793 884	·792 975	·792 064	·791 151	·790 236	·789 319
·460	·788 400	·787 479	·786 556	·785 631	·784 704	·783 775	·782 844	·781 911	·780 976	·780 039
·470	·779 100	·778 159	·777 216	·776 271	·775 324	·774 375	·773 424	·772 471	·771 516	·770 559
·480	·769 600	·768 639	·767 676	·766 711	·765 744	·764 775	·763 804	·762 831	·761 856	·760 879
·490	·759 900	·758 919	·757 936	·756 951	·755 964	·754 975	·753 984	·752 991	·751 996	·750 999

TABLE VIII. *Values of* $1-r^2$ *for* $r = \cdot001$ *to* $\cdot999$.

Values of $1-r^2$.

r	·000	·001	·002	·003	·004	·005	·006	·007	·008	·009
·500	·750 000	·748 999	·747 996	·746 991	·745 984	·744 975	·743 964	·742 951	·741 936	·740 919
·510	·739 900	·738 879	·737 856	·736 831	·735 804	·734 775	·733 744	·732 711	·731 676	·730 639
·520	·729 600	·728 559	·727 516	·726 471	·725 424	·724 375	·723 324	·722 271	·721 216	·720 159
·530	·719 100	·718 039	·716 976	·715 911	·714 844	·713 775	·712 704	·711 631	·710 556	·709 479
·540	·708 400	·707 319	·706 236	·705 151	·704 064	·702 975	·701 884	·700 791	·699 696	·698 599
·550	·697 500	·696 399	·695 296	·694 191	·693 084	·691 975	·690 864	·689 751	·688 636	·687 519
·560	·686 400	·685 279	·684 156	·683 031	·681 904	·680 775	·679 644	·678 511	·677 376	·676 239
·570	·675 100	·673 959	·672 816	·671 671	·670 524	·669 375	·668 224	·667 071	·665 916	·664 759
·580	·663 600	·662 439	·661 276	·660 111	·658 944	·657 775	·656 604	·655 431	·654 256	·653 079
·590	·651 900	·650 719	·649 536	·648 351	·647 164	·645 975	·644 784	·643 591	·642 396	·641 199
·600	·640 000	·638 799	·637 596	·636 391	·635 184	·633 975	·632 764	·631 551	·630 336	·629 119
·610	·627 900	·626 679	·625 456	·624 231	·623 004	·621·775	·620 544	·619 311	·618 076	·616 839
·620	·615 600	·614 359	·613 116	·611 871	·610 624	·609 375	·608 124	·606 871	·605 616	·604 359
·630	·603 100	·601 839	·600 576	·599 311	·598 044	·596 775	·595 504	·594 231	·592 956	·591 679
·640	·590 400	·589 119	·587 836	·586 551	·585 264	·583 975	·582 684	·581 391	·580 096	·578 799
·650	·577 500	·576 199	·574 896	·573 591	·572 284	·570 975	·569 664	·568 351	·567 036	·565 719
·660	·564 400	·563 079	·561 756	·560 431	·559 104	·557 775	·556 444	·555 111	·553 776	·552 439
·670	·551 100	·549 759	·548 416	·547 071	·545 724	·544 375	·543 024	·541 671	·540 316	·538 959
·680	·537 600	·536 239	·534 876	·533 511	·532 144	·530 775	·529 404	·528 031	·526 656	·525 279
·690	·523 900	·522 519	·521 136	·519 751	·518 364	·516 975	·515 584	·514 191	·512 796	·511 399
·700	·510 000	·508 599	·507 196	·505 791	·504 384	·502 975	·501 564	·500 151	·498 736	·497 319
·710	·495 900	·494 479	·493 056	·491 631	·490 204	·488 775	·487 344	·485 911	·484 476	·483 039
·720	·481 600	·480 159	·478 716	·477 271	·475 824	·474 375	·472 924	·471 471	·470 016	·468 559
·730	·467 100	·465 639	·464 176	·462 711	·461 244	·459 775	·458 304	·456 831	·455 356	·453 879
·740	·452 400	·450 919	·449 436	·447 951	·446 464	·444 975	·443 484	·441 991	·440 496	·438 999
·750	·437 500	·435 999	·434 496	·432 991	·431 484	·429 975	·428 464	·426 951	·425 436	·423 919
·760	·422 400	·420 879	·419 356	·417 831	·416 304	·414 775	·413 244	·411 711	·410 176	·408 639
·770	·407 100	·405 559	·404 016	·402 471	·400 924	·399 375	·397 824	·396 271	·394 716	·393 159
·780	·391 600	·390 039	·388 476	·386 911	·385 344	·383 775	·382 204	·380 631	·379 056	·377 479
·790	·375 900	·374 319	·372 736	·371 151	·369 564	·367 975	·366 384	·364 791	·363 196	·361 599
·800	·360 000	·358 399	·356 796	·355 191	·353 584	·351 975	·350 364	·348 751	·347 136	·345 519
·810	·343 900	·342 279	·340 656	·339 031	·337 404	·335 775	·334 144	·332 511	·330 876	·329 239
·820	·327 600	·325 959	·324 316	·322 671	·321 024	·319 375	·317 724	·316 071	·314 416	·312 759
·830	·311 100	·309 439	·307 776	·306 111	·304 444	·302 775	·301 104	·299 431	·297 756	·296 079
·840	·294 400	·292 719	·291 036	·289 351	·287 664	·285 975	·284 284	·282 591	·280 896	·279 199
·850	·277 500	·275 799	·274 096	·272 391	·270 684	·268 975	·267 264	·265 551	·263 836	·262 119
·860	·260 400	·258 679	·256 956	·255 231	·253 504	·251 775	·250 044	·248 311	·246 576	·244 839
·870	·243 100	·241 359	·239 616	·237 871	·236 124	·234 375	·232 624	·230 871	·229 116	·227 359
·880	·225 600	·223 839	·222 076	·220 311	·218 544	·216 775	·215 004	·213 231	·211 456	·209 679
·890	·207 900	·206 119	·204 336	·202 551	·200 764	·198 975	·197 184	·195 391	·193 596	·191 799
·900	·190 000	·188 199	·186 396	·184 591	·182 784	·180 975	·179 164	·177 351	·175 536	·173 719
·910	·171 900	·170 079	·168 256	·166 431	·164 604	·162 775	·160 944	·159 111	·157 276	·155 439
·920	·153 600	·151 759	·149 916	·148 071	·146 224	·144 375	·142 524	·140 671	·138 816	·136 959
·930	·135 100	·133 239	·131 376	·129 511	·127 644	·125 775	·123 904	·122 031	·120 156	·118 279
·940	·116 400	·114 519	·112 636	·110 751	·108 864	·106 975	·105 084	·103 191	·101 296	·099 399
·950	·097 500	·095 599	·093 696	·091 791	·089 884	·087 975	·086 064	·084 151	·082 236	·080 319
·960	·078 400	·076 479	·074 556	·072 631	·070 704	·068 775	·066 844	·064 911	·062 976	·061 039
·970	·059 100	·057 159	·055 216	·053 271	·051 324	·049 375	·047 424	·045 471	·043 516	·041 559
·980	·039 600	·037 639	·035 676	·033 711	·031 744	·029 775	·027 804	·025 831	·023 856	·021 879
·990	019 900	·017 919	·015 936	·013 951	·011 964	·009 975	·007 984	·005 991	·003 996	·001 999

TABLE IX. *Values of the Incomplete Normal Moment Function* $\mu_n(x)$.

A. *Odd Moments* $m_n(x) = \mu_n(x)/\{(n-1)(n-3)(n-5)\ldots 2\}$.

x	$m_1(x)$	$m_3(x)$	$m_5(x)$	$m_7(x)$	$m_9(x)$
0·0	·0000000	·0000000	·0000000	·0000000	·0000000
0·1	·0019897	·0000050	·0000000	·0000000	·0000000
0·2	·0078996	·0000787	·0000005	·0000000	·0000000
0·3	·0175545	·0003920	·0000059	·0000001	·0000000
0·4	·0306721	·0012105	·0000321	·0000006	·0000000
0·5	·0468770	·0028688	·0001183	·0000037	·0000001
0·6	·0657177	·0057372	·0003390	·0000151	·0000005
0·7	·0866883	·0101861	·0008146	·0000493	·0000024
0·8	·1092507	·0165494	·0017172	·0001350	·0000086
0·9	·1328570	·0250925	·0032702	·0003242	·0000259
1·0	·1569716	·0359862	·0057399	·0006988	·0000687
1·1	·1810901	·0492895	·0094199	·0013795	·0001634
1·2	·2047562	·0649423	·0146092	·0025293	·0003549
1·3	·2275737	·0827672	·0215865	·0043539	·0007135
1·4	·2492148	·1024819	·0305828	·0070957	·0013414
1·5	·2694247	·1237174	·0417570	·0110219	·0023776
1·6	·2880214	·1460428	·0551764	·0164068	·0040005
1·7	·3048932	·1689923	·0708039	·0235098	·0064248
1·8	·3199921	·1920929	·0884945	·0325513	·0098944
1·9	·3333265	·2148899	·1080009	·0436894	·0146688
2·0	·3449513	·2369694	·1289874	·0569995	·0210055
2·1	·3549587	·2579749	·1510502	·0724606	·0291380
2·2	·3634677	·2776192	·1737425	·0899486	·0392533
2·3	·3706152	·2956902	·1966019	·1092390	·0514703
2·4	·3765478	·3120515	·2191769	·1300173	·0658224
2·5	·3814140	·3266380	·2410506	·1518971	·0822459
2·6	·3853593	·3394489	·2618602	·1744437	·1005767
2·7	·3885213	·3505370	·2813106	·1972006	·1205553
2·8	·3910268	·3599983	·2991823	·2197160	·1418391
2·9	·3929897	·3679593	·3153329	·2415682	·1640231
3·0	·3945104	·3745671	·3296916	·2623860	·1866637
3·1	·3956755	·3799784	·3422662	·2818638	·2093055
3·2	·3965582	·3843517	·3531029	·2997718	·2315079
3·3	·3972197	·3878403	·3623049	·3159582	·2528687
3·4	·3977101	·3905878	·3700046	·3303476	·2730432
3·5	·3980696	·3927244	·3763548	·3429335	·2917571
3·6	·3983304	·3943653	·3815183	·3537687	·3088145
3·7	·3985175	·3956099	·3856585	·3629529	·3240979
3·8	·3986503	·3965425	·3889331	·3706199	·3375646
3·9	·3987436	·3972329	·3914881	·3769253	·3492376
4·0	·3988085	·3977378	·3934552	·3820351	·3591947
4·1	·3988530	·3981028	·3949499	·3861165	·3675554
4·2	·3988833	·3983635	·3960708	·3893304	·3744677
4·3	·3989037	·3985475	·3969007	·3918258	·3800964
4·4	·3989173	·3986759	·3975073	·3937367	·3846117
4·5	·3989263	·3987645	·3979452	·3951801	·3881809
4·6	·3989321	·3988248	·3982573	·3962557	·3909614
4·7	·3989359	·3988656	·3984770	·3970466	·3930967
4·8	·3989383	·3988927	·3986298	·3976205	·3947135
4·9	·3989398	·3989106	·3987348	·3980315	·3959207
5·0	·3989408	·3989222	·3988061	·3983221	·3968097
∞	·3989423	·3989423	·3989423	·3989423	·3989423

TABLE IX. *Values of the Incomplete Normal Moment Function.*

B. *Even Moments* $m_n(x) = \mu_n(x)/\{(n-1)(n-3)(n-5)\ldots 1\}$.

x	$m_2(x)$	$m_4(x)$	$m_6(x)$	$m_8(x)$	$m_{10}(x)$
0·0	·0000000	·0000000	·0000000	·0000000	·0000000
0·1	·0001325	·0000002	·0000000	·0000000	·0000000
0·2	·0010512	·0000084	·0000000	·0000000	·0000000
0·3	·0034951	·0000626	·0000008	·0000000	·0000000
0·4	·0081136	·0002572	·0000058	·0000001	·0000000
0·5	·0154298	·0007604	·0000270	·0000008	·0000001
0·6	·0258121	·0018200	·0000925	·0000037	·0000001
0·7	·0394585	·0037575	·0002588	·0000139	·0000006
0·8	·0563914	·0069507	·0006223	·0000437	·0000025
0·9	·0764632	·0118045	·0013297	·0001177	·0000086
1·0	·0993740	·0187171	·0025857	·0002812	·0000251
1·1	·1246965	·0280428	·0046525	·0006094	·0000658
1·2	·1519070	·0400559	·0078427	·0012160	·0001558
1·3	·1804203	·0549214	·0125028	·0022617	·0003386
1·4	·2096248	·0726741	·0189894	·0039577	·0006842
1·5	·2389164	·0932091	·0276408	·0065653	·0012964
1·6	·2677274	·1162835	·0387442	·0103869	·0023209
1·7	·2955511	·1415300	·0525059	·0157516	·0039494
1·8	·3219594	·1684803	·0690258	·0229926	·0064207
1·9	·3466134	·1965937	·0882796	·0324204	·0100147
2·0	·3692680	·2252921	·1101113	·0442938	·0150415
2·1	·3897700	·2539927	·1342371	·0587910	·0218224
2·2	·4080525	·2821413	·1602593	·0759866	·0306667
2·3	·4241237	·3092387	·1876903	·0958345	·0418437
2·4	·4380556	·3348616	·2159821	·1181613	·0555560
2·5	·4499695	·3586763	·2445598	·1426700	·0719132
2·6	·4600231	·3804450	·2728554	·1689546	·0909136
2·7	·4683965	·4000247	·3003387	·1965228	·1124320
2·8	·4752816	·4173616	·3265431	·2248263	·1362197
2·9	·4808719	·4324798	·3510842	·2532933	·1619132
3·0	·4853546	·4454679	·3736720	·2813629	·1890538
3·1	·4889053	·4564647	·3941138	·3085150	·2171145
3·2	·4916838	·4656432	·4123121	·3342962	·2455315
3·3	·4938321	·4731975	·4282552	·3583379	·2737379
3·4	·4954736	·4793298	·4420056	·3803672	·3011962
3·5	·4967130	·4842409	·4536843	·4002102	·3274261
3·6	·4976381	·4881218	·4634555	·4177877	·3520261
3·7	·4983205	·4911484	·4715111	·4331061	·3746880
3·8	·4988183	·4934784	·4780568	·4462441	·3952025
3·9	·4991771	·4952491	·4833001	·4573366	·4134583
4·0	·4994330	·4965779	·4874418	·4665592	·4294345
4·1	·4996133	·4975627	·4906683	·4741120	·4431886
4·2	·4997391	·4982835	·4931479	·4802063	·4548407
4·3	·4998258	·4988045	·4950279	·4850521	·4645574
4·4	·4998849	·4991766	·4964343	·4888500	·4725352
4·5	·4999247	·4994392	·4974729	·4917846	·4789861
4·6	·4999512	·4996222	·4982298	·4940207	·4841246
4·7	·4999688	·4997483	·4987744	·4957010	·4881574
4·8	·4999802	·4998342	·4991613	·4969464	·4912765
4·9	·4999876	·4998919	·4994326	·4978572	·4936544
5·0	·4999923	·4999303	·4996206	·4985144	·4954417
∞	·5000000	·5000000	·5000000	·5000000	·5000000

TABLE X. *Diagram of Generalised 'Probable Error.'*

Table of Generalised 'Probable Errors.'

Number of Variables	Probable Error
1	0·674,4898
2	1·177,4062
3	1·538,1667
4	1·832,1239
5	2·086,0146
6	2·312,5982
7	2·519,0869
8	2·710,0022
9	2·888,3962
10	3·056,4366
11	3·215,7402

Diagram for Value of Probable Error for *n* Variables.

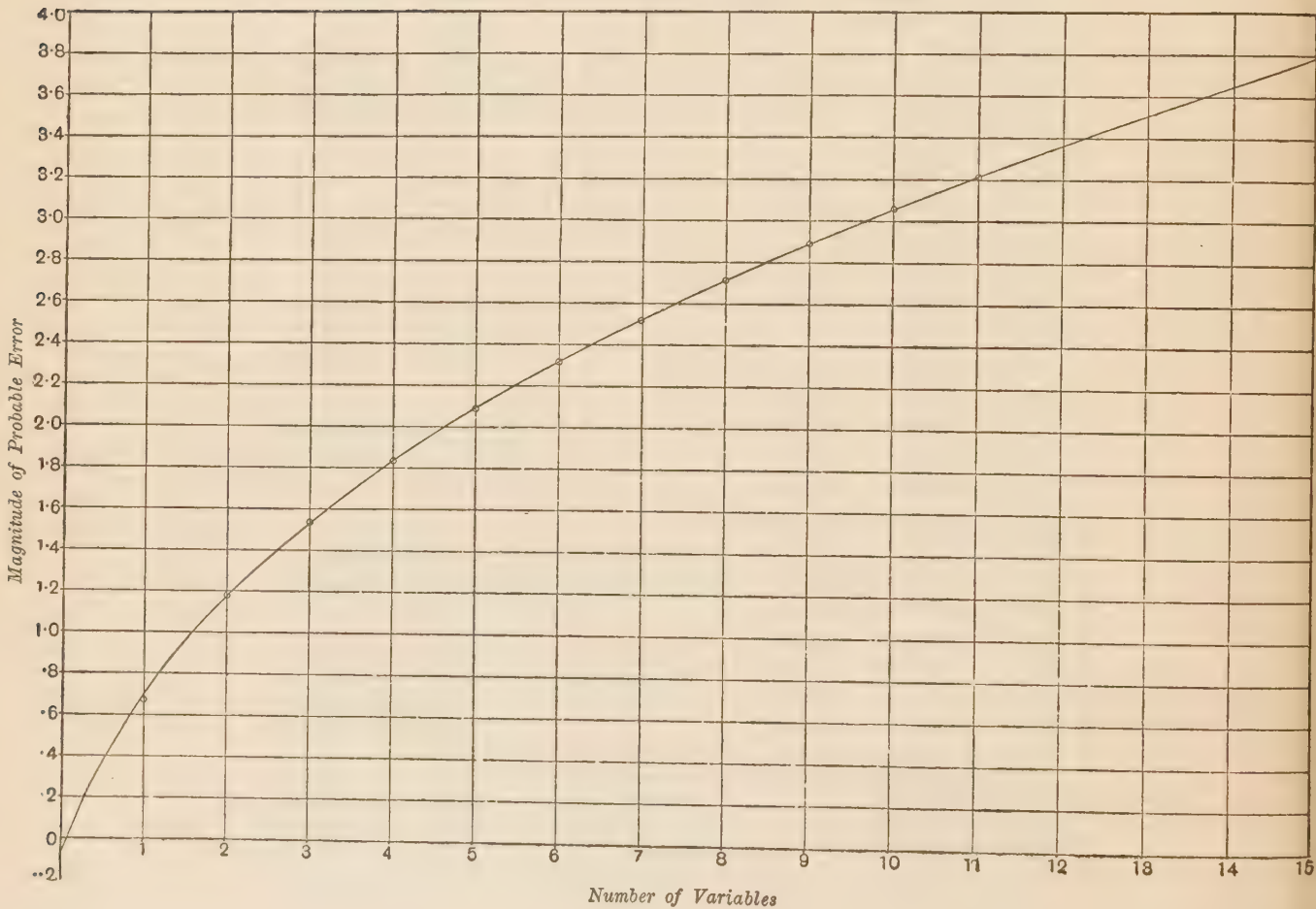


TABLE XI. *Constants of Normal Curve from Moments of Tail about Stump.*

Values of the Functions ψ_1 and ψ_2 required to determine the Constants of a Normal Frequency Distribution from the Moments of its Truncated Tail.

h'	ψ_1	ψ_2	ψ_3	h'	ψ_1	ψ_2	ψ_3
0·00	·571	1·253	2·000	1·1	·734	1·977	7·371
0·01	·573	1·259	2·016	1·2	·746	2·051	8·690
0·02	·574	1·265	2·032	1·3	·757	2·126	10·331
0·03	·576	1·271	2·049	1·4	·767	2·202	12·383
0·04	·578	1·276	2·066	1·5	·777	2·280	14·968
0·05	·580	1·282	2·083	1·6	·787	2·358	18·248
0·06	·581	1·288	2·100	1·7	·796	2·437	22·439
0·07	·583	1·294	2·118	1·8	·804	2·517	27·832
0·08	·585	1·300	2·136	1·9	·813	2·598	34·823
0·09	·587	1·305	2·155	2·0	·820	2·679	43·956
0·1	·588	1·311	2·173	2·1	·828	2·762	55·977
0·2	·605	1·371	2·377	2·2	·835	2·845	71·925
0·3	·622	1·432	2·617	2·3	·842	2·929	93·248
0·4	·638	1·495	2·902	2·4	·848	3·013	121·988
0·5	·653	1·560	3·241	2·5	·854	3·098	161·038
0·6	·668	1·626	3·646	2·6	·860	3·184	214·537
0·7	·682	1·693	4·133	2·7	·866	3·270	288·434
0·8	·696	1·762	4·720	2·8	·871	3·357	391·374
0·9	·709	1·833	5·433	2·9	·876	3·445	535·963
1·0	·722	1·904	6·303	3·0	·880	3·532	740·796
1·1	·734	1·977	7·371	3·5	·888	3·969	[4299·226]

Let d equal distance of centroid of tail from stump, Σ = standard deviation of the tail about its mean, and n = its area.

(i) Find ψ_1 from $\psi_1 = \Sigma^2/d^2$. Hence from table determine h'.

(ii) From this value of h' find ψ_2, then $\sigma = d \times \psi_2$ gives the standard deviation of the uncurtailed normal curve.

(iii) $h = h' \times \sigma$ gives the origin of the uncurtailed normal curve.

(iv) Knowing h', Table II gives $\frac{1}{2}(1+\alpha)$ and therefore the ratio $\frac{1}{2}(1-\alpha)$ of tail to total area of curve N, or $N = n/\frac{1}{2}(1-\alpha)$. For many purposes it is sufficient to use $N = n \times \psi_3$.

TABLE XII. *Test for Goodness of Fit. Values of P.*

χ^2	$n'=3$	$n'=4$	$n'=5$	$n'=6$	$n'=7$	$n'=8$	$n'=9$	$n'=10$	$n'=11$
1	·606531	·801253	·909796	·962566	·985612	·994829	·998249	·999438	·999828
2	·367879	·572407	·735759	·849146	·919699	·959840	·981012	·991468	·996340
3	·223130	·391625	·557825	·699986	·808847	·885002	·934357	·964295	·981424
4	·135335	·261464	·406006	·549416	·676676	·779778	·857123	·911413	·947347
5	·082085	·171797	·287298	·415880	·543813	·659963	·757576	·834308	·891178
6	·049787	·111610	·199148	·306219	·423190	·539750	·647232	·739919	·815263
7	·030197	·071897	·135888	·220640	·320847	·428880	·536632	·637119	·725444
8	·018316	·046012	·091578	·156236	·238103	·332594	·433470	·534146	·628837
9	·011109	·029291	·061099	·109064	·173578	·252656	·342296	·437274	·532104
10	·006738	·018566	·040428	·075235	·124652	·188573	·265026	·350485	·440493
11	·004087	·011726	·026564	·051380	·088376	·138619	·201699	·275709	·357518
12	·002479	·007383	·017351	·034787	·061969	·100558	·151204	·213308	·285057
13	·001503	·004637	·011276	·023379	·043036	·072109	·111850	·162607	·223672
14	·000912	·002905	·007295	·015609	·029636	·051181	·081765	·122325	·172992
15	·000553	·001817	·004701	·010363	·020256	·036000	·059145	·090937	·132061
16	·000335	·001134	·003019	·006844	·013754	·025116	·042380	·066881	·099632
17	·000203	·000707	·001933	·004500	·009283	·017396	·030109	·048716	·074364
18	·000123	·000440	·001234	·002947	·006232	·011970	·021226	·035174	·054964
19	·000075	·000273	·000786	·001922	·004164	·008187	·014860	·025193	·040263
20	·000045	·000170	·000499	·001250	·002769	·005570	·010336	·017913	·029253
21	·000028	·000105	·000317	·000810	·001835	·003770	·007147	·012650	·021093
22	·000017	·000065	·000200	·000524	·001211	·002541	·004916	·008880	·015105
23	·000010	·000040	·000127	·000338	·000796	·001705	·003364	·006197	·010747
24	·000006	·000025	·000080	·000217	·000522	·001139	·002292	·004301	·007600
25	·000004	·000016	·000050	·000139	·000341	·000759	·001554	·002971	·005345
26	·000002	·000010	·000032	·000090	·000223	·000504	·001050	·002043	·003740
27	·000001	·000006	·000020	·000057	·000145	·000333	·000707	·001399	·002604
28	·000001	·000004	·000012	·000037	·000094	·000220	·000474	·000954	·001805
29	·000001	·000002	·000008	·000023	·000061	·000145	·000317	·000648	·001246
30	·000000	·000001	·000005	·000015	·000039	·000095	·000211	·000439	·000857
40	·000000	·000000	·000000	·000000	·000001	·000001	·000003	·000008	·000017
50	·000000	·000000	·000000	·000000	·000000	·000000	·000000	·000000	·000000
60	·000000	·000000	·000000	·000000	·000000	·000000	·000000	·000000	·000000
70	·000000	·000000	·000000	·000000	·000000	·000000	·000000	·000000	·000000

TABLE XII.—(*continued*).

χ^2	$n'=12$	$n'=13$	$n'=14$	$n'=15$	$n'=16$	$n'=17$	$n'=18$	$n'=19$	$n'=20$
1	·999950	·999986	·999997	·999999	1·	1·	1·	1·	1·
2	·998496	·999406	·999774	·999917	·999970	·999990	·999997	·999999	1·
3	·990726	·995544	·997934	·999074	·999598	·999830	·999931	·999972	·999989
4	·969917	·983436	·991191	·995466	·997737	·998903	·999483	·999763	·999894
5	·931167	·957979	·975193	·985813	·992127	·995754	·997771	·998860	·999431
6	·873365	·916082	·946153	·966491	·979749	·988095	·993187	·996197	·997929
7	·799073	·857613	·902151	·934711	·957650	·973260	·983549	·990125	·994213
8	·713304	·785131	·843601	·889327	·923783	·948867	·966547	·978637	·986671
9	·621892	·702931	·772943	·831051	·877517	·913414	·940261	·959743	·973479
10	·530387	·615960	·693934	·762183	·819739	·866628	·903610	·931906	·952946
11	·443263	·528919	·610817	·686036	·752594	·809485	·856564	·894357	·923839
12	·362642	·445680	·527643	·606303	·679028	·743980	·800136	·847237	·885624
13	·293326	·369041	·447812	·526524	·602298	·672758	·736186	·791573	·838571
14	·232993	·300708	·373844	·449711	·525529	·598714	·667102	·729091	·783691
15	·182498	·241436	·307354	·378154	·451418	·524638	·595482	·661967	·722598
16	·141130	·191236	·249129	·313374	·382051	·452961	·523834	·592547	·657277
17	·107876	·149597	·199304	·256178	·318864	·385597	·454366	·523105	·589868
18	·081581	·115691	·157520	·206781	·262666	·323897	·388841	·455653	·522438
19	·061094	·088529	·123104	·164949	·213734	·268663	·328532	·391823	·456836
20	·045341	·067086	·095210	·130141	·171932	·220220	·274229	·332819	·394578
21	·033371	·050380	·072929	·101632	·136830	·178510	·226291	·279413	·336801
22	·024374	·037520	·055362	·078614	·107804	·143191	·184719	·231985	·284256
23	·017676	·027726	·041677	·060270	·084140	·113735	·149251	·190590	·237342
24	·012733	·020341	·031130	·045822	·065093	·089504	·119435	·155028	·196152
25	·009117	·014822	·023084	·034566	·049943	·069824	·094710	·124915	·160542
26	·006490	·010734	·017001	·025887	·038023	·054028	·074461	·099758	·130189
27	·004595	·007727	·012441	·019254	·028736	·041483	·058068	·078995	·104653
28	·003238	·005532	·009050	·014228	·021569	·031620	·044938	·062055	·083428
29	·002270	·003940	·006546	·010450	·016085	·023936	·034526	·048379	·065985
30	·001585	·002792	·004710	·007632	·011921	·018002	·026345	·037446	·051798
40	·000036	·000072	·000138	·000255	·000453	·000778	·001294	·002087	·003272
50	·000001	·000001	·000003	·000006	·000012	·000023	·000042	·000075	·000131
60	·000000	·000000	·000000	·000000	·000000	·000001	·000001	·000002	·000004
70	·000000	·000000	·000000	·000000	·000000	·000000	·000000	·000000	·000000

TABLE XII. *Test for Goodness of Fit. Values of* **P.**

χ^2	$n'=21$	$n'=22$	$n'=23$	$n'=24$	$n'=25$	$n'=26$	$n'=27$	$n'=28$	$n'=29$	$n'=30$
1	1·	1·	1·	1·	1·	1·	1·	1·	1·	1·
2	1·	1·	1·	1·	1·	1·	1·	1·	1·	1·
3	·999996	·999998	·999999	1·	1·	1·	1·	1·	1·	1·
4	·999954	·999980	·999992	·999997	·999999	1·	1·	1·	1·	1·
5	·999722	·999868	·999939	·999972	·999987	·999994	·999998	·999999	1·	1·
6	·998898	·999427	·999708	·999855	·999929	·999966	·999984	·999993	·999997	·999999
7	·996685	·998142	·998980	·999452	·999711	·999851	·999924	·999962	·999981	·999991
8	·991868	·995143	·997160	·998371	·999085	·999494	·999726	·999853	·999924	·999960
9	·982907	·989214	·993331	·995957	·997595	·998596	·999194	·999546	·999748	·999863
10	·968171	·978912	·986304	·991277	·994547	·996653	·997981	·998803	·999302	·999599
11	·946223	·962787	·974749	·983189	·989012	·992946	·995549	·997239	·998315	·998988
12	·916076	·939617	·957379	·970470	·979908	·986567	·991173	·994294	·996372	·997728
13	·877384	·908624	·933161	·951990	·966121	·976501	·983974	·989247	·992900	·995384
14	·830496	·869599	·901479	·926871	·946650	·961732	·973000	·981254	·987189	·991377
15	·776408	·822952	862238	·894634	·920759	·941383	·957334	·969432	·978436	·985015
16	·716624	·769650	·815886	·855268	·888076	·914828	·936203	·952947	·965819	·975536
17	·652974	·711106	·763362	·809251	·848662	·881793	·909083	·931122	·948589	·962181
18	·587408	·649004	·705988	·757489	·803008	·842390	·875773	·903519	·926149	·944272
19	·521826	·585140	·645328	·701224	·751990	·797120	·836430	·870001	·898136	·921288
20	·457930	·521261	·583040	·641912	·696776	·746825	·791556	·830756	·864464	·892927
21	·397132	·458944	·520738	·581087	·638725	·692609	·741964	·786288	·825349	·859149
22	·340511	·399510	·459889	·520252	·579267	·635744	·688697	·737377	·781291	·820189
23	·288795	·343979	·401730	·460771	·519798	·577564	·632947	·685013	·733041	·776543
24	·242392	·293058	·347229	·403808	·461597	·519373	·575965	·630316	·681535	·728932
25	·201431	·247164	·297075	·350285	405760	·462373	·518975	·574462	·627835	·678248
26	·165812	·206449	·251682	·300866	·353165	·407598	·463105	·518600	·573045	·625491
27	·135264	·170853	·211226	·255967	·304453	·355884	·409333	·463794	·518247	·571705
28	·109399	·140151	·175681	·215781	·260040	·307853	·358458	·410973	·464447	·517913
29	·087759	·114002	·144861	·180310	·220131	·263916	·311082	·360899	·412528	·465066
30	·069854	·091988	·118464	·149402	·184752	·224289	·267611	·314154	·363218	·414004
40	·004995	·007437	·010812	·015369	·021387	·029164	·039012	·051237	·066128	·083937
50	·000221	·000365	·000586	·000921	·001416	·002131	·003144	·004551	·006467	·009032
60	·000007	·000013	·000022	·000038	·000064	·000104	·000168	·000264	·000407	·000618
70	·000000	·000000	·000001	·000001	·000002	·000004	·000007	·000011	·000019	·000030

TABLE XIII. *Auxiliary Table* A.

χ^2	$\log\left\{\chi\sqrt{\frac{2}{\pi}}e^{-\frac{1}{2}\chi^2}\right\}$	$\log e^{-\frac{1}{2}\chi^2}$	χ^2	$\log\left\{\chi\sqrt{\frac{2}{\pi}}e^{-\frac{1}{2}\chi^2}\right\}$	$\log e^{-\frac{1}{2}\chi^2}$
1	$\bar{1}\cdot68479282$	$\bar{1}\cdot78285276$	*51*	$\bar{11}\cdot68121586$	$\bar{12}\cdot92549071$
2	$\bar{1}\cdot61816058$	$\bar{1}\cdot56570552$	*52*	$\bar{11}\cdot46828520$	$\bar{12}\cdot70834347$
3	$\bar{1}\cdot48905897$	$\bar{1}\cdot34855828$	*53*	$\bar{11}\cdot25527422$	$\bar{12}\cdot49119623$
4	$\bar{1}\cdot33438109$	$\bar{1}\cdot13141104$	*54*	$\bar{11}\cdot04218593$	$\bar{12}\cdot27404899$
5	$\bar{1}\cdot16568886$	$\bar{2}\cdot91426380$	*55*	$\bar{12}\cdot82902315$	$\bar{12}\cdot05690175$
6	$\bar{2}\cdot98813224$	$\bar{2}\cdot69711655$	*56*	$\bar{12}\cdot61578858$	$\bar{13}\cdot83975450$
7	$\bar{2}\cdot80445839$	$\bar{2}\cdot47996931$	*57*	$\bar{12}\cdot40248475$	$\bar{13}\cdot62260726$
8	$\bar{2}\cdot61630713$	$\bar{2}\cdot26282207$	*58*	$\bar{12}\cdot18911408$	$\bar{13}\cdot40546002$
9	$\bar{2}\cdot42473615$	$\bar{2}\cdot04567483$	*59*	$\bar{13}\cdot97567885$	$\bar{13}\cdot18831278$
10	$\bar{2}\cdot23046765$	$\bar{3}\cdot82852759$	*60*	$\bar{13}\cdot76218123$	$\bar{14}\cdot97116554$
11	$\bar{2}\cdot03401675$	$\bar{3}\cdot61138035$	*61*	$\bar{13}\cdot54862328$	$\bar{14}\cdot75401830$
12	$\bar{3}\cdot83576379$	$\bar{3}\cdot39423311$	*62*	$\bar{13}\cdot33500696$	$\bar{14}\cdot53687106$
13	$\bar{3}\cdot63599760$	$\bar{3}\cdot17708587$	*63*	$\bar{13}\cdot12133415$	$\bar{14}\cdot31972382$
14	$\bar{3}\cdot43494271$	$\bar{4}\cdot95993863$	*64*	$\bar{14}\cdot90760662$	$\bar{14}\cdot10257658$
15	$\bar{3}\cdot23277708$	$\bar{4}\cdot74279139$	*65*	$\bar{14}\cdot69382607$	$\bar{15}\cdot88542934$
16	$\bar{3}\cdot02964420$	$\bar{4}\cdot52564414$	*66*	$\bar{14}\cdot48099412$	$\bar{15}\cdot66828209$
17	$\bar{4}\cdot82566143$	$\bar{4}\cdot30849690$	*67*	$\bar{14}\cdot26611232$	$\bar{15}\cdot45113485$
18	$\bar{4}\cdot62092598$	$\bar{4}\cdot09134966$	*68*	$\bar{14}\cdot05218213$	$\bar{15}\cdot23398761$
19	$\bar{4}\cdot41551928$	$\bar{5}\cdot87420242$	*69*	$\bar{15}\cdot83820498$	$\bar{15}\cdot01684037$
20	$\bar{4}\cdot20951024$	$\bar{5}\cdot65705518$	*70*	$\bar{15}\cdot62418221$	$\bar{16}\cdot79969313$
21	$\bar{4}\cdot00295765$	$\bar{5}\cdot43990794$	*71*	$\bar{15}\cdot41011512$	$\bar{16}\cdot58254589$
22	$\bar{5}\cdot79591210$	$\bar{5}\cdot22276070$	*72*	$\bar{15}\cdot19600496$	$\bar{16}\cdot36539865$
23	$\bar{5}\cdot58841744$	$\bar{5}\cdot00561346$	*73*	$\bar{16}\cdot98185290$	$\bar{16}\cdot14825141$
24	$\bar{5}\cdot38051190$	$\bar{6}\cdot78846622$	*74*	$\bar{16}\cdot76766009$	$\bar{17}\cdot93110417$
25	$\bar{5}\cdot17222904$	$\bar{6}\cdot57131898$	*75*	$\bar{16}\cdot55342762$	$\bar{17}\cdot71395693$
26	$\bar{6}\cdot96359847$	$\bar{6}\cdot35417173$	*76*	$\bar{16}\cdot33915654$	$\bar{17}\cdot49680968$
27	$\bar{6}\cdot75464644$	$\bar{6}\cdot13702449$	*77*	$\bar{16}\cdot12484787$	$\bar{17}\cdot27966244$
28	$\bar{6}\cdot54539633$	$\bar{7}\cdot91987725$	*78*	$\bar{17}\cdot91050256$	$\bar{17}\cdot06251520$
29	$\bar{6}\cdot33586907$	$\bar{7}\cdot70273001$	*79*	$\bar{17}\cdot69612157$	$\bar{18}\cdot84536796$
30	$\bar{6}\cdot12608346$	$\bar{7}\cdot48558277$	*80*	$\bar{17}\cdot48170578$	$\bar{18}\cdot62822072$
31	$\bar{7}\cdot91605644$	$\bar{7}\cdot26843553$	*81*	$\bar{17}\cdot26725605$	$\bar{18}\cdot41107348$
32	$\bar{7}\cdot70580334$	$\bar{7}\cdot05128829$	*82*	$\bar{17}\cdot05277323$	$\bar{18}\cdot19392624$
33	$\bar{7}\cdot49533808$	$\bar{8}\cdot83414105$	*83*	$\bar{18}\cdot83825810$	$\bar{19}\cdot97677900$
34	$\bar{7}\cdot28467333$	$\bar{8}\cdot61699381$	*84*	$\bar{18}\cdot62371146$	$\bar{19}\cdot75963176$
35	$\bar{7}\cdot07382065$	$\bar{8}\cdot39984657$	*85*	$\bar{18}\cdot40913404$	$\bar{19}\cdot54248452$
36	$\bar{8}\cdot86279064$	$\bar{8}\cdot18269932$	*86*	$\bar{18}\cdot19452656$	$\bar{19}\cdot32533727$
37	$\bar{8}\cdot65159301$	$\bar{9}\cdot96555208$	*87*	$\bar{19}\cdot97988972$	$\bar{19}\cdot10819003$
38	$\bar{8}\cdot44023670$	$\bar{9}\cdot74840484$	*88*	$\bar{19}\cdot76522419$	$\bar{20}\cdot89104279$
39	$\bar{8}\cdot22872997$	$\bar{9}\cdot53125760$	*89*	$\bar{19}\cdot55053062$	$\bar{20}\cdot67389555$
40	$\bar{8}\cdot01708042$	$\bar{9}\cdot31411036$	*90*	$\bar{19}\cdot33580963$	$\bar{20}\cdot45674831$
41	$\bar{9}\cdot80529511$	$\bar{9}\cdot09696312$	*91*	$\bar{19}\cdot12106183$	$\bar{20}\cdot23960107$
42	$\bar{9}\cdot59338058$	$\bar{10}\cdot87981588$	*92*	$\bar{20}\cdot90628780$	$\bar{20}\cdot02245383$
43	$\bar{9}\cdot38134293$	$\bar{10}\cdot66266864$	*93*	$\bar{20}\cdot69148812$	$\bar{21}\cdot80530659$
44	$\bar{9}\cdot16918780$	$\bar{10}\cdot44552140$	*94*	$\bar{20}\cdot47666333$	$\bar{21}\cdot58815935$
45	$\bar{10}\cdot95692047$	$\bar{10}\cdot22837416$	*95*	$\bar{20}\cdot26181397$	$\bar{21}\cdot37101211$
46	$\bar{10}\cdot74454589$	$\bar{10}\cdot01122691$	*96*	$\bar{20}\cdot04694054$	$\bar{21}\cdot15386486$
47	$\bar{10}\cdot53206866$	$\bar{11}\cdot79407967$	*97*	$\bar{21}\cdot83204355$	$\bar{22}\cdot93671762$
48	$\bar{10}\cdot31949311$	$\bar{11}\cdot57693243$	*98*	$\bar{21}\cdot61712348$	$\bar{22}\cdot71957038$
49	$\bar{10}\cdot10682329$	$\bar{11}\cdot35978519$	*99*	$\bar{21}\cdot40218080$	$\bar{22}\cdot50242314$
50	$\bar{11}\cdot89406301$	$\bar{11}\cdot14263795$	*100*	$\bar{21}\cdot18721596$	$\bar{22}\cdot28527590$

TABLES XIV—XVI. *Auxiliary Tables* B, C *and* D.

TABLE XIV (B).

Table of colog $[n]$:—$[n] = n(n-2)(n-4)\ldots\ldots$

n odd nos.	colog $[n]$	n even nos.	colog $[n]$
1	·00000000	*2*	$\bar{1}$·69897000
3	$\bar{1}$·52287875	*4*	$\bar{1}$·09691001
5	$\bar{2}$·82390874	*6*	$\bar{2}$·31875876
7	$\bar{3}$·97881070	*8*	$\bar{3}$·41566878
9	$\bar{3}$·02456819	*10*	$\bar{4}$·41566878
11	$\bar{5}$·98317551	*12*	$\bar{5}$·33648753
13	$\bar{6}$·86923215	*14*	$\bar{6}$·19035949
15	$\bar{7}$·69314089	*16*	$\bar{8}$·98623951
17	$\bar{8}$·46269197	*18*	$\bar{9}$·73096701
19	$\bar{9}$·18393837	*20*	$\bar{10}$·42993701
21	$\bar{11}$·86171908	*22*	$\bar{11}$·08751433
23	$\bar{12}$·49999124	*24*	$\bar{13}$·70730309
25	$\bar{13}$·10205123	*26*	$\bar{14}$·29232974
27	$\bar{15}$·67068747	*28*	$\bar{16}$·84517171
29	$\bar{16}$·20828947	*30*	$\bar{17}$·36805045
31	$\bar{18}$·71692778	*32*	$\bar{19}$·86290048
33	$\bar{19}$·19841384	*34*	$\bar{20}$·33142156
35	$\bar{21}$·65434579	*36*	$\bar{22}$·77511906
37	$\bar{22}$·08614407	*38*	$\bar{23}$·19533546
39	$\bar{24}$·49507946	*40*	$\bar{25}$·59327547
41	$\bar{26}$·88229561	*42*	$\bar{27}$·97002618
43	$\bar{27}$·24882715	*44*	$\bar{28}$·32657350
45	$\bar{29}$·59561464	*46*	$\bar{30}$·66381567
47	$\bar{31}$·92351678	*48*	$\bar{32}$·98257443
49	$\bar{32}$·23332070	*50*	$\bar{33}$·28360443
51	$\bar{34}$·52575052	*52*	$\bar{35}$·56760109
53	$\bar{36}$·80147465	*54*	$\bar{37}$·83520733
55	$\bar{37}$·06111196	*56*	$\bar{38}$·08701930
57	$\bar{39}$·30523711	*58*	$\bar{40}$·32359131
59	$\bar{41}$·53438510	*60*	$\bar{42}$·54544006
61	$\bar{43}$·74905526	*62*	$\bar{44}$·75304837
63	$\bar{45}$·94971471	*64*	$\bar{46}$·94686839
65	$\bar{46}$·13680135	*66*	$\bar{47}$·12732446
67	$\bar{48}$·31072655	*68*	$\bar{49}$·29481554
69	$\bar{50}$·47187746	*70*	$\bar{51}$·44971750
71	$\bar{52}$·62061911	*72*	$\bar{53}$·59238501
73	$\bar{54}$·75729625	*74*	$\bar{55}$·72315329
75	$\bar{56}$·88223499	*76*	$\bar{57}$·84233970
77	$\bar{58}$·99574426	*78*	$\bar{59}$·95024509
79	$\bar{59}$·09811717	*80*	$\bar{60}$·04715511
81	$\bar{61}$·18963215	*82*	$\bar{62}$·13334125
83	$\bar{63}$·27055406	*84*	$\bar{64}$·20906197
85	$\bar{65}$·34113514	*86*	$\bar{66}$·27456352
87	$\bar{67}$·40161588	*88*	$\bar{68}$·33008084
89	$\bar{69}$·45222588	*90*	$\bar{70}$·37583833
91	$\bar{71}$·49318448	*92*	$\bar{72}$·41205051
93	$\bar{73}$·52470154	*94*	$\bar{74}$·43892265
95	$\bar{75}$·54697793	*96*	$\bar{76}$·45665142
97	$\bar{77}$·56020620	*98*	$\bar{78}$·46542534
99	$\bar{79}$·56457100	*100*	$\bar{80}$·46542534

TABLE XV (C).

χ^2	$\sqrt{\frac{2}{\pi}}\int_{\chi}^{\infty} e^{-\frac{1}{2}\chi^2}\,d\chi$
1	·3173106
2	·1572992
3	·0832646
4	·0455003
5	·0253474
6	·0143060
7	·0081506
8	·0046776
9	·0026998
10	·0015654
11	·0009112
12	·0005321
13	·0003115
14	·0001828
15	·0001076
16	·0000634
17	·0000374
18	·0000221
19	·0000132
20	·0000078
21	·0000046
22	·0000027
23	·0000016
24	·0000011
25	·0000007
26	·0000004
27	·0000003
28	·0000002
29	·0000001
30	·0000000

TABLE XVI (D).

Function	Log. Function
$e^{-\frac{1}{2}}$	$\bar{1}$·7828527590
$\sqrt{\frac{2}{\pi}}$	$\bar{1}$·9019400615

TABLE XVII.

Values of $(-\log P)$ *corresponding to given values of* χ^2 *in a fourfold table.*

(Extension of Table XII for $n' = 4$.)

χ^2	$-\log P$	χ^2	$-\log P$	χ^2	$-\log P$	χ^2	$-\log P$	χ^2	$-\log P$	χ^2	$-\log P$
1	0·096	*26*	5·021	*50*	10·097	*1100*	237·439	*2600*	562·973	*13500*	2929·521
2	0·242	*27*	5·230	*60*	12·231	*1150*	248·287	*2700*	584·680	*14000*	3038·086
3	0·407	*28*	5·440	*70*	14·370	*1200*	259·135	*2800*	606·387	*14500*	3146·652
4	0·583	*29*	5·650	*80*	16·513	*1250*	269·983	*2900*	628·094	*15000*	3255·219
5	0·765	*30*	5·860	*90*	18·659	*1300*	280·832	*3000*	649·801	*15500*	3363·785
6	0·952	*31*	6·071	*100*	20·809	*1350*	291·681	*3500*	758·341	*16000*	3472·352
7	1·143	*32*	6·281	*150*	31·579	*1400*	302·531	*4000*	866·886	*16500*	3580·919
8	1·337	*33*	6·492	*200*	42·375	*1450*	313·381	*4500*	975·434	*17000*	3689·486
9	1·533	*34*	6·703	*250*	53·184	*1500*	324·231	*5000*	1083·995	*17500*	3798·053
10	1·731	*35*	6·914	*300*	64·002	*1550*	335·081	*5500*	1192·538	*18000*	3906·621
11	1·931	*36*	7·126	*350*	74·826	*1600*	345·931	*6000*	1301·092	*18500*	4015·188
12	2·132	*37*	7·337	*400*	85·655	*1650*	356·782	*6500*	1409·649	*19000*	4123·756
13	2·334	*38*	7·549	*450*	96·487	*1700*	367·633	*7000*	1518·206	*19500*	4232·324
14	2·537	*39*	7·761	*500*	107·321	*1750*	378·484	*7500*	1626·765	*20000*	4340·892
15	2·741	*40*	7·972	*550*	118·158	*1800*	389·335	*8000*	1735·324	*20500*	4449·461
16	2·945	*41*	8·184	*600*	128·997	*1850*	400·187	*8500*	1843·885	*21000*	4558·029
17	3·151	*42*	8·397	*650*	139·837	*1900*	411·038	*9000*	1952·446	*21500*	4666·597
18	3·357	*43*	8·609	*700*	150·678	*1950*	421·890	*9500*	2061·008	*22000*	4775·166
19	3·564	*44*	8·821	*750*	161·520	*2000*	432·742	*10000*	2169·570	*22500*	4883·735
20	3·770	*45*	9·034	*800*	172·364	*2050*	443·594	*10500*	2278·133	*23000*	4992·304
21	3·978	*46*	9·246	*850*	183·208	*2100*	454·446	*11000*	2386·697	*23500*	5100·873
22	4·186	*47*	9·459	*900*	194·053	*2200*	476·151	*11500*	2495·261	*24000*	5209·442
23	4·394	*48*	9·672	*950*	204·899	*2300*	497·856	*12000*	2603·825	*24500*	5318·011
24	4·602	*49*	9·885	*1000*	215·745	*2400*	519·561	*12500*	2712·390	*25000*	5426·580
25	4·811	*50*	10·097	*1050*	226·592	*2500*	541·267	*13000*	2820·955		
26	5·021			*1100*	237·439	*2600*	562·973	*13500*	2929·521		

TABLE XVIII.

Values of $(-\log P)$, *entering with* r *and* $_0\sigma_r$.

Values of $_0\sigma_r$.

Values of r.	*·01*	*·02*	*·03*	*·04*	*·05*	*·06*	*·07*	*·08*
0·05	6·248	1·907	1·020	0·675	0·498	0·392	0·322	0·273
0·075	13·228	3·760	1·908	1·217	0·874	0·674	0·545	0·456
0·1	22·924	6·267	3·076	1·910	1·343	1·019	0·814	0·675
0·15	50·687	13·329	6·298	3·784	2·586	1·916	1·498	1·218
0·2	90·035	23·254	10·771	6·343	4·259	3·100	2·384	1·924
0·3	206·348	52·453	23·836	13·758	9·057	6·478	4·906	3·903
0·4	380·266	96·013	43·254	24·726	16·112	11·407	8·552	6·686
0·5	626·428	157·607	70·669	40·177	26·025	18·312	13·642	10·597
0·6	970·879	243·753	108·980	61·747	39·845	27·922	20·713	16·020
0·7	1463·946	367·033	163·781	92·579	59·584	41·634	30·792	23·740
0·8	2220·267	556·100	247·801	139·832	89·819	62·625	46·209	35·539
0·9	3607·924	902·949	401·907	226·479	145·241	101·085	74·442	57·134
0·95	5056·547	1265·013	562·757	316·904	203·069	141·207	103·886	79·671

TABLE XIX.

Values of χ^2 corresponding to the values of $(-\log P)$ in Table XVIII.

Values of ${}_0\sigma_r$.

Values of r.	·01	·02	·03	·04	·05	·06	·07	·08
0·05	31·84	10·88	6·36	4·51	3·52	2·91	2·48	2·19
0·075	64·66	19·95	10·89	7·38	5·58	4·51	3·78	3·28
0·1	109·82	31·93	16·64	10·90	8·03	6·35	5·26	4·51
0·15	238·45	65·13	32·08	20·07	14·24	10·93	8·82	7·39
0·2	422·29	111·35	53·16	32·29	22·35	16·75	13·25	10·97
0·3	956·68	246·62	114·05	67·14	45·11	32·93	25·45	20·64
0·4	1758·21	447·81	204·07	118·18	78·13	56·14	42·73	33·92
0·5	2892·33	731·95	330·80	189·82	124·22	88·38	66·60	52·34
0·6	4479·02	1129·10	507·65	289·58	188·28	133·02	99·55	77·70
0·7	6750·09	1697·24	760·43	431·96	279·58	196·57	146·35	113·61
0·8	10233·49	2568·34	1147·76	649·98	419·22	293·64	217·74	168·34
0·9	16624·37	4166·12	1857·93	1049·48	674·92	471·22	348·23	268·26
0·95	23295·86	5833·82	2599·00	1466·24	941·56	656·32	484·15	372·37

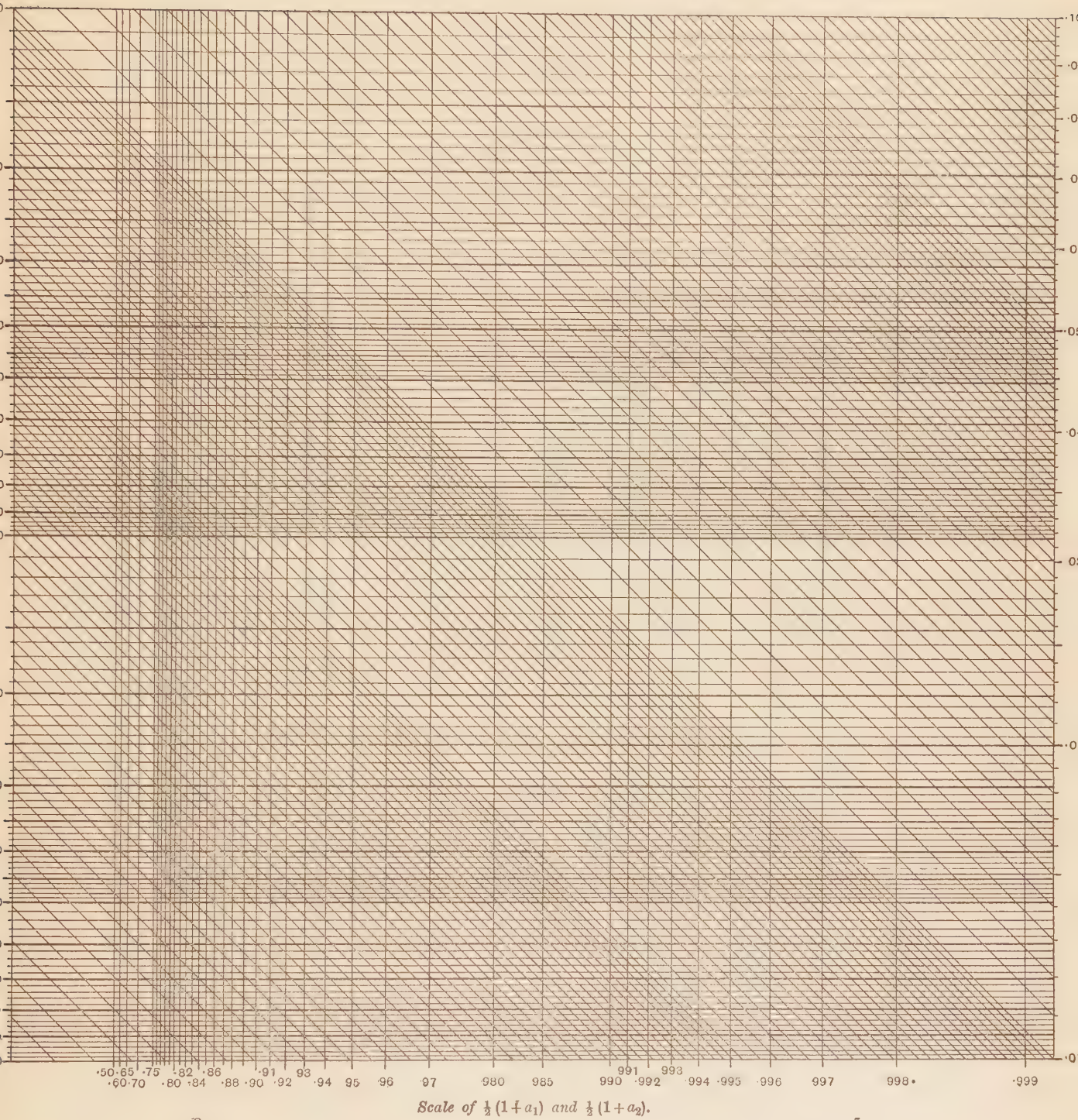
TABLE XX.

Values of $\log \chi^2$ corresponding to values of r and ${}_0\sigma_r$ in Tables XVII and XVIII.

Values of ${}_0\sigma_r$.

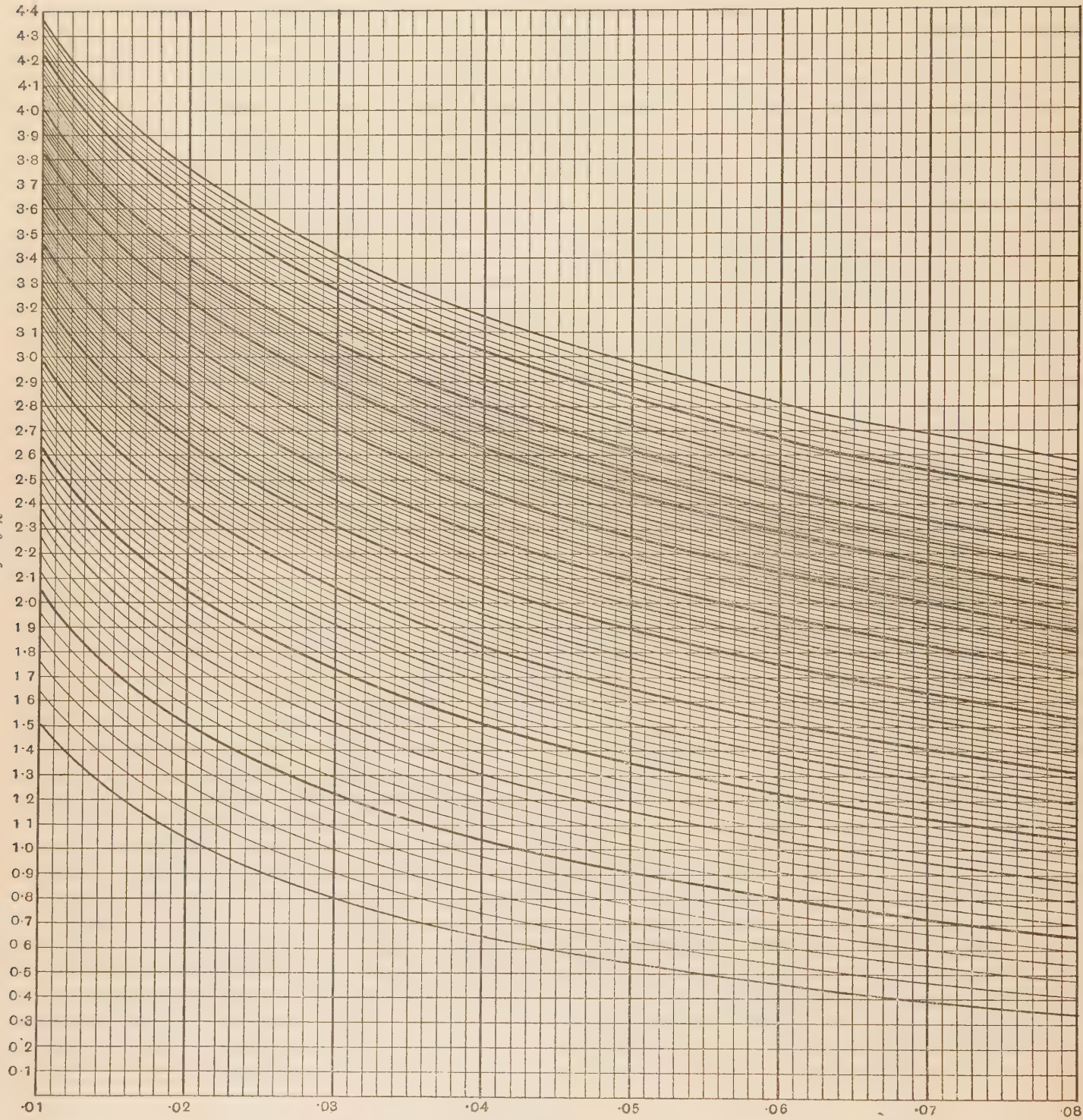
Values of r.	·01	·02	·03	·04	·05	·06	·07	·08
0·05	1·5030	1·0366	0·8035	0·6542	0·5465	0·4639	0·3945	0·3404
0·075	1·8106	1·2999	1·0370	0·8681	0·7466	0·6542	0·5775	0·5159
0·1	2·0407	1·5042	1·2212	1·0374	0·9047	0·8028	0·7210	0·6522
0·15	2·3774	1·8138	1·5062	1·3025	1·1535	1·0386	0·9455	0·8686
0·2	2·6256	2·0467	1·7256	1·5091	1·3493	1·2240	1·1222	1·0400
0·3	2·9808	2·3920	2·0571	1·8270	1·6543	1·5176	1·4057	1·3148
0·4	3·2451	2·6511	2·3098	2·0725	1·8928	1·7493	1·6307	1·5305
0·5	3·4612	2·8645	2·5196	2·2783	2·0942	1·9464	1·8235	1·7188
0·6	3·6512	3·0527	2·7056	2·4618	2·2748	2·1239	1·9980	1·8904
0·7	3·8293	3·2297	2·8811	2·6354	2·4465	2·2935	2·1654	2·0554
0·8	4·0100	3·4097	3·0598	2·8129	2·6224	2·4678	2·3379	2·2262
0·9	4·2207	3·6197	3·2690	3·0210	2·8293	2·6732	2·5419	2·4286
0·95	4·3673	3·7660	3·4148	3·1662	2·9738	2·8171	2·6850	2·5710

XXI. *Abac to determine* ${}_0\sigma_r$.



Scale of $\frac{1}{2}(1+a_1)$ *and* $\frac{1}{2}(1+a_2)$.

XXII. *Abac to determine* r_P.



Approximate values of Probable Error of r from a four-fold Correlation Table (to be used with χ_1 of Table V).

TABLE XXIII. *Values of χ_r for Values of r.*

r	χ_r	r	χ_r	r	χ_r	r	χ_r	r	χ_r
·00	1·0000	·20	·9717	·40	·8845	·60	·7298	·80	·4843
·01	·9999	·21	·9688	·41	·8785	·61	·7200	·81	·4687
·02	·9997	·22	·9657	·42	·8723	·62	·7099	·82	·4526
·03	·9994	·23	·9625	·43	·8659	·63	·6997	·83	·4362
·04	·9989	·24	·9591	·44	·8594	·64	·6892	·84	·4192
·05	·9982	·25	·9556	·45	·8527	·65	·6785	·85	·4018
·06	·9975	·26	·9520	·46	·8458	·66	·6675	·86	·3838
·07	·9966	·27	·9482	·47	·8388	·67	·6563	·87	·3652
·08	·9955	·28	·9442	·48	·8315	·68	·6448	·88	·3461
·09	·9943	·29	·9401	·49	·8241	·69	·6331	·89	·3262
·10	·9930	·30	·9358	·50	·8165	·70	·6211	·90	·3057
·11	·9915	·31	·9314	·51	·8087	·71	·6088	·91	·2843
·12	·9899	·32	·9268	·52	·8007	·72	·5962	·92	·2620
·13	·9881	·33	·9221	·53	·7926	·73	·5834	·93	·2387
·14	·9862	·34	·9172	·54	·7842	·74	·5702	·94	·2142
·15	·9841	·35	·9122	·55	·7756	·75	·5568	·95	·1882
·16	·9819	·36	·9070	·56	·7669	·76	·5430	·96	·1605
·17	·9796	·37	·9016	·57	·7579	·77	·5288	·97	·1305
·18	·9771	·38	·8961	·58	·7488	·78	·5144	·98	·0972
·19	·9745	·39	·8904	·59	·7394	·79	·4995	·99	·0585
								1·00	·0000

TABLE XXIV.

Values of χ_α for Values of $\frac{1}{2}(1+\alpha)$.

$\frac{1}{2}(1+\alpha)$	χ_α	$\frac{1}{2}(1+\alpha)$	χ_α	$\frac{1}{2}(1+\alpha)$	χ_α	$\frac{1}{2}(1+\alpha)$	χ_α
·50	1·2533	·65	1·2877	·80	1·4288	·95	2·1132
·51	1·2535	·66	1·2928	·81	1·4457	·96	2·2740
·52	1·2539	·67	1·2984	·82	1·4641	·97	2·5071
·53	1·2546	·68	1·3044	·83	1·4844	·98	2·8915
·54	1·2556	·69	1·3109	·84	1·5067	·985	3·2097
·55	1·2569	·70	1·3180	·85	1·5315	·990	3·7333
·56	1·2585	·71	1·3256	·86	1·5590	·991	3·8854
·57	1·2604	·72	1·3338	·87	1·5897	·992	4·0639
·58	1·2626	·73	1·3427	·88	1·6245	·993	4·2784
·59	1·2652	·74	1·3523	·89	1·6640	·994	4·5419
·60	1·2680	·75	1·3626	·90	1·7094	·995	4·8779
·61	1·2712	·76	1·3738	·91	1·7623	·996	5·3278
·62	1·2748	·77	1·3859	·92	1·8249	·997	5·9776
·63	1·2787	·78	1·3990	·93	1·9003	·998	7·0465
·64	1·2830	·79	1·4133	·94	1·9937	·999	9·3870

TABLE XXV.

$$\text{Values of } y = \frac{n-2}{n-3}\cdot\frac{n-4}{n-5}\cdots\begin{cases}\cdots\frac{4}{3}\cdot\frac{2}{\pi} \text{ if } n \text{ be even}\\ \cdots\frac{3}{2}\cdot\frac{1}{2} \text{ if } n \text{ be odd}\end{cases}\int_{-\infty}^{z}(1+z^2)^{-\frac{n}{2}}\,.\,dz.$$

Or, the probability that the mean of a sample of n, drawn at random from a normal population, will not exceed (in algebraic sense) the mean of the population by more than z times the standard deviation of the sample.

z	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=8$	$n=9$	$n=10$
·1	·53173	·54975+	·56324	·57438	·58404	·59267	·60052	·60776	·61450
·2	·56283	·59806	·62405−	·64520	·66329	·67920	·69347	·70644	·71835−
·3	·59277	·64367	·68038	·70958	·73396	·75493	·77329	·78960	·80422
·4	·62112	·68570	·73088	·76574	·79396	·81749	·83747	·85466	·86961
·5	·64758	·72361	·77491	·81305−	·84281	·86672	·88627	·90249	·91607
·6	·67202	·75725−	·81245+	·85182	·88129	·90398	·92178	·93594	·94730
·7	·69440	·78673	·84394	·88295−	·91085+	·93138	·94677	·95847	·96744
·8	·71478	·81235−	·87005−	·90758	·93317	·95112	·96396	·97325+	·98005+
·9	·73326	·83448	·89154	·92688	·94983	·96517	·97560	·98279	·98780
1·0	·75000	·85355+	·90915+	·94194	·96221	·97509	·98343	·98890	·99252
1·1	·76515−	·86997	·92358	·95367	·97137	·98208	·98867	·99279	·99539
1·2	·77886	·88411	·93540	·96282	·97818	·98702	·99220	·99528	·99713
1·3	·79129	·89631	·94512	·96998	·98324	·99051	·99458	·99688	·99819
1·4	·80257	·90687	·95312	·97559	·98703	·99301	·99619	·99791	·99885−
1·5	·81283	·91603	·95975−	·98003	·98988	·99480	·99730	·99859	·99926
1·6	·82219	·92400	·96525+	·98355−	·99204	·99610	·99806	·99903	·99951
1·7	·83075−	·93097	·96985+	·98636	·99369	·99704	·99860	·99933	·99968
1·8	·83859	·93708	·97372	·98862	·99496	·99774	·99897	·99953	·99978
1·9	·84579	·94246	·97698	·99045−	·99595−	·99826	·99924	·99967	·99985+
2·0	·85242	·94721	·97974	·99193	·99672	·99864	·99943	·99976	·99990
2·1	·85854	·95143	·98210	·99315+	·99732	·99894	·99957	·99983	·99993
2·2	·86420	·95518	·98411	·99415+	·99780	·99916	·99968	·99988	·99995
2·3	·86945+	·95854	·98585−	·99498	·99818	·99933	·99975+	·99991	·99996
2·4	·87433	·96154	·98734	·99568	·99849	·99946	·99981	·99993	·99997
2·5	·87888	·96424	·98864	·99625+	·99874	·99957	·99985−	·99995	·99998
2·6	·88312	·96667	·98978	·99674	·99894	·99965−	·99988	·99996	·99999
2·7	·88709	·96887	·99077	·99715+	·99910	·99971	·99991	·99997	·99999
2·8	·89081	·97087	·99163	·99750	·99924	·99976	·99993	·99998	·99999
2·9	·89430	·97269	·99240	·99780	·99935−	·99980	·99994	·99998	·99999
3·0	·89758	·97434	·99308	·99806	·99944	·99984	·99995+	·99999	1·00000

TABLE XXVI.

Table for use in plotting Type III Curves, i.e.

$$y = y_0 e^{-p\frac{x}{a}}\left(1+\frac{x}{a}\right)^p.$$

X	$\log_{10}(1+X) - X\log_{10}e$	X	$\log_{10}(1+X) - X\log_{10}e$
−·95	·888 450	+·80	·092 163
−·90	·609 135	+·85	·101 979
−·85	·454 759	+·90	·112 111
−·80	·351 534	+·95	·122 545
−·75	·276 339	+1·00	·133 265
−·70	·218 873	+1·05	·144 255
−·65	·173 641	+1·10	·155 505
−·60	·137 363	+1·15	·167 000
−·55	·107 926	+1·20	·178 731
−·50	·083 883	+1·25	·190 685
−·45	·064 205	+1·30	·202 855
−·40	·048 131	+1·35	·215 230
−·35	·035 084	+1·40	·227 801
−·30	·024 614	+1·45	·240 561
−·25	·016 365	+1·50	·253 502
−·20	·010 051	+1·55	·266 616
−·15	·005 437	+1·60	·279 898
−·10	·002 328	+1·65	·293 340
−·05	·000 562	+1·70	·306 937
·00	·000 000	+1·75	·320 683
+·05	·000 525	+1·80	·334 572
+·10	·002 037	+1·85	·348 600
+·15	·004 446	+1·90	·362 761
+·20	·007 678	+1·95	·377 052
+·25	·011 664	+2·00	·391 468
+·30	·016 345	+2·05	·406 004
+·35	·021 669	+2·10	·420 657
+·40	·027 590	+2·15	·435 422
+·45	·034 065	+2·20	·450 298
+·50	·041 056	+2·25	·465 279
+·55	·048 530	+2·30	·480 363
+·60	·056 457	+2·35	·495 547
+·65	·064 808	+2·40	·510 828
+·70	·073 557	+2·45	·526 202
+·75	·082 683	+2·50	·541 668

TABLE XXVII.

Powers of Natural Numbers.

n	n^2	n^3	n^4	n^5	n^6	n^7	n
1	1	1	1	1	1	1	*1*
2	4	8	16	32	64	128	*2*
3	9	27	81	243	729	2187	*3*
4	16	64	256	1024	4096	16384	*4*
5	25	125	625	3125	15625	78125	*5*
6	36	216	1296	7776	46656	279936	*6*
7	49	343	2401	16807	117649	823543	*7*
8	64	512	4096	32768	262144	2097152	*8*
9	81	729	6561	59049	531441	4782969	*9*
10	100	1000	10000	100000	1000000	10000000	*10*
11	121	1331	14641	161051	1771561	19487171	*11*
12	144	1728	20736	248832	2985984	35831808	*12*
13	169	2197	28561	371293	4826809	62748517	*13*
14	196	2744	38416	537824	7529536	105413504	*14*
15	225	3375	50625	759375	11390625	170859375	*15*
16	256	4096	65536	1048576	16777216	268435456	*16*
17	289	4913	83521	1419857	24137569	410338673	*17*
18	324	5832	104976	1889568	34012224	612220032	*18*
19	361	6859	130321	2476099	47045881	893871739	*19*
20	400	8000	160000	3200000	64000000	1280000000	*20*
21	441	9261	194481	4084101	85766121	1801088541	*21*
22	484	10648	234256	5153632	113379904	2494357888	*22*
23	529	12167	279841	6436343	148035889	3404825447	*23*
24	576	13824	331776	7962624	191102976	4586471424	*24*
25	625	15625	390625	9765625	244140625	6103515625	*25*
26	676	17576	456976	11881376	308915776	8031810176	*26*
27	729	19683	531441	14348907	387420489	10460353203	*27*
28	784	21952	614656	17210368	481890304	13492928512	*28*
29	841	24389	707281	20511149	594823321	17249876309	*29*
30	900	27000	810000	24300000	729000000	21870000000	*30*
31	961	29791	923521	28629151	887503681	27512614111	*31*
32	1024	32768	1048576	33554432	1073741824	34359738368	*32*
33	1089	35937	1185921	39135393	1291467969	42618442977	*33*
34	1156	39304	1336336	45435424	1544804416	52523350144	*34*
35	1225	42875	1500625	52521875	1838265625	64339296875	*35*
36	1296	46656	1679616	60466176	2176782336	78364164096	*36*
37	1369	50653	1874161	69343957	2565726409	94931877133	*37*
38	1444	54872	2085136	79235168	3010936384	114415582592	*38*
39	1521	59319	2313441	90224199	3518743761	137231006679	*39*
40	1600	64000	2560000	102400000	4096000000	163840000000	*40*
41	1681	68921	2825761	115856201	4750104241	194754273881	*41*
42	1764	74088	3111696	130691232	5489031744	230539333248	*42*
43	1849	79507	3418801	147008443	6321363049	271818611107	*43*
44	1936	85184	3748096	164916224	7256313856	319277809664	*44*
45	2025	91125	4100625	184528125	8303765625	373669453125	*45*
46	2116	97336	4477456	205962976	9474296896	435817657216	*46*
47	2209	103823	4879681	229345007	10779215329	506623120463	*47*
48	2304	110592	5308416	254803968	12230590464	587068342272	*48*
49	2401	117649	5764801	282475249	13841287201	678223072849	*49*
50	2500	125000	6250000	312500000	15625000000	781250000000	*50*

TABLE XXVII.—(*continued*).

Powers of Natural Numbers.

n	n^2	n^3	n^4	n^5	n^6	n^7	n
51	2601	132651	6765201	345025251	17596287801	897410677851	51
52	2704	140608	7311616	380204032	19770609664	1028071702528	52
53	2809	148877	7890481	418195493	22164361129	1174711139837	53
54	2916	157464	8503056	459165024	24794911296	1338925209984	54
55	3025	166375	9150625	503284375	27680640625	1522435234375	55
56	3136	175616	9834496	550731776	30840979456	1727094849536	56
57	3249	185193	10556001	601692057	34296447249	1954897493193	57
58	3364	195112	11316496	656356768	38068692544	2207984167552	58
59	3481	205379	12117361	714924299	42180533641	2488651484819	59
60	3600	216000	12960000	777600000	46656000000	2799360000000	60
61	3721	226981	13845841	844596301	51520374361	3142742836021	61
62	3844	238328	14776336	916132832	56800235584	3521614606208	62
63	3969	250047	15752961	992436543	62523502209	3938980639167	63
64	4096	262144	16777216	1073741824	68719476736	4398046511104	64
65	4225	274625	17850625	1160290625	75418890625	4902227890625	65
66	4356	287496	18974736	1252332576	82653950016	5455160701056	66
67	4489	300763	20151121	1350125107	90458382169	6060711605323	67
68	4624	314432	21381376	1453933568	98867482624	6722988818432	68
69	4761	328509	22667121	1564031349	107918163081	7446353252589	69
70	4900	343000	24010000	1680700000	117649000000	8235430000000	70
71	5041	357911	25411681	1804229351	128100283921	9095120158391	71
72	5184	373248	26873856	1934917632	139314069504	10030613004288	72
73	5329	389017	28398241	2073071593	151334226289	11047398519097	73
74	5476	405224	29986576	2219006624	164206490176	12151280273024	74
75	5625	421875	31640625	2373046875	177978515625	13348388671875	75
76	5776	438976	33362176	2535525376	192699928576	14645194571776	76
77	5929	456533	35153041	2706784157	208422380089	16048523266853	77
78	6084	474552	37015056	2887174368	225199600704	17565568854912	78
79	6241	493039	38950081	3077056399	243087455521	19203908986159	79
80	6400	512000	40960000	3276800000	262144000000	20971520000000	80
81	6561	531441	43046721	3486784401	282429536481	22876792454961	81
82	6724	551368	45212176	3707398432	304006671424	24928547056768	82
83	6889	571787	47458321	3939040643	326940373369	27136050989627	83
84	7056	592704	49787136	4182119424	351298031616	29509034655744	84
85	7225	614125	52200625	4437053125	377149515625	32057708828125	85
86	7396	636056	54700816	4704270176	404567235136	34792782221696	86
87	7569	658503	57289761	4984209207	433626201009	37725479487783	87
88	7744	681472	59969536	5277319168	464404086784	40867559636992	88
89	7921	704969	62742241	5584059449	496981290961	44231334895529	89
90	8100	729000	65610000	5904900000	531441000000	47829690000000	90
91	8281	753571	68574961	6240321451	567869252041	51676101935731	91
92	8464	778688	71639296	6590815232	606355001344	55784660123648	92
93	8649	804357	74805201	6956883693	646990183449	60170087060757	93
94	8836	830584	78074896	7339040224	689869781056	64847759419264	94
95	9025	857375	81450625	7737809375	735091890625	69833729609375	95
96	9216	884736	84934656	8153726976	782757789696	75144747810816	96
97	9409	912673	88529281	8587340257	832972004929	80798284478113	97
98	9604	941192	92236816	9039207968	885842380864	86812553324672	98
99	9801	970299	96059601	9509900499	941480149401	93206534790699	99
100	10000	1000000	100000000	10000000000	1000000000000	100000000000000	100

TABLE XXVIII.

Sums of Powers of Natural Numbers.

n	$S(n)$	$S(n^2)$	$S(n^3)$	$S(n^4)$	$S(n^5)$	$S(n^6)$	$S(n^7)$	n
1	1	1	1	1	1	1	1	1
2	3	5	9	17	33	65	129	2
3	6	14	36	98	276	794	2316	3
4	10	30	100	354	1300	4890	18700	4
5	15	55	225	979	4425	20515	96825	5
6	21	91	441	2275	12201	67171	376761	6
7	28	140	784	4676	29008	184820	1200304	7
8	36	204	1296	8772	61776	446964	3297456	8
9	45	285	2025	15333	120825	978405	8080425	9
10	55	385	3025	25333	220825	1978405	18080425	10
11	66	506	4356	39974	381876	3749966	37567596	11
12	78	650	6084	60710	630708	6735950	73399404	12
13	91	819	8281	89271	1002001	11562759	136147921	13
14	105	1015	11025	127687	1539825	19092295	241561425	14
15	120	1240	14400	178312	2299200	30482920	412420800	15
16	136	1496	18496	243848	3347776	47260136	680856256	16
17	153	1785	23409	327369	4767633	71397705	1091194929	17
18	171	2109	29241	432345	6657201	105409929	1703414961	18
19	190	2470	36100	562666	9133300	152455810	2597286700	19
20	210	2870	44100	722666	12333300	216455810	3877286700	20
21	231	3311	53361	917147	16417401	302221931	5678375241	21
22	253	3795	64009	1151403	21571033	415601835	8172733129	22
23	276	4324	76176	1431244	28007376	563637724	11577558576	23
24	300	4900	90000	1763020	35970000	754740700	16164030000	24
25	325	5525	105625	2153645	45735625	998881325	22267545625	25
26	351	6201	123201	2610621	57617001	1307797101	30299355801	26
27	378	6930	142884	3142062	71965908	1695217590	40759709004	27
28	406	7714	164836	3756718	89176276	2177107894	54252637516	28
29	435	8555	189225	4463999	109687425	2771931215	71502513825	29
30	465	9455	216225	5273999	133987425	3500931215	93372513825	30
31	496	10416	246016	6197520	162616576	4388434896	120885127936	31
32	528	11440	278784	7246096	196171008	5462176720	155244866304	32
33	561	12529	314721	8432017	135306401	6753644689	197863309281	33
34	595	13685	354025	9768353	180741825	8298449105	250386659425	34
35	630	14910	396900	11268978	333263700	10136714730	314725956300	35
36	666	16206	443556	12948594	393729876	12313497066	393090120396	36
37	703	17575	494209	14822755	463073833	14879223475	488021997529	37
38	741	19019	549081	16907891	542309001	17890159859	602437580121	38
39	780	20540	608400	19221332	632533200	21408903620	739668586800	39
40	820	22140	672400	21781332	734933200	25504903620	903508586800	40
41	861	23821	741321	24607093	850789401	30255007861	1098262860681	41
42	903	25585	815409	27718789	981480633	35744039605	1328802193929	42
43	946	27434	894916	31137590	1128489076	42065402654	1600620805036	43
44	990	29370	980100	34885686	1293405300	49321716510	1919898614700	44
45	1035	31395	1071225	38986311	1477933425	57625482135	2293568067825	45
46	1081	33511	1168561	43463767	1683896401	67099779031	2729385725041	46
47	1128	35720	1272384	48343448	1913241408	77878994360	3236008845504	47
48	1176	38024	1382976	53651864	2168045376	90109584824	3823077187776	48
49	1225	40425	1500625	59416665	2450520625	103950872025	4501300260625	49
50	1275	42925	1625625	65666665	2763020625	119575872025	5282550260625	50

TABLE XXVIII.—(*continued*).

Sums of Powers of Natural Numbers.

n	$S(n)$	$S(n^2)$	$S(n^3)$	$S(n^4)$	$S(n^5)$	$S(n^6)$	$S(n^7)$	n
51	1326	45526	1758276	72431866	3108045876	137172159826	6179960938476	51
52	1378	48230	1898884	79743482	3488249908	156942769490	7208032641004	52
53	1431	51039	2047761	87633963	3906445401	179107130619	8382743780841	53
54	1485	53955	2205225	96137019	4365610425	203902041915	9721668990825	54
55	1540	56980	2371600	105287644	4868894800	231582682540	11244104225200	55
56	1596	60116	2547216	115122140	5419626576	262423661996	12971199074736	56
57	1653	63365	2732409	125678141	6021318633	296720109245	14926096567929	57
58	1711	66729	2927521	136994637	6677675401	334788801789	17134080735481	58
59	1770	70210	3132900	149111998	7392599700	376969335430	19622732220300	59
60	1830	73810	3348900	162071998	8170199700	423625335430	22422092220300	60
61	1891	77531	3575881	175917839	9014796001	475145709791	25564835056321	61
62	1953	81375	3814209	190694175	9930928833	531945945375	29086449662529	62
63	2016	85344	4064256	206447136	10923365376	594469447584	33025430301696	63
64	2080	89440	4326400	223224352	11997107200	663188924320	37423476812800	64
65	2145	93665	4601025	241074977	13157397825	738607814945	42325704703425	65
66	2211	98021	4888521	260049713	14409730401	821261764961	47780865404481	66
67	2278	102510	5189284	280200834	15759855508	911720147130	53841577009804	67
68	2346	107134	5503716	301582210	17213789076	1010587629754	60564565828236	68
69	2415	111895	5832225	324249331	18777820425	1118505792835	68010919080825	69
70	2485	116795	6175225	348259331	20458520425	1236154792835	76246349080825	70
71	2556	121836	6533136	373671012	22262749776	1364255076756	85341469239216	71
72	2628	127020	6906384	400544868	24197667408	1503569146260	95372082243504	72
73	2701	132349	7295401	428943109	26270739001	1654903372549	106419480762601	73
74	2775	137825	7700625	458929685	28489745625	1819109862725	118570761035625	74
75	2850	143450	8122500	490570310	30862792500	1997088378350	131919149707500	75
76	2926	149226	8561476	523932486	33398317876	2189788306926	146564344279276	76
77	3003	155155	9018009	559085527	36105102033	2398210687015	162612867546129	77
78	3081	161239	9492561	596100583	38992276401	2623410287719	180178436401041	78
79	3160	167480	9985600	635050664	42069332800	2866497743240	199382345387200	79
80	3240	173880	10497600	676010664	45346132800	3128641743240	220353865387200	80
81	3321	180441	11029041	719057385	48832917201	3411071279721	243230657842161	81
82	3403	187165	11580409	764269561	52540315633	3715077951145	268159204898929	82
83	3486	194054	12152196	811727882	56479356276	4042018324514	295295255888556	83
84	3570	201110	12744900	861515018	60661475700	4393316356130	324804290544300	84
85	3655	208335	13359025	913715643	65098528825	4770465871755	356861999372425	85
86	3741	215731	13995081	968416459	69802799001	5175033106891	391654781594121	86
87	3828	223300	14653584	1025706220	74787008208	5608659307900	429380261081904	87
88	3916	231044	15335056	1085675756	80064327376	6073063394684	470247820718896	88
89	4005	238965	16040025	1148417997	85648386825	6570044685645	514479155614425	89
90	4095	247065	16769025	1214027997	91553286825	7101485685645	562308845614425	90
91	4186	255346	17522596	1282602958	97793608276	7669354937686	613984947550156	91
92	4278	263810	18301284	1354242254	104384423508	8275709939030	669769607673804	92
93	4371	272459	19105641	1429047455	111341307201	8922700122479	729939694734561	93
94	4465	281295	19936225	1507122351	118680347425	9612569903535	794787454153825	94
95	4560	290320	20793600	1588572976	126418156800	10347661794160	864621183763200	95
96	4656	299536	21678336	1673507632	134571883776	11130419583856	939765931574016	96
97	4753	308945	22591009	1762036913	143159224033	11963391588785	1020564216052129	97
98	4851	318549	23532201	1854273729	152198432001	12849233969649	1107376769376801	98
99	4950	328350	24502500	1950333330	161708332500	13790714119050	1200583304167500	99
100	5050	338350	25502500	2050333330	171708332500	14790714119050	1300583304167500	100

TABLE XXIX. *Tetrachoric Functions for Fourfold Correlation Tables.*

$\frac{1}{2}(1-\alpha)$	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	h
·001	·00337	·00736	+·01175	+·01391	+·01134	+·00415	3·09023
·002	·00634	·01290	+·01885	+·01968	+·01269	+·00053	2·87816
·003	·00915	·01778	+·02446	+·02335	+·01228	−·00328	2·74778
·004	·01185	·02222	+·02918	+·02587	+·01111	−·00687	2·65207
·005	·01446	·02634	+·03326	+·02764	+·00952	−·01017	2·57583
·006	·01700	·03020	+·03686	+·02887	+·00770	−·01318	2·51214
·007	·01949	·03386	+·04008	+·02970	+·00575	−·01592	2·45726
·008	·02192	·03734	+·04298	+·03021	+·00371	−·01841	2·40892
·009	·02431	·04066	+·04561	+·03047	+·00164	−·02067	2·36562
·010	·02665	·04384	+·04800	+·03053	−·00044	−·02271	2·32635
·011	·02896	·04690	+·05020	+·03041	−·00253	−·02457	2·29037
·012	·03123	·04985	+·05221	+·03014	−·00460	−·02625	2·25713
·013	·03348	·05270	+·05406	+·02975	−·00664	−·02777	2·22621
·014	·03569	·05545	+·05577	+·02926	−·00866	−·02914	2·19729
·015	·03787	·05811	+·05735	+·02867	−·01064	−·03037	2·17009
·016	·04003	·06069	+·05880	+·02801	−·01259	−·03147	2·14441
·017	·04216	·06320	+·06015	+·02727	−·01449	−·03246	2·12007
·018	·04427	·06564	+·06139	+·02647	−·01636	−·03334	2·09693
·019	·04635	·06801	+·06254	+·02562	−·01818	−·03411	2·07485
·020	·04842	·07031	+·06361	+·02472	−·01996	−·03479	2·05375
·021	·05046	·07256	+·06459	+·02378	−·02170	−·03538	2·03352
·022	·05249	·07475	+·06549	+·02280	−·02340	−·03589	2·01409
·023	·05449	·07688	+·06633	+·02179	−·02505	−·03632	1·99539
·024	·05648	·07897	+·06709	+·02074	−·02666	−·03667	1·97737
·025	·05845	·08100	+·06780	+·01968	−·02823	−·03696	1·95996
·026	·06040	·08299	+·06844	+·01858	−·02976	−·03718	1·94313
·027	·06233	·08493	+·06903	+·01747	−·03125	−·03734	1·92684
·028	·06425	·08682	+·06956	+·01634	−·03270	−·03744	1·91104
·029	·06615	·08868	+·07005	+·01520	−·03411	−·03749	1·89570
·030	·06804	·09049	+·07048	+·01404	−·03547	−·03749	1·88079
·031	·06992	·09227	+·07087	+·01287	−·03680	−·03744	1·86630
·032	·07177	·09400	+·07122	+·01168	−·03810	−·03734	1·85218
·033	·07362	·09570	+·07153	+·01049	−·03935	−·03720	1·83842
·034	·07545	·09737	+·07179	+·00929	−·04057	−·03702	1·82501
·035	·07727	·09900	+·07202	+·00809	−·04176	−·03680	1·81191
·036	·07908	·10060	+·07221	+·00688	−·04291	−·03654	1·79912
·037	·08087	·10216	+·07237	+·00566	−·04402	−·03624	1·78661
·038	·08265	·10370	+·07249	+·00444	−·04510	−·03592	1·77438
·039	·08442	·10520	+·07258	+·00322	−·04615	−·03556	1·76241
·040	·08617	·10668	+·07264	+·00200	−·04717	−·03517	1·75069
·041	·08792	·10812	+·07268	+·00077	−·04815	−·03475	1·73920
·042	·08965	·10954	+·07268	−·00045	−·04910	−·03431	1·72793
·043	·09137	·11093	+·07266	−·00167	−·05003	−·03384	1·71689
·044	·09309	·11229	+·07261	−·00290	−·05092	−·03335	1·70604
·045	·09479	·11363	+·07253	−·00412	−·05178	−·03283	1·69540
·046	·09648	·11495	+·07243	−·00534	−·05261	−·03229	1·68494
·047	·09816	·11623	+·07231	−·00656	−·05342	−·03173	1·67466
·048	·09983	·11750	+·07217	−·00778	−·05420	−·03115	1·66456
·049	·10149	·11874	+·07200	−·00899	−·05495	−·03055	1·65463
·050	·10314	·11996	+·07181	−·01020	−·05567	−·02994	1·64485

τ_1, τ_2 and h are essentially positive.

TABLE XXIX.—(*continued*).

$\frac{1}{2}(1-a)$	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	h
·051	·10478	·12115	+·07160	−·01140	−·05637	−·02931	1·63523
·052	·10641	·12232	+·07138	−·01260	−·05704	−·02866	1·62576
·053	·10803	·12347	+·07113	−·01380	−·05769	−·02799	1·61644
·054	·10964	·12460	+·07087	−·01499	−·05831	−·02732	1·60725
·055	·11124	·12571	+·07058	−·01618	−·05891	−·02662	1·59819
·056	·11284	·12680	+·07028	−·01736	−·05949	−·02592	1·58927
·057	·11442	·12787	+·06997	−·01854	−·06004	−·02520	1·58047
·058	·11600	·12892	+·06964	−·01971	−·06057	−·02447	1·57179
·059	·11756	·12995	+·06929	−·02087	−·06107	−·02373	1·56322
·060	·11912	·13096	+·06893	−·02203	−·06155	−·02298	1·55477
·061	·12067	·13196	+·06855	−·02318	−·06202	−·02222	1·54643
·062	·12222	·13293	+·06816	−·02433	−·06246	−·02145	1·53820
·063	·12375	·13389	+·06775	−·02547	−·06288	−·02068	1·53007
·064	·12528	·13483	+·06734	−·02660	−·06328	−·01989	1·52204
·065	·12679	·13575	+·06690	−·02773	−·06365	−·01910	1·51410
·066	·12830	·13666	+·06646	−·02884	−·06401	−·01830	1·50626
·067	·12981	·13754	+·06601	−·02996	−·06435	−·01749	1·49851
·068	·13130	·13842	+·06554	−·03106	−·06467	−·01668	1·49085
·069	·13279	·13927	+·06506	−·03216	−·06498	−·01586	1·48328
·070	·13427	·14011	+·06457	−·03325	−·06526	−·01504	1·47579
·071	·13574	·14094	+·06407	−·03433	−·06552	−·01421	1·46838
·072	·13720	·14175	+·06356	−·03541	−·06577	−·01337	1·46106
·073	·13866	·14254	+·06304	−·03648	−·06600	−·01253	1·45381
·074	·14011	·14332	+·06251	−·03754	−·06621	−·01169	1·44663
·075	·14156	·14409	+·06197	−·03859	−·06641	−·01085	1·43953
·076	·14299	·14484	+·06142	−·03963	−·06659	−·01000	1·43250
·077	·14442	·14558	+·06086	−·04067	−·06675	−·00915	1·42554
·078	·14584	·14630	+·06029	−·04170	−·06690	−·00829	1·41865
·079	·14726	·14701	+·05971	−·04272	−·06703	−·00743	1·41183
·080	·14867	·14771	+·05913	−·04374	−·06715	−·00658	1·40507
·081	·15007	·14839	+·05854	−·04474	−·06725	−·00572	1·39838
·082	·15146	·14906	+·05794	−·04574	−·06733	−·00485	1·39174
·083	·15285	·14971	+·05733	−·04673	−·06741	−·00399	1·38517
·084	·15423	·15036	+·05671	−·04771	−·06746	−·00312	1·37866
·085	·15561	·15099	+·05609	−·04869	−·06751	−·00226	1·37220
·086	·15698	·15160	+·05546	−·04965	−·06753	−·00139	1·36581
·087	·15834	·15221	+·05483	−·05061	−·06755	−·00053	1·35946
·088	·15970	·15280	+·05418	−·05156	−·06755	+·00034	1·35317
·089	·16105	·15339	+·05353	−·05250	−·06754	+·00120	1·34694
·090	·16239	·15396	+·05288	−·05344	−·06751	+·00207	1·34076
·091	·16373	·15451	+·05222	−·05436	−·06748	+·00294	1·33462
·092	·16506	·15506	+·05155	−·05528	−·06743	+·00380	1·32854
·093	·16639	·15560	+·05088	−·05619	−·06736	+·00467	1·32251
·094	·16770	·15612	+·05020	−·05709	−·06729	+·00553	1·31652
·095	·16902	·15663	+·04952	−·05798	−·06720	+·00639	1·31058
·096	·17033	·15713	+·04883	−·05887	−·06710	+·00725	1·30469
·097	·17163	·15763	+·04813	−·05975	−·06699	+·00811	1·29884
·098	·17292	·15811	+·04744	−·06061	−·06687	+·00897	1·29303
·099	·17421	·15858	+·04673	−·06148	−·06674	+·00982	1·28727
·100	·17550	·15904	+·04602	−·06233	−·06660	+·01068	1·28155

TABLE XXIX. *Tetrachoric Functions for Fourfold Correlation Tables.*

$\frac{1}{2}(1-a)$	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	h
·101	·17678	·15948	+·04531	−·06317	−·06644	+·01153	1·27587
·102	·17805	·15992	+·04459	−·06401	−·06628	+·01238	1·27024
·103	·17932	·16035	+·04387	−·06484	−·06610	+·01322	1·26464
·104	·18058	·16077	+·04315	−·06566	−·06592	+·01407	1·25908
·105	·18184	·16118	+·04242	−·06647	−·06572	+·01491	1·25357
·106	·18309	·16158	+·04169	−·06727	−·06551	+·01575	1·24808
·107	·18433	·16197	+·04095	−·06807	−·06530	+·01659	1·24264
·108	·18557	·16235	+·04021	−·06886	−·06507	+·01742	1·23723
·109	·18681	·16272	+·03947	−·06964	−·06484	+·01825	1·23186
·110	·18804	·16308	+·03872	−·07041	−·06459	+·01908	1·22653
·111	·18926	·16343	+·03797	−·07117	−·06434	+·01990	1·22123
·112	·19048	·16378	+·03721	−·07193	−·06408	+·02072	1·21596
·113	·19169	·16411	+·03646	−·07268	−·06381	+·02154	1·21073
·114	·19290	·16443	+·03570	−·07342	−·06353	+·02235	1·20553
·115	·19410	·16475	+·03493	−·07415	−·06324	+·02316	1·20036
·116	·19530	·16506	+·03417	−·07488	−·06294	+·02397	1·19522
·117	·19649	·16536	+·03340	−·07559	−·06264	+·02477	1·19012
·118	·19768	·16565	+·03263	−·07630	−·06233	+·02557	1·18504
·119	·19886	·16593	+·03186	−·07700	−·06201	+·02636	1·18000
·120	·20004	·16620	+·03108	−·07770	−·06168	+·02716	1·17499
·121	·20121	·16647	+·03030	−·07838	−·06134	+·02794	1·17000
·122	·20238	·16672	+·02952	−·07906	−·06100	+·02873	1·16505
·123	·20354	·16697	+·02874	−·07973	−·06065	+·02950	1·16012
·124	·20470	·16721	+·02796	−·08039	−·06029	+·03028	1·15522
·125	·20585	·16745	+·02717	−·08105	−·05992	+·03105	1·15035
·126	·20700	·16767	+·02638	−·08169	−·05955	+·03181	1·14551
·127	·20814	·16789	+·02559	−·08233	−·05917	+·03257	1·14069
·128	·20928	·16810	+·02480	−·08297	−·05878	+·03333	1·13590
·129	·21042	·16830	+·02401	−·08359	−·05839	+·03408	1·13113
·130	·21155	·16849	+·02321	−·08421	−·05799	+·03483	1·12639
·131	·21267	·16868	+·02241	−·08482	−·05758	+·03557	1·12168
·132	·21379	·16886	+·02162	−·08542	−·05717	+·03631	1·11699
·133	·21490	·16903	+·02082	−·08601	−·05675	+·03704	1·11232
·134	·21601	·16919	+·02001	−·08660	−·05632	+·03777	1·10768
·135	·21712	·16935	+·01921	−·08718	−·05589	+·03850	1·10306
·136	·21822	·16950	+·01841	−·08775	−·05546	+·03921	1·09847
·137	·21932	·16964	+·01760	−·08831	−·05501	+·03993	1·09390
·138	·22041	·16978	+·01680	−·08887	−·05456	+·04064	1·08935
·139	·22149	·16990	+·01599	−·08942	−·05411	+·04134	1·08482
·140	·22258	·17003	+·01518	−·08996	−·05365	+·04204	1·08032
·141	·22365	·17014	+·01437	−·09050	−·05318	+·04273	1·07584
·142	·22473	·17025	+·01356	−·09103	−·05271	+·04342	1·07138
·143	·22580	·17035	+·01275	−·09155	−·05224	+·04410	1·06694
·144	·22686	·17044	+·01194	−·09206	−·05176	+·04478	1·06252
·145	·22792	·17053	+·01113	−·09257	−·05127	+·04545	1·05812
·146	·22898	·17061	+·01032	−·09307	−·05078	+·04612	1·05374
·147	·23003	·17069	+·00950	−·09356	−·05028	+·04678	1·04939
·148	·23108	·17076	+·00869	−·09405	−·04978	+·04744	1·04505
·149	·23212	·17082	+·00788	−·09452	−·04928	+·04809	1·04073
·150	·23316	·17087	+·00706	−·09499	−·04877	+·04874	1·03643

TABLE XXIX.—(*continued*).

$\frac{1}{2}(1-a)$	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	h
·151	·23419	·17092	+·00625	−·09546	−·04825	+·04938	1·03215
·152	·23522	·17097	+·00543	−·09592	−·04774	+·05002	1·02789
·153	·23625	·17100	+·00462	−·09637	−·04721	+·05065	1·02365
·154	·23727	·17103	+·00380	−·09681	−·04669	+·05127	1·01943
·155	·23829	·17106	+·00298	−·09725	−·04615	+·05189	1·01522
·156	·23930	·17108	+·00217	−·09768	−·04562	+·05250	1·01103
·157	·24031	·17109	+·00135	−·09810	−·04508	+·05311	1·00686
·158	·24131	·17110	+·00053	−·09852	−·04454	+·05371	1·00271
·159	·24232	·17110	−·00028	−·09892	−·04399	+·05431	·99858
·160	·24331	·17109	−·00110	−·09933	−·04344	+·05490	·99446
·161	·24430	·17108	−·00191	−·09972	−·04288	+·05549	·99036
·162	·24529	·17107	−·00273	−·10011	−·04232	+·05607	·98627
·163	·24628	·17104	−·00355	−·10049	−·04176	+·05664	·98220
·164	·24726	·17102	−·00436	−·10087	−·04120	+·05721	·97815
·165	·24823	·17098	−·00518	−·10124	−·04063	+·05778	·97411
·166	·24921	·17094	−·00599	−·10160	−·04006	+·05834	·97009
·167	·25017	·17090	−·00681	−·10196	−·03948	+·05889	·96609
·168	·25114	·17085	−·00762	−·10231	−·03890	+·05943	·96210
·169	·25210	·17080	−·00844	−·10265	−·03832	+·05998	·95812
·170	·25305	·17073	−·00925	−·10299	−·03774	+·06051	·95417
·171	·25401	·17067	−·01007	−·10332	−·03715	+·06104	·95022
·172	·25495	·17060	−·01088	−·10364	−·03656	+·06156	·94629
·173	·25590	·17052	−·01169	−·10396	−·03597	+·06208	·94238
·174	·25684	·17044	−·01251	−·10427	−·03537	+·06260	·93848
·175	·25778	·17035	−·01332	−·10458	−·03478	+·06310	·93459
·176	·25871	·17026	−·01413	−·10487	−·03417	+·06360	·93072
·177	·25964	·17016	−·01494	−·10517	−·03357	+·06410	·92686
·178	·26056	·17006	−·01575	−·10545	−·03296	+·06459	·92301
·179	·26148	·16995	−·01656	−·10573	−·03236	+·06507	·91918
·180	·26240	·16984	−·01737	−·10601	−·03175	+·06555	·91537
·181	·26331	·16972	−·01817	−·10627	−·03113	+·06603	·91156
·182	·26422	·16960	−·01898	−·10653	−·03052	+·06649	·90777
·183	·26513	·16948	−·01979	−·10679	−·02990	+·06695	·90399
·184	·26603	·16934	−·02059	−·10704	−·02928	+·06741	·90023
·185	·26693	·16921	−·02140	−·10728	−·02866	+·06786	·89647
·186	·26782	·16907	−·02220	−·10752	−·02803	+·06830	·89273
·187	·26871	·16892	−·02300	−·10775	−·02741	+·06874	·88901
·188	·26960	·16877	−·02380	−·10798	−·02678	+·06917	·88529
·189	·27049	·16861	−·02460	−·10819	−·02615	+·06960	·88159
·190	·27137	·16845	−·02540	−·10841	−·02552	+·07002	·87790
·191	·27224	·16829	−·02620	−·10861	−·02489	+·07044	·87422
·192	·27311	·16812	−·02700	−·10882	−·02425	+·07085	·87055
·193	·27398	·16795	−·02779	−·10901	−·02362	+·07125	·86689
·194	·27485	·16777	−·02859	−·10920	−·02298	+·07165	·86325
·195	·27571	·16759	−·02938	−·10939	−·02234	+·07204	·85962
·196	·27657	·16740	−·03018	−·10956	−·02170	+·07243	·85600
·197	·27742	·16721	−·03097	−·10974	−·02106	+·07281	·85239
·198	·27827	·16701	−·03176	−·10990	−·02041	+·07319	·84879
·199	·27912	·16681	−·03255	−·11007	−·01977	+·07356	·84520
·200	·27996	·16661	−·03334	−·11022	−·01912	+·07392	·84162

TABLE XXIX. *Tetrachoric Functions for Fourfold Correlation Tables.*

$\frac{1}{2}(1-a)$	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	h
·201	·28080	·16640	−·03412	−·11037	−·01848	+·07428	·83805
·202	·28164	·16619	−·03491	−·11051	−·01783	+·07464	·83450
·203	·28247	·16597	−·03569	−·11065	−·01718	+·07498	·83095
·204	·28330	·16575	−·03648	−·11079	−·01653	+·07532	·82742
·205	·28413	·16553	−·03726	−·11091	−·01587	+·07566	·82389
·206	·28495	·16530	−·03804	−·11104	−·01522	+·07599	·82038
·207	·28577	·16506	−·03882	−·11115	−·01457	+·07632	·81687
·208	·28658	·16483	−·03959	−·11126	−·01391	+·07664	·81338
·209	·28739	·16459	−·04037	−·11137	−·01326	+·07695	·80990
·210	·28820	·16434	−·04114	−·11147	−·01260	+·07726	·80642
·211	·28901	·16409	−·04192	−·11157	−·01194	+·07756	·80296
·212	·28981	·16384	−·04269	−·11166	−·01129	+·07786	·79950
·213	·29060	·16358	−·04346	−·11174	−·01063	+·07815	·79606
·214	·29140	·16332	−·04423	−·11182	−·00997	+·07844	·79262
·215	·29219	·16305	−·04499	−·11189	−·00931	+·07872	·78919
·216	·29298	·16279	−·04576	−·11196	−·00865	+·07899	·78577
·217	·29376	·16251	−·04652	−·11203	−·00799	+·07926	·78237
·218	·29454	·16224	−·04728	−·11208	−·00733	+·07952	·77897
·219	·29532	·16196	−·04804	−·11214	−·00667	+·07978	·77557
·220	·29609	·16167	−·04880	−·11218	−·00600	+·08004	·77219
·221	·29686	·16139	−·04956	−·11223	−·00534	+·08028	·76882
·222	·29763	·16110	−·05031	−·11226	−·00468	+·08052	·76546
·223	·29840	·16080	−·05107	−·11230	−·00402	+·08076	·76210
·224	·29916	·16050	−·05182	−·11233	−·00335	+·08099	·75875
·225	·29991	·16020	−·05257	−·11235	−·00269	+·08122	·75541
·226	·30067	·15990	−·05332	−·11237	−·00203	+·08144	·75208
·227	·30142	·15959	−·05406	−·11238	−·00136	+·08165	·74876
·228	·30216	·15927	−·05481	−·11239	−·00070	+·08186	·74545
·229	·30291	·15896	−·05555	−·11239	−·00004	+·08207	·74214
·230	·30365	·15864	−·05629	−·11239	+·00063	+·08226	·73885
·231	·30439	·15832	−·05703	−·11238	+·00129	+·08246	·73556
·232	·30512	·15799	−·05777	−·11237	+·00195	+·08265	·73228
·233	·30585	·15766	−·05851	−·11235	+·00262	+·08283	·72900
·234	·30658	·15733	−·05924	−·11233	+·00328	+·08301	·72574
·235	·30730	·15699	−·05997	−·11230	+·00394	+·08318	·72248
·236	·30802	·15665	−·06070	−·11227	+·00461	+·08334	·71923
·237	·30874	·15631	−·06143	−·11224	+·00527	+·08351	·71599
·238	·30945	·15596	−·06215	−·11220	+·00593	+·08366	·71275
·239	·31017	·15561	−·06288	−·11215	+·00659	+·08381	·70952
·240	·31087	·15526	−·06360	−·11210	+·00726	+·08396	·70630
·241	·31158	·15490	−·06432	−·11205	+·00792	+·08410	·70309
·242	·31228	·15454	−·06504	−·11199	+·00858	+·08423	·69988
·243	·31298	·15418	−·06576	−·11192	+·00924	+·08436	·69668
·244	·31367	·15382	−·06647	−·11185	+·00990	+·08449	·69349
·245	·31436	·15345	−·06718	−·11178	+·01056	+·08461	·69031
·246	·31505	·15308	−·06789	−·11170	+·01122	+·08472	·68713
·247	·31574	·15270	−·06860	−·11162	+·01188	+·08483	·68396
·248	·31642	·15232	−·06931	−·11154	+·01253	+·08494	·68080
·249	·31710	·15194	−·07001	−·11145	+·01319	+·08504	·67764
·250	·31778	·15156	−·07071	−·11135	+·01385	+·08513	·67449

TABLE XXIX.—(*continued*).

$\frac{1}{2}(1-\alpha)$	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	h
·251	·31845	·15117	−·07141	−·11125	+·01450	+·08522	·67135
·252	·31912	·15078	−·07211	−·11115	+·01516	+·08530	·66821
·253	·31979	·15039	−·07280	−·11104	+·01581	+·08538	·66508
·254	·32045	·14999	−·07350	−·11093	+·01647	+·08546	·66196
·255	·32111	·14959	−·07419	−·11081	+·01712	+·08553	·65884
·256	·32177	·14919	−·07488	−·11069	+·01777	+·08559	·65573
·257	·32242	·14879	−·07557	−·11056	+·01842	+·08565	·65262
·258	·32307	·14838	−·07625	−·11043	+·01907	+·08571	·64952
·259	·32372	·14797	−·07693	−·11030	+·01972	+·08575	·64643
·260	·32437	·14756	−·07761	−·11016	+·02037	+·08580	·64335
·261	·32501	·14714	−·07829	−·11002	+·02102	+·08584	·64027
·262	·32565	·14672	−·07897	−·10987	+·02166	+·08587	·63719
·263	·32628	·14630	−·07964	−·10972	+·02231	+·08590	·63412
·264	·32691	·14588	−·08031	−·10956	+·02295	+·08593	·63106
·265	·32754	·14545	−·08098	−·10940	+·02360	+·08595	·62801
·266	·32817	·14502	−·08165	−·10924	+·02424	+·08596	·62496
·267	·32879	·14459	−·08231	−·10907	+·02488	+·08597	·62191
·268	·32941	·14415	−·08298	−·10890	+·02552	+·08598	·61887
·269	·33003	·14372	−·08364	−·10873	+·02616	+·08598	·61584
·270	·33065	·14328	−·08429	−·10855	+·02680	+·08598	·61281
·271	·33126	·14283	−·08495	−·10837	+·02743	+·08597	·60979
·272	·33187	·14239	−·08560	−·10818	+·02807	+·08596	·60678
·273	·33247	·14194	−·08625	−·10799	+·02870	+·08594	·60376
·274	·33307	·14149	−·08690	−·10779	+·02933	+·08591	·60076
·275	·33367	·14104	−·08755	−·10759	+·02997	+·08589	·59776
·276	·33427	·14058	−·08819	−·10739	+·03060	+·08586	·59477
·277	·33486	·14012	−·08883	−·10718	+·03122	+·08582	·59178
·278	·33545	·13966	−·08947	−·10697	+·03185	+·08578	·58879
·279	·33604	·13920	−·09011	−·10676	+·03248	+·08573	·58581
·280	·33662	·13873	−·09074	−·10654	+·03310	+·08568	·58284
·281	·33720	·13826	−·09137	−·10632	+·03372	+·08563	·57987
·282	·33778	·13779	−·09200	−·10609	+·03434	+·08557	·57691
·283	·33836	·13732	−·09263	−·10587	+·03496	+·08551	·57395
·284	·33893	·13685	−·09325	−·10563	+·03558	+·08544	·57100
·285	·33950	·13637	−·09388	−·10540	+·03620	+·08536	·56805
·286	·34007	·13589	−·09450	−·10516	+·03681	+·08529	·56511
·287	·34063	·13541	−·09511	−·10491	+·03743	+·08521	·56217
·288	·34119	·13492	−·09573	−·10466	+·03804	+·08512	·55924
·289	·34175	·13443	−·09634	−·10441	+·03865	+·08503	·55631
·290	·34230	·13394	−·09695	−·10416	+·03926	+·08494	·55338
·291	·34286	·13345	−·09756	−·10390	+·03987	+·08484	·55047
·292	·34341	·13296	−·09816	−·10364	+·04047	+·08473	·54755
·293	·34395	·13246	−·09876	−·10337	+·04107	+·08463	·54464
·294	·34449	·13196	−·09936	−·10310	+·04168	+·08451	·54174
·295	·34503	·13146	−·09996	−·10283	+·04228	+·08440	·53884
·296	·34557	·13096	−·10056	−·10256	+·04287	+·08428	·53594
·297	·34611	·13046	−·10115	−·10228	+·04347	+·08415	·53305
·298	·34664	·12995	−·10174	−·10199	+·04407	+·08402	·53016
·299	·34717	·12944	−·10233	−·10171	+·04466	+·08389	·52728
·300	·34769	·12893	−·10291	−·10142	+·04525	+·08375	·52440

TABLE XXIX. *Tetrachoric Functions for Fourfold Correlation Tables.*

$\frac{1}{2}(1-a)$	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	h
·301	·34822	·12841	−·10349	−·10113	+·04584	+·08361	·52153
·302	·34874	·12790	−·10407	−·10083	+·04643	+·08347	·51866
·303	·34925	·12738	−·10465	−·10053	+·04701	+·08332	·51579
·304	·34977	·12686	−·10522	−·10023	+·04759	+·08316	·51293
·305	·35028	·12634	−·10580	−·09992	+·04818	+·08301	·51007
·306	·35079	·12581	−·10636	−·09961	+·04876	+·08284	·50722
·307	·35129	·12529	−·10693	−·09930	+·04933	+·08268	·50437
·308	·35180	·12476	−·10750	−·09899	+·04991	+·08251	·50153
·309	·35230	·12423	−·10806	−·09867	+·05048	+·08233	·49869
·310	·35279	·12370	−·10862	−·09834	+·05105	+·08216	·49585
·311	·35329	·12316	−·10917	−·09802	+·05162	+·08197	·49302
·312	·35378	·12263	−·10973	−·09769	+·05219	+·08179	·49019
·313	·35427	·12209	−·11028	−·09736	+·05276	+·08160	·48736
·314	·35475	·12155	−·11082	−·09703	+·05332	+·08140	·48454
·315	·35524	·12101	−·11137	−·09669	+·05388	+·08121	·48173
·316	·35572	·12046	−·11191	−·09635	+·05444	+·08101	·47891
·317	·35620	·11992	−·11245	−·09600	+·05500	+·08080	·47610
·318	·35667	·11937	−·11299	−·09566	+·05555	+·08059	·47330
·319	·35714	·11882	−·11353	−·09531	+·05610	+·08038	·47050
·320	·35761	·11827	−·11406	−·09495	+·05665	+·08016	·46770
·321	·35808	·11771	−·11459	−·09460	+·05720	+·07994	·46490
·322	·35854	·11716	−·11512	−·09424	+·05775	+·07972	·46211
·323	·35900	·11660	−·11564	−·09388	+·05829	+·07949	·45933
·324	·35946	·11604	−·11616	−·09351	+·05883	+·07926	·45654
·325	·35991	·11548	−·11668	−·09315	+·05937	+·07902	·45376
·326	·36037	·11492	−·11720	−·09278	+·05991	+·07878	·45099
·327	·36082	·11436	−·11771	−·09240	+·06044	+·07854	·44821
·328	·36126	·11379	−·11822	−·09203	+·06097	+·07829	·44544
·329	·36171	·11322	−·11873	−·09165	+·06150	+·07804	·44268
·330	·36215	·11265	−·11923	−·09127	+·06203	+·07779	·43991
·331	·36259	·11208	−·11974	−·09088	+·06255	+·07753	·43715
·332	·36302	·11151	−·12024	−·09049	+·06308	+·07727	·43440
·333	·36346	·11093	−·12073	−·09010	+·06360	+·07701	·43164
·334	·36389	·11036	−·12123	−·08971	+·06412	+·07674	·42889
·335	·36431	·10978	−·12172	−·08932	+·06463	+·07647	·42615
·336	·36474	·10920	−·12221	−·08892	+·06514	+·07620	·42340
·337	·36516	·10862	−·12270	−·08852	+·06565	+·07592	·42066
·338	·36558	·10804	−·12318	−·08811	+·06616	+·07564	·41793
·339	·36600	·10745	−·12366	−·08771	+·06667	+·07535	·41519
·340	·36641	·10687	−·12414	−·08730	+·06717	+·07507	·41246
·341	·36682	·10628	−·12461	−·08689	+·06767	+·07477	·40974
·342	·36723	·10569	−·12509	−·08647	+·06817	+·07448	·40701
·343	·36764	·10510	−·12555	−·08606	+·06867	+·07418	·40429
·344	·36804	·10451	−·12602	−·08564	+·06916	+·07388	·40157
·345	·36844	·10391	−·12649	−·08522	+·06965	+·07358	·39886
·346	·36884	·10332	−·12695	−·08479	+·07014	+·07327	·39614
·347	·36923	·10272	−·12741	−·08437	+·07062	+·07296	·39343
·348	·36962	·10212	−·12786	−·08394	+·07110	+·07264	·39073
·349	·37001	·10152	−·12831	−·08351	+·07158	+·07232	·38802
·350	·37040	·10092	−·12876	−·08307	+·07206	+·07200	·38532

TABLE XXIX.—(*continued*).

$\frac{1}{2}(1-\alpha)$	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	h
·351	·37078	·10032	− ·12921	− ·08264	+ ·07254	+ ·07168	·38262
·352	·37116	·09971	− ·12966	− ·08220	+ ·07301	+ ·07135	·37993
·353	·37154	·09911	− ·13010	− ·08176	+ ·07348	+ ·07102	·37723
·354	·37192	·09850	− ·13054	− ·08131	+ ·07395	+ ·07069	·37454
·355	·37229	·09789	− ·13097	− ·08087	+ ·07441	+ ·07035	·37186
·356	·37266	·09728	− ·13140	− ·08042	+ ·07487	+ ·07002	·36917
·357	·37303	·09667	− ·13183	− ·07997	+ ·07533	+ ·06967	·36649
·358	·37340	·09606	− ·13226	− ·07952	+ ·07579	+ ·06933	·36381
·359	·37376	·09544	− ·13269	− ·07906	+ ·07624	+ ·06898	·36113
·360	·37412	·09483	− ·13311	− ·07861	+ ·07669	+ ·06863	·35846
·361	·37447	·09421	− ·13353	− ·07815	+ ·07714	+ ·06827	·35579
·362	·37483	·09359	− ·13394	− ·07768	+ ·07758	+ ·06792	·35312
·363	·37518	·09297	− ·13436	− ·07722	+ ·07803	+ ·06756	·35045
·364	·37553	·09235	− ·13477	− ·07675	+ ·07847	+ ·06719	·34779
·365	·37588	·09173	− ·13517	− ·07629	+ ·07890	+ ·06683	·34513
·366	·37622	·09111	− ·13558	− ·07582	+ ·07934	+ ·06646	·34247
·367	·37656	·09048	− ·13598	− ·07534	+ ·07977	+ ·06609	·33981
·368	·37690	·08985	− ·13638	− ·07487	+ ·08020	+ ·06571	·33715
·369	·37724	·08923	− ·13677	− ·07439	+ ·08062	+ ·06534	·33450
·370	·37757	·08860	− ·13717	− ·07391	+ ·08105	+ ·06496	·33185
·371	·37790	·08797	− ·13756	− ·07343	+ ·08147	+ ·06458	·32921
·372	·37823	·08734	− ·13794	− ·07295	+ ·08188	+ ·06419	·32656
·373	·37855	·08671	− ·13833	− ·07246	+ ·08230	+ ·06380	·32392
·374	·37888	·08607	− ·13871	− ·07198	+ ·08271	+ ·06341	·32128
·375	·37920	·08544	− ·13909	− ·07149	+ ·08312	+ ·06302	·31864
·376	·37951	·08480	− ·13946	− ·07100	+ ·08352	+ ·06262	·31600
·377	·37983	·08416	− ·13984	− ·07050	+ ·08392	+ ·06222	·31337
·378	·38014	·08353	− ·14021	− ·07001	+ ·08432	+ ·06182	·31074
·379	·38045	·08289	− ·14057	− ·06951	+ ·08472	+ ·06142	·30811
·380	·38076	·08225	− ·14094	− ·06901	+ ·08512	+ ·06101	·30548
·381	·38106	·08160	− ·14130	− ·06851	+ ·08551	+ ·06061	·30286
·382	·38136	·08096	− ·14166	− ·06801	+ ·08589	+ ·06019	·30023
·383	·38166	·08032	− ·14201	− ·06750	+ ·08628	+ ·05978	·29761
·384	·38196	·07967	− ·14236	− ·06700	+ ·08666	+ ·05936	·29499
·385	·38225	·07903	− ·14271	− ·06649	+ ·08704	+ ·05895	·29237
·386	·38254	·07838	− ·14306	− ·06598	+ ·08742	+ ·05853	·28976
·387	·38283	·07773	− ·14340	− ·06547	+ ·08779	+ ·05810	·28715
·388	·38312	·07708	− ·14374	− ·06495	+ ·08816	+ ·05766	·28454
·389	·38340	·07643	− ·14408	− ·06444	+ ·08853	+ ·05725	·28193
·390	·38368	·07578	− ·14442	− ·06392	+ ·08889	+ ·05682	·27932
·391	·38396	·07513	− ·14475	− ·06340	+ ·08925	+ ·05638	·27671
·392	·38423	·07447	− ·14508	− ·06288	+ ·08961	+ ·05595	·27411
·393	·38451	·07382	− ·14540	− ·06236	+ ·08997	+ ·05551	·27151
·394	·38478	·07316	− ·14573	− ·06183	+ ·09032	+ ·05507	·26891
·395	·38504	·07251	− ·14604	− ·06131	+ ·09067	+ ·05463	·26631
·396	·38531	·07185	− ·14636	− ·06078	+ ·09101	+ ·05419	·26371
·397	·38557	·07119	− ·14668	− ·06025	+ ·09136	+ ·05374	·26112
·398	·38583	·07053	− ·14699	− ·05972	+ ·09170	+ ·05329	·25853
·399	·38609	·06987	− ·14730	− ·05919	+ ·09203	+ ·05284	·25594
·400	·38634	·06921	− ·14760	− ·05866	+ ·09237	+ ·05239	·25335

TABLE XXIX. *Tetrachoric Functions for Fourfold Correlation Tables.*

$\frac{1}{2}(1-\alpha)$	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	h
·401	·38659	·06855	−·14790	−·05812	+·09270	+·05193	·25076
·402	·38684	·06789	−·14820	−·05758	+·09303	+·05148	·24817
·403	·38709	·06722	−·14850	−·05705	+·09335	+·05102	·24559
·404	·38734	·06656	−·14879	−·05651	+·09367	+·05056	·24301
·405	·38758	·06589	−·14908	−·05596	+·09399	+·05010	·24043
·406	·38782	·06522	−·14937	−·05542	+·09430	+·04963	·23785
·407	·38805	·06456	−·14965	−·05488	+·09462	+·04916	·23527
·408	·38829	·06389	−·14993	−·05433	+·09493	+·04869	·23269
·409	·38852	·06322	−·15021	−·05378	+·09523	+·04822	·23012
·410	·38875	·06255	−·15049	−·05323	+·09553	+·04775	·22754
·411	·38897	·06188	−·15076	−·05268	+·09583	+·04728	·22497
·412	·38920	·06121	−·15103	−·05213	+·09613	+·04680	·22240
·413	·38942	·06053	−·15130	−·05158	+·09642	+·04632	·21983
·414	·38964	·05986	−·15156	−·05102	+·09671	+·04584	·21727
·415	·38985	·05919	−·15182	−·05047	+·09700	+·04536	·21470
·416	·39007	·05851	−·15208	−·04991	+·09728	+·04488	·21214
·417	·39028	·05784	−·15233	−·04935	+·09756	+·04439	·20957
·418	·39049	·05716	−·15258	−·04879	+·09784	+·04390	·20701
·419	·39069	·05648	−·15283	−·04823	+·09811	+·04341	·20445
·420	·39089	·05580	−·15308	−·04767	+·09838	+·04292	·20189
·421	·39109	·05513	−·15332	−·04711	+·09865	+·04243	·19934
·422	·39129	·05445	−·15356	−·04654	+·09891	+·04194	·19678
·423	·39149	·05377	−·15380	−·04598	+·09918	+·04144	·19422
·424	·39168	·05309	−·15403	−·04541	+·09943	+·04094	·19167
·425	·39187	·05240	−·15426	−·04484	+·09969	+·04044	·18912
·426	·39206	·05172	−·15449	−·04427	+·09994	+·03994	·18657
·427	·39224	·05104	−·15471	−·04370	+·10019	+·03944	·18402
·428	·39243	·05036	−·15493	−·04313	+·10043	+·03894	·18147
·429	·39261	·04967	−·15515	−·04256	+·10067	+·03843	·17892
·430	·39279	·04899	−·15537	−·04198	+·10091	+·03793	·17637
·431	·39296	·04830	−·15558	−·04141	+·10115	+·03742	·17383
·432	·39313	·04761	−·15579	−·04083	+·10138	+·03691	·17128
·433	·39330	·04693	−·15599	−·04026	+·10161	+·03640	·16874
·434	·39347	·04624	−·15620	−·03968	+·10183	+·03589	·16620
·435	·39364	·04555	−·15640	−·03910	+·10205	+·03537	·16366
·436	·39380	·04486	−·15659	−·03852	+·10227	+·03486	·16112
·437	·39396	·04418	−·15679	−·03794	+·10249	+·03434	·15858
·438	·39411	·04349	−·15698	−·03735	+·10270	+·03382	·15604
·439	·39427	·04280	−·15717	−·03677	+·10291	+·03330	·15351
·440	·39442	·04211	−·15735	−·03619	+·10311	+·03278	·15097
·441	·39457	·04141	−·15753	−·03560	+·10331	+·03226	·14843
·442	·39472	·04072	−·15771	−·03502	+·10351	+·03174	·14590
·443	·39486	·04003	−·15789	−·03443	+·10371	+·03121	·14337
·444	·39501	·03934	−·15806	−·03384	+·10390	+·03069	·14084
·445	·39514	·03864	−·15823	−·03325	+·10409	+·03016	·13830
·446	·39528	·03795	−·15840	−·03266	+·10427	+·02963	·13577
·447	·39542	·03726	−·15856	−·03207	+·10446	+·02910	·13324
·448	·39555	·03656	−·15872	−·03148	+·10463	+·02858	·13072
·449	·39568	·03587	−·15888	−·03089	+·10481	+·02804	·12819
·450	·39580	·03517	−·15904	−·03030	+·10498	+·02751	·12566

TABLE XXIX.—(*continued*).

$\frac{1}{2}(1-a)$	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	h
·451	·39593	·03447	− ·15919	− ·02970	+ ·10515	+ ·02698	·12314
·452	·39605	·03378	− ·15934	− ·02911	+ ·10532	+ ·02644	·12061
·453	·39617	·03308	− ·15948	− ·02851	+ ·10548	+ ·02591	·11809
·454	·39629	·03238	− ·15962	− ·02792	+ ·10564	+ ·02537	·11556
·455	·39640	·03168	− ·15976	− ·02732	+ ·10579	+ ·02484	·11304
·456	·39651	·03099	− ·15990	− ·02673	+ ·10594	+ ·02430	·11052
·457	·39662	·03029	− ·16003	− ·02613	+ ·10609	+ ·02376	·10799
·458	·39673	·02959	− ·16016	− ·02553	+ ·10624	+ ·02322	·10547
·459	·39683	·02889	− ·16029	− ·02493	+ ·10638	+ ·02268	·10295
·460	·39694	·02819	− ·16041	− ·02433	+ ·10652	+ ·02214	·10043
·461	·39703	·02749	− ·16053	− ·02373	+ ·10665	+ ·02159	·09791
·462	·39713	·02679	− ·16065	− ·02313	+ ·10678	+ ·02105	·09540
·463	·39723	·02609	− ·16077	− ·02253	+ ·10691	+ ·02051	·09288
·464	·39732	·02539	− ·16088	− ·02193	+ ·10704	+ ·01996	·09036
·465	·39741	·02469	− ·16099	− ·02132	+ ·10716	+ ·01941	·08784
·466	·39749	·02398	− ·16109	− ·02072	+ ·10727	+ ·01887	·08533
·467	·39758	·02328	− ·16120	− ·02012	+ ·10739	+ ·01832	·08281
·468	·39766	·02258	− ·16130	− ·01951	+ ·10750	+ ·01777	·08030
·469	·39774	·02188	− ·16139	− ·01891	+ ·10761	+ ·01722	·07778
·470	·39781	·02117	− ·16149	− ·01830	+ ·10771	+ ·01668	·07527
·471	·39789	·02047	− ·16158	− ·01770	+ ·10781	+ ·01613	·07276
·472	·39796	·01977	− ·16166	− ·01709	+ ·10791	+ ·01558	·07024
·473	·39803	·01906	− ·16175	− ·01648	+ ·10801	+ ·01502	·06773
·474	·39809	·01836	− ·16183	− ·01588	+ ·10810	+ ·01447	·06522
·475	·39816	·01765	− ·16191	− ·01527	+ ·10818	+ ·01392	·06271
·476	·39822	·01695	− ·16198	− ·01466	+ ·10827	+ ·01337	·06020
·477	·39828	·01625	− ·16206	− ·01405	+ ·10835	+ ·01281	·05768
·478	·39834	·01554	− ·16212	− ·01344	+ ·10842	+ ·01226	·05517
·479	·39839	·01484	− ·16219	− ·01284	+ ·10850	+ ·01171	·05266
·480	·39844	·01413	− ·16225	− ·01223	+ ·10857	+ ·01115	·05015
·481	·39849	·01342	− ·16231	− ·01162	+ ·10864	+ ·01060	·04764
·482	·39854	·01272	− ·16237	− ·01101	+ ·10870	+ ·01004	·04513
·483	·39858	·01201	− ·16242	− ·01040	+ ·10876	+ ·00949	·04263
·484	·39862	·01131	− ·16247	− ·00979	+ ·10882	+ ·00893	·04012
·485	·39866	·01060	− ·16252	− ·00918	+ ·10887	+ ·00837	·03761
·486	·39870	·00990	− ·16257	− ·00857	+ ·10892	+ ·00782	·03510
·487	·39873	·00919	− ·16261	− ·00796	+ ·10896	+ ·00726	·03259
·488	·39876	·00848	− ·16265	− ·00734	+ ·10901	+ ·00670	·03008
·489	·39879	·00778	− ·16268	− ·00673	+ ·10905	+ ·00614	·02758
·490	·39882	·00707	− ·16271	− ·00612	+ ·10908	+ ·00559	·02507
·491	·39884	·00636	− ·16274	− ·00551	+ ·10912	+ ·00503	·02256
·492	·39886	·00566	− ·16277	− ·00490	+ ·10914	+ ·00447	·02005
·493	·39888	·00495	− ·16279	− ·00429	+ ·10917	+ ·00391	·01755
·494	·39890	·00424	− ·16281	− ·00367	+ ·10919	+ ·00335	·01504
·495	·39891	·00354	− ·16283	− ·00306	+ ·10921	+ ·00279	·01253
·496	·39892	·00283	− ·16284	− ·00245	+ ·10923	+ ·00224	·01003
·497	·39893	·00212	− ·16285	− ·00184	+ ·10924	+ ·00168	·00752
·498	·39894	·00141	− ·16286	− ·00122	+ ·10925	+ ·00112	·00501
·499	·39894	·00071	− ·16287	− ·00061	+ ·10925	+ ·00056	·00251
·500	·39894	·00000	− ·16287	·00000	+ ·10925	·00000	·00000

TABLE XXX. *Supplementary Tables for determining High*

$r = \cdot 80$.

$h=$	0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1·0	1·1	1·2
$k=0·0$	·3976	·3766	·3538	·3294	·3039	·2778	·2515	·2254	·2001	·1759	·1531	·1320	·1127
0·1	·3766	·3583	·3380	·3162	·2930	·2689	·2445	·2200	·1960	·1728	·1509	·1304	·1116
0·2	·3538	·3380	·3204	·3011	·2804	·2586	·2361	·2134	·1909	·1690	·1481	·1284	·1102
0·3	·3294	·3162	·3011	·2843	·2661	·2466	·2263	·2056	·1848	·1643	·1446	·1258	·1083
0·4	·3039	·2930	·2804	·2661	·2503	·2332	·2152	·1965	·1775	·1587	·1402	·1226	·1060
0·5	·2778	·2689	·2586	·2466	·2332	·2186	·2028	·1862	·1692	·1520	·1351	·1187	·1031
0·6	·2515	·2445	·2361	·2263	·2152	·2028	·1893	·1748	·1598	·1444	·1291	·1140	·0995
0·7	·2254	·2200	·2134	·2056	·1965	·1862	·1748	·1625	·1494	·1359	·1222	·1086	·0954
0·8	·2001	·1960	·1909	·1848	·1775	·1692	·1598	·1494	·1383	·1266	·1146	·1025	·0906
0·9	·1759	·1728	·1690	·1643	·1587	·1520	·1444	·1359	·1266	·1167	·1064	·0958	·0852
1·0	·1531	·1509	·1481	·1446	·1402	·1351	·1291	·1222	·1146	·1064	·0976	·0886	·0794
1·1	·1320	·1304	·1284	·1258	·1226	·1187	·1140	·1086	·1025	·0958	·0886	·0809	·0731
1·2	·1127	·1116	·1102	·1083	·1060	·1031	·0995	·0954	·0906	·0852	·0794	·0731	·0665
1·3	·0953	·0946	·0936	·0923	·0906	·0885	·0859	·0828	·0791	·0749	·0702	·0652	·0597
1·4	·0798	·0793	·0787	·0778	·0766	·0751	·0733	·0710	·0682	·0650	·0614	·0574	·0530
1·5	·0662	·0659	·0655	·0649	·0641	·0631	·0618	·0601	·0581	·0557	·0529	·0498	·0464
1·6	·0545	·0543	·0540	·0536	·0531	·0524	·0515	·0503	·0489	·0471	·0451	·0427	·0401
1·7	·0444	·0443	·0441	·0438	·0435	·0430	·0424	·0416	·0406	·0394	·0379	·0362	·0342
1·8	·0358	·0357	·0357	·0355	·0353	·0350	·0346	·0341	·0334	·0325	·0315	·0302	·0287
1·9	·0287	·0286	·0286	·0285	·0283	·0281	·0279	·0275	·0271	·0265	·0258	·0249	·0238
2·0	·0227	·0227	·0227	·0226	·0225	·0224	·0223	·0220	·0217	·0213	·0209	·0202	·0195
2·1	·0178	·0178	·0178	·0178	·0177	·0177	·0176	·0174	·0172	·0170	·0167	·0163	·0158
2·2	·0139	·0139	·0139	·0139	·0138	·0138	·0137	·0137	·0135	·0134	·0132	·0129	·0126
2·3	·0107	·0107	·0107	·0107	·0107	·0107	·0106	·0106	·0105	·0104	·0103	·0101	·0099
2·4	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0081	·0081	·0080	·0079	·0078	·0077
2·5	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0061	·0061	·0061	·0060	·0059
2·6	·0047	·0047	·0047	·0047	·0047	·0047	·0046	·0046	·0046	·0046	·0046	·0045	·0045

$r = \cdot 85$.

$h=$	0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1·0	1·1	1·2
$k=0·0$	·4117	·3905	·3670	·3417	·3149	·2873	·2595	·2319	·2052	·1798	·1560	·1341	·1141
0·1	·3905	·3723	·3518	·3292	·3050	·2796	·2537	·2277	·2022	·1777	·1546	·1332	·1136
0·2	·3670	·3518	·3342	·3145	·2930	·2702	·2464	·2222	·1983	·1749	·1527	·1319	·1127
0·3	·3417	·3292	·3145	·2978	·2791	·2588	·2374	·2154	·1931	·1712	·1501	·1301	·1116
0·4	·3149	·3050	·2930	·2791	·2632	·2457	·2268	·2070	·1867	·1665	·1467	·1277	·1099
0·5	·2873	·2796	·2702	·2588	·2457	·2309	·2146	·1972	·1790	·1606	·1423	·1246	·1078
0·6	·2595	·2537	·2464	·2374	·2268	·2146	·2008	·1859	·1700	·1535	·1370	·1206	·1049
0·7	·2319	·2277	·2222	·2154	·2070	·1972	·1859	·1733	·1597	·1453	·1306	·1158	·1014
0·8	·2052	·2022	·1983	·1931	·1867	·1790	·1700	·1597	·1483	·1360	·1232	·1101	·0971
0·9	·1798	·1777	·1749	·1712	·1665	·1606	·1535	·1453	·1360	·1258	·1149	·1035	·0920
1·0	·1560	·1546	·1527	·1501	·1467	·1423	·1370	·1306	·1232	·1149	·1058	·0962	·0862
1·1	·1341	·1332	·1319	·1301	·1277	·1246	·1206	·1158	·1101	·1035	·0962	·0882	·0798
1·2	·1141	·1136	·1127	·1116	·1099	·1078	·1049	·1014	·0971	·0920	·0862	·0798	·0729
1·3	·0963	·0959	·0954	·0947	·0936	·0921	·0901	·0876	·0845	·0807	·0763	·0712	·0656
1·4	·0805	·0803	·0800	·0795	·0788	·0778	·0765	·0748	·0725	·0698	·0665	·0626	·0583
1·5	·0666	·0665	·0664	·0661	·0656	·0650	·0642	·0630	·0615	·0595	·0571	·0543	·0510
1·6	·0547	·0547	·0546	·0544	·0541	·0538	·0532	·0525	·0514	·0501	·0484	·0464	·0439
1·7	·0445	·0445	·0444	·0443	·0442	·0440	·0436	·0432	·0425	·0416	·0405	·0390	·0373
1·8	·0359	·0359	·0359	·0358	·0357	·0356	·0354	·0351	·0347	·0341	·0334	·0324	·0312
1·9	·0287	·0287	·0287	·0287	·0286	·0285	·0284	·0283	·0280	·0276	·0272	·0265	·0257
2·0	·0227	·0227	·0227	·0227	·0227	·0227	·0226	·0225	·0224	·0221	·0218	·0214	·0209
2·1	·0179	·0179	·0179	·0178	·0178	·0178	·0178	·0177	·0176	·0175	·0173	·0171	·0167
2·2	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0138	·0138	·0137	·0136	·0135	·0133
2·3	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0106	·0106	·0105	·0104
2·4	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0081	·0081	·0081	·0080
2·5	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0061	·0061
2·6	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0046	·0046	·0046	·0046

Correlations from Tetrachoric Groupings.

$r = \cdot 80$.

$h=$	1·3	1·4	1·5	1·6	1·7	1·8	1·9	2·0	2·1	2·2	2·3	2·4	2·5	2·6
$k=0·0$	·0953	·0798	·0662	·0545	·0444	·0358	·0287	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·1	·0946	·0793	·0659	·0543	·0443	·0357	·0286	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·2	·0936	·0787	·0655	·0540	·0441	·0357	·0286	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·3	·0923	·0778	·0649	·0536	·0438	·0355	·0285	·0226	·0178	·0139	·0107	·0082	·0062	·0047
0·4	·0906	·0766	·0641	·0531	·0435	·0353	·0283	·0225	·0177	·0138	·0107	·0082	·0062	·0047
0·5	·0885	·0751	·0631	·0524	·0430	·0350	·0281	·0224	·0177	·0138	·0107	·0082	·0062	·0047
0·6	·0859	·0733	·0618	·0515	·0424	·0346	·0279	·0223	·0176	·0137	·0106	·0082	·0062	·0046
0·7	·0828	·0710	·0601	·0503	·0416	·0341	·0275	·0220	·0174	·0137	·0106	·0081	·0062	·0046
0·8	·0791	·0682	·0581	·0489	·0406	·0334	·0271	·0217	·0172	·0135	·0105	·0081	·0061	·0046
0·9	·0749	·0650	·0557	·0471	·0394	·0325	·0265	·0213	·0170	·0134	·0104	·0080	·0061	·0046
1·0	·0702	·0614	·0529	·0451	·0379	·0315	·0258	·0209	·0167	·0132	·0103	·0079	·0061	·0046
1·1	·0652	·0574	·0498	·0427	·0362	·0302	·0249	·0202	·0163	·0129	·0101	·0078	·0060	·0045
1·2	·0597	·0530	·0464	·0401	·0342	·0287	·0238	·0195	·0158	·0126	·0099	·0077	·0059	·0045
1·3	·0541	·0484	·0427	·0372	·0319	·0271	·0226	·0186	·0151	·0121	·0096	·0075	·0058	·0044
1·4	·0484	·0436	·0388	·0341	·0295	·0252	·0212	·0176	·0144	·0117	·0093	·0073	·0057	·0043
1·5	·0427	·0388	·0348	·0309	·0270	·0232	·0197	·0165	·0136	·0111	·0089	·0070	·0055	·0042
1·6	·0372	·0341	·0309	·0276	·0243	·0211	·0181	·0153	·0127	·0104	·0084	·0067	·0053	·0041
1·7	·0319	·0295	·0270	·0243	·0216	·0190	·0164	·0140	·0117	·0097	·0079	·0063	·0050	·0039
1·8	·0271	·0252	·0232	·0211	·0190	·0168	·0146	·0126	·0107	·0089	·0073	·0059	·0047	·0037
1·9	·0226	·0212	·0197	·0181	·0164	·0146	·0129	·0112	·0096	·0081	·0067	·0055	·0044	·0035
2·0	·0186	·0176	·0165	·0153	·0140	·0126	·0112	·0098	·0085	·0072	·0060	·0050	·0040	·0032
2·1	·0151	·0144	·0136	·0127	·0117	·0107	·0096	·0085	·0074	·0064	·0054	·0045	·0037	·0030
2·2	·0121	·0117	·0111	·0104	·0097	·0089	·0081	·0072	·0064	·0055	·0047	·0040	·0033	·0027
2·3	·0096	·0093	·0089	·0084	·0079	·0073	·0067	·0060	·0054	·0047	·0041	·0035	·0029	·0024
2·4	·0075	·0073	·0070	·0067	·0063	·0059	·0055	·0050	·0045	·0040	·0035	·0030	·0025	·0021
2·5	·0058	·0057	·0055	·0053	·0050	·0047	·0044	·0040	·0037	·0033	·0029	·0025	·0022	·0018
2·6	·0044	·0043	·0042	·0041	·0039	·0037	·0035	·0032	·0030	·0027	·0024	·0021	·0018	·0016

$r = \cdot 85$.

$h=$	1·3	1·4	1·5	1·6	1·7	1·8	1·9	2·0	2·1	2·2	2·3	2·4	2·5	2·6
$k=0·0$	·0963	·0805	·0666	·0547	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·1	·0959	·0803	·0665	·0547	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·2	·0954	·0800	·0664	·0546	·0444	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·3	·0947	·0795	·0661	·0544	·0443	·0358	·0287	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·4	·0936	·0788	·0656	·0541	·0442	·0357	·0286	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·5	·0921	·0778	·0650	·0538	·0440	·0356	·0285	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·6	·0901	·0765	·0642	·0532	·0436	·0354	·0284	·0226	·0178	·0139	·0107	·0082	·0062	·0047
0·7	·0876	·0748	·0630	·0525	·0432	·0351	·0283	·0225	·0177	·0138	·0107	·0082	·0062	·0047
0·8	·0845	·0725	·0615	·0514	·0425	·0347	·0280	·0224	·0176	·0138	·0107	·0082	·0062	·0047
0·9	·0807	·0698	·0595	·0501	·0416	·0341	·0276	·0221	·0175	·0137	·0106	·0081	·0062	·0046
1·0	·0763	·0665	·0571	·0484	·0405	·0334	·0272	·0218	·0173	·0136	·0106	·0081	·0062	·0046
1·1	·0712	·0626	·0543	·0464	·0390	·0324	·0265	·0214	·0171	·0135	·0105	·0081	·0061	·0046
1·2	·0656	·0583	·0510	·0439	·0373	·0312	·0257	·0209	·0167	·0133	·0104	·0080	·0061	·0046
1·3	·0597	·0535	·0473	·0411	·0352	·0297	·0247	·0202	·0163	·0130	·0102	·0079	·0060	·0046
1·4	·0535	·0485	·0432	·0380	·0329	·0280	·0234	·0194	·0157	·0126	·0100	·0078	·0060	·0045
1·5	·0473	·0432	·0390	·0346	·0302	·0260	·0220	·0183	·0150	·0121	·0097	·0076	·0058	·0045
1·6	·0411	·0380	·0346	·0311	·0274	·0239	·0204	·0172	·0142	·0116	·0093	·0073	·0057	·0044
1·7	·0352	·0329	·0302	·0274	·0245	·0216	·0186	·0159	·0133	·0109	·0088	·0070	·0055	·0043
1·8	·0297	·0280	·0260	·0239	·0216	·0192	·0168	·0144	·0122	·0102	·0083	·0067	·0053	·0041
1·9	·0247	·0234	·0220	·0204	·0186	·0168	·0149	·0129	·0111	·0093	·0077	·0063	·0050	·0039
2·0	·0202	·0194	·0183	·0172	·0159	·0144	·0129	·0114	·0099	·0084	·0070	·0058	·0047	·0037
2·1	·0163	·0157	·0150	·0142	·0133	·0122	·0111	·0099	·0087	·0075	·0063	·0053	·0043	·0034
2·2	·0130	·0126	·0121	·0116	·0109	·0102	·0093	·0084	·0075	·0065	·0056	·0047	·0039	·0032
2·3	·0102	·0100	·0097	·0093	·0088	·0083	·0077	·0070	·0063	·0056	·0049	·0042	·0035	·0029
2·4	·0079	·0078	·0076	·0073	·0070	·0067	·0063	·0058	·0053	·0047	·0042	·0036	·0031	·0025
2·5	·0060	·0060	·0058	·0057	·0055	·0053	·0050	·0047	·0043	·0039	·0035	·0031	·0026	·0022
2·6	·0046	·0045	·0045	·0044	·0043	·0041	·0039	·0037	·0034	·0032	·0029	·0025	·0022	·0019

TABLE XXX. *Supplementary Tables for determining High*

$r = \cdot 90$.

$h=$	0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1·0	1·1	1·2
$k=0·0$	·4282	·4067	·3822	·3552	·3266	·2969	·2670	·2377	·2094	·1827	·1579	·1353	·1149
0·1	·4067	·3887	·3678	·3441	·3183	·2910	·2630	·2350	·2077	·1817	·1574	·1350	·1147
0·2	·3822	·3678	·3504	·3302	·3076	·2830	·2573	·2311	·2052	·1801	·1564	·1345	·1144
0·3	·3552	·3441	·3302	·3135	·2943	·2728	·2498	·2258	·2016	·1778	·1550	·1336	·1140
0·4	·3266	·3183	·3076	·2943	·2784	·2602	·2401	·2187	·1966	·1744	·1528	·1322	·1132
0·5	·2969	·2910	·2830	·2728	·2602	·2453	·2284	·2097	·1900	·1698	·1497	·1302	·1119
0·6	·2670	·2630	·2573	·2498	·2401	·2284	·2145	·1988	·1817	·1637	·1454	·1274	·1101
0·7	·2377	·2350	·2311	·2258	·2187	·2097	·1988	·1860	·1717	·1561	·1399	·1236	·1075
0·8	·2094	·2077	·2052	·2016	·1966	·1900	·1817	·1717	·1600	·1470	·1331	·1186	·1041
0·9	·1827	·1817	·1801	·1778	·1744	·1698	·1637	·1561	·1470	·1365	·1249	·1124	·0997
1·0	·1579	·1574	·1564	·1550	·1528	·1497	·1454	·1399	·1331	·1249	·1155	·1052	·0942
1·1	·1353	·1350	·1345	·1336	·1322	·1302	·1274	·1236	·1186	·1124	·1052	·0969	·0878
1·2	·1149	·1147	·1144	·1140	·1132	·1119	·1101	·1075	·1041	·0997	·0942	·0878	·0806
1·3	·0967	·0966	·0965	·0962	·0958	·0950	·0939	·0923	·0900	·0869	·0830	·0783	·0727
1·4	·0807	·0807	·0806	·0805	·0802	·0798	·0792	·0782	·0767	·0747	·0720	·0686	·0645
1·5	·0668	·0668	·0667	·0667	·0665	·0663	·0660	·0654	·0645	·0632	·0614	·0591	·0562
1·6	·0548	·0548	·0548	·0547	·0547	·0546	·0544	·0540	·0535	·0528	·0516	·0501	·0481
1·7	·0446	·0446	·0446	·0445	·0445	·0445	·0444	·0442	·0439	·0435	·0428	·0418	·0405
1·8	·0359	·0359	·0359	·0359	·0359	·0359	·0358	·0357	·0356	·0353	·0350	·0344	·0338
1·9	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0286	·0286	·0284	·0282	·0279	·0274
2·0	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0226	·0225	·0223	·0220
2·1	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0178	·0178	·0178	·0177	·0176	·0175
2·2	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0138	·0138	·0137
2·3	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0106
2·4	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082
2·5	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062
2·6	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047

$r = \cdot 95$.

$h=$	0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1·0	1·1	1·2
$k=0·0$	·4495	·4271	·4005	·3705	·3385	·3055	·2729	·2414	·2116	·1840	·1586	·1357	·1151
0·1	·4271	·4099	·3880	·3622	·3333	·3026	·2713	·2407	·2113	·1839	·1586	·1356	·1151
0·2	·4005	·3880	·3712	·3500	·3252	·2976	·2685	·2392	·2106	·1835	·1585	·1356	·1150
0·3	·3705	·3622	·3500	·3338	·3135	·2898	·2637	·2365	·2092	·1829	·1582	·1355	·1150
0·4	·3385	·3333	·3252	·3135	·2980	·2787	·2564	·2320	·2067	·1816	·1576	·1352	·1149
0·5	·3055	·3026	·2976	·2898	·2787	·2640	·2459	·2250	·2024	·1792	·1563	·1346	·1147
0·6	·2729	·2713	·2685	·2637	·2564	·2459	·2321	·2153	·1960	·1753	·1542	·1335	·1141
0·7	·2414	·2407	·2392	·2365	·2320	·2250	·2153	·2025	·1870	·1694	·1506	·1315	·1131
0·8	·2116	·2113	·2106	·2092	·2067	·2024	·1960	·1870	·1753	·1611	·1452	·1283	·1113
0·9	·1840	·1839	·1835	·1829	·1816	·1792	·1753	·1694	·1611	·1505	·1377	·1234	·1084
1·0	·1586	·1586	·1585	·1582	·1576	·1563	·1542	·1506	·1452	·1377	·1281	·1167	·1041
1·1	·1357	·1356	·1356	·1355	·1352	·1346	·1335	·1315	·1283	·1234	·1167	·1082	·0981
1·2	·1151	·1151	·1150	·1150	·1149	·1147	·1141	·1131	·1113	·1084	·1041	·0981	·0906
1·3	·0968	·0968	·0968	·0968	·0967	·0966	·0964	·0959	·0950	·0934	·0908	·0870	·0818
1·4	·0808	·0808	·0808	·0808	·0807	·0807	·0806	·0804	·0800	·0792	·0778	·0755	·0721
1·5	·0668	·0668	·0668	·0668	·0668	·0668	·0668	·0667	·0665	·0661	·0654	·0642	·0622
1·6	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0547	·0545	·0542	·0536	·0525
1·7	·0446	·0446	·0446	·0446	·0446	·0446	·0446	·0445	·0445	·0445	·0443	·0440	·0435
1·8	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0358	·0357	·0355
1·9	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0286	·0285
2·0	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227
2·1	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0178
2·2	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139
2·3	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107
2·4	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082
2·5	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062
2·6	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047

Correlations from Tetrachoric Groupings.

$r = \cdot 90$.

$h=$	1·3	1·4	1·5	1·6	1·7	1·8	1·9	2·0	2·1	2·2	2·3	2·4	2·5	2·6
$k=0·0$	·0967	·0807	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·1	·0966	·0807	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·2	·0965	·0806	·0667	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·3	·0962	·0805	·0667	·0547	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·4	·0958	·0802	·0665	·0547	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·5	·0950	·0798	·0663	·0546	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·6	·0939	·0792	·0660	·0544	·0444	·0358	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·7	·0923	·0782	·0654	·0540	·0442	·0357	·0286	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·8	·0900	·0767	·0645	·0535	·0439	·0356	·0286	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·9	·0869	·0747	·0632	·0528	·0435	·0353	·0284	·0226	·0178	·0139	·0107	·0082	·0062	·0047
1·0	·0830	·0720	·0614	·0516	·0428	·0350	·0282	·0225	·0177	·0138	·0107	·0082	·0062	·0047
1·1	·0783	·0686	·0591	·0501	·0418	·0344	·0279	·0223	·0176	·0138	·0107	·0082	·0062	·0047
1·2	·0727	·0645	·0562	·0481	·0405	·0338	·0274	·0220	·0175	·0137	·0106	·0082	·0062	·0047
1·3	·0664	·0596	·0526	·0456	·0388	·0325	·0267	·0216	·0173	·0136	·0106	·0081	·0062	·0046
1·4	·0596	·0543	·0485	·0426	·0367	·0310	·0258	·0211	·0169	·0134	·0105	·0081	·0062	·0046
1·5	·0526	·0485	·0439	·0391	·0341	·0292	·0246	·0203	·0164	·0131	·0103	·0080	·0061	·0046
1·6	·0456	·0426	·0391	·0353	·0312	·0271	·0231	·0193	·0158	·0127	·0101	·0079	·0060	·0046
1·7	·0388	·0367	·0341	·0312	·0281	·0247	·0214	·0181	·0150	·0122	·0098	·0077	·0059	·0045
1·8	·0325	·0310	·0292	·0271	·0247	·0221	·0194	·0167	·0140	·0116	·0094	·0074	·0058	·0044
1·9	·0267	·0258	·0246	·0231	·0214	·0194	·0173	·0151	·0129	·0108	·0088	·0071	·0056	·0043
2·0	·0216	·0211	·0203	·0193	·0181	·0167	·0151	·0134	·0116	·0099	·0082	·0067	·0053	·0042
2·1	·0173	·0169	·0164	·0158	·0150	·0140	·0129	·0116	·0102	·0088	·0075	·0062	·0050	·0040
2·2	·0136	·0134	·0131	·0127	·0122	·0116	·0108	·0099	·0088	·0078	·0067	·0056	·0046	·0037
2·3	·0106	·0105	·0103	·0101	·0098	·0094	·0088	·0082	·0075	·0067	·0058	·0050	·0042	·0034
2·4	·0081	·0081	·0080	·0079	·0077	·0074	·0071	·0067	·0062	·0056	·0050	·0044	·0037	·0031
2·5	·0062	·0062	·0061	·0060	·0059	·0058	·0056	·0053	·0050	·0046	·0042	·0037	·0032	·0027
2·6	·0046	·0046	·0046	·0046	·0045	·0044	·0043	·0042	·0040	·0037	·0034	·0031	·0027	·0024

$r = \cdot 95$.

$h=$	1·3	1·4	1·5	1·6	1·7	1·8	1·9	2·0	2·1	2·2	2·3	2·4	2·5	2·6
$k=0·0$	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·1	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·2	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·3	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·4	·0967	·0807	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·5	·0966	·0807	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·6	·0964	·0806	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·7	·0959	·0804	·0667	·0548	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·8	·0950	·0800	·0665	·0547	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·9	·0934	·0792	·0661	·0545	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
1·0	·0908	·0778	·0654	·0542	·0443	·0358	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
1·1	·0870	·0755	·0642	·0536	·0440	·0357	·0286	·0227	·0179	·0139	·0107	·0082	·0062	·0047
1·2	·0818	·0721	·0622	·0525	·0435	·0355	·0285	·0227	·0178	·0139	·0107	·0082	·0062	·0047
1·3	·0752	·0676	·0593	·0508	·0426	·0350	·0283	·0226	·0178	·0139	·0107	·0082	·0062	·0047
1·4	·0676	·0619	·0554	·0483	·0411	·0342	·0279	·0224	·0177	·0139	·0107	·0082	·0062	·0047
1·5	·0593	·0554	·0505	·0450	·0390	·0330	·0273	·0221	·0176	·0138	·0107	·0082	·0062	·0047
1·6	·0508	·0483	·0450	·0409	·0362	·0312	·0263	·0215	·0173	·0137	·0106	·0082	·0062	·0047
1·7	·0426	·0411	·0390	·0362	·0328	·0289	·0248	·0207	·0169	·0135	·0105	·0081	·0062	·0047
1·8	·0350	·0342	·0330	·0312	·0289	·0261	·0229	·0195	·0162	·0131	·0104	·0080	·0062	·0046
1·9	·0283	·0279	·0273	·0263	·0248	·0229	·0205	·0179	·0152	·0125	·0101	·0079	·0061	·0046
2·0	·0226	·0224	·0221	·0215	·0207	·0195	·0179	·0160	·0139	·0117	·0096	·0077	·0060	·0046
2·1	·0178	·0177	·0176	·0173	·0169	·0162	·0152	·0139	·0124	·0107	·0090	·0073	·0058	·0045
2·2	·0139	·0139	·0138	·0137	·0135	·0131	·0125	·0117	·0107	·0095	·0082	·0068	·0055	·0043
2·3	·0107	·0107	·0107	·0106	·0105	·0104	·0101	·0096	·0090	·0082	·0072	·0062	·0051	·0041
2·4	·0082	·0082	·0082	·0082	·0081	·0080	·0079	·0077	·0073	·0068	·0062	·0054	·0046	·0038
2·5	·0062	·0062	·0062	·0062	·0062	·0062	·0061	·0060	·0058	·0055	·0051	·0046	·0040	·0034
2·6	·0047	·0047	·0047	·0047	·0047	·0046	·0046	·0046	·0045	·0043	·0041	·0038	·0034	·0030

TABLE XXX. *Supplementary Tables for determining High*

$r = 1{\cdot}00$.

$h=$	0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1·0	1·1	1·2
$k=0{\cdot}0$	·5000	·4602	·4207	·3821	·3446	·3085	·2743	·2420	·2119	·1841	·1587	·1357	·1151
0·1	·4602	·4602	·4207	·3821	·3446	·3085	·2743	·2420	·2119	·1841	·1587	·1357	·1151
0·2	·4207	·4207	·4207	·3821	·3446	·3085	·2743	·2420	·2119	·1841	·1587	·1357	·1151
0·3	·3821	·3821	·3821	·3821	·3446	·3085	·2743	·2420	·2119	·1841	·1587	·1357	·1151
0·4	·3446	·3446	·3446	·3446	·3446	·3085	·2743	·2420	·2119	·1841	·1587	·1357	·1151
0·5	·3085	·3085	·3085	·3085	·3085	·3085	·2743	·2420	·2119	·1841	·1587	·1357	·1151
0·6	·2743	·2743	·2743	·2743	·2743	·2743	·2743	·2420	·2119	·1841	·1587	·1357	·1151
0·7	·2420	·2420	·2420	·2420	·2420	·2420	·2420	·2420	·2119	·1841	·1587	·1357	·1151
0·8	·2119	·2119	·2119	·2119	·2119	·2119	·2119	·2119	·2119	·1841	·1587	·1357	·1151
0·9	·1841	·1841	·1841	·1841	·1841	·1841	·1841	·1841	·1841	·1841	·1587	·1357	·1151
1·0	·1587	·1587	·1587	·1587	·1587	·1587	·1587	·1587	·1587	·1587	·1587	·1357	·1151
1·1	·1357	·1357	·1357	·1357	·1357	·1357	·1357	·1357	·1357	·1357	·1357	·1357	·1151
1·2	·1151	·1151	·1151	·1151	·1151	·1151	·1151	·1151	·1151	·1151	·1151	·1151	·1151
1·3	·0968	·0968	·0968	·0968	·0968	·0968	·0968	·0968	·0968	·0968	·0968	·0968	·0968
1·4	·0808	·0808	·0808	·0808	·0808	·0808	·0808	·0808	·0808	·0808	·0808	·0808	·0808
1·5	·0668	·0668	·0668	·0668	·0668	·0668	·0668	·0668	·0668	·0668	·0668	·0668	·0668
1·6	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0548
1·7	·0446	·0446	·0446	·0446	·0446	·0446	·0446	·0446	·0446	·0446	·0446	·0446	·0446
1·8	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359
1·9	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287
2·0	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0228
2·1	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179
2·2	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139
2·3	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107
2·4	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082
2·5	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062
2·6	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047

Correlations from Tetrachoric Groupings.

$r = 1{\cdot}00$.

$h=$	1·3	1·4	1·5	1·6	1·7	1·8	1·9	2·0	2·1	2·2	2·3	2·4	2·5	2·6
$k=0{\cdot}0$	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·1	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·2	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·3	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·4	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·5	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·6	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·7	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·8	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·9	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·0	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·1	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·2	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·3	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·4	·0808	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·5	·0668	·0668	·0668	·0548	·0446	·0359	·0287	·0228	0179	·0139	·0107	·0082	·0062	·0047
1·6	·0548	·0548	·0548	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·7	·0446	·0446	·0446	·0446	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·8	·0359	·0359	·0359	·0359	·0359	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·9	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
2·0	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0179	·0139	·0107	·0082	·0062	·0047
2·1	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0139	·0107	·0082	·0062	·0047
2·2	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0107	·0082	·0062	·0047
2·3	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0082	·0062	·0047
2·4	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0062	·0047
2·5	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0047
2·6	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047

TABLE XXXI. *The* Γ-*Function.*

	Log Γ(*p*), Negative Characteristic, $\bar{1}$									
p	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
1·00	·999,9999	7497	5001	2512	0030	·998,7555	5087	2627	0173	$-\overline{7727}$
1·01	·997,5287	2855	0430	$-\overline{8011}$	$-\overline{5600}$	·996,3196	0798	$-\overline{8408}$	$-\overline{6025}$	$\overline{3648}$
1·02	·995,1279	$-\overline{8916}$	$-\overline{6561}$	$-\overline{4212}$	$-\overline{1870}$	·993,9535	7207	4886	2572	0265
1·03	·992,7964	5671	3384	1104	$-\overline{8831}$	·991,6564	4305	2052	$-\overline{9806}$	$-\overline{7567}$
1·04	·990,5334	3108	0889	$-\overline{8677}$	$-\overline{6471}$	·989,4273	2080	$-\overline{9895}$	$-\overline{7716}$	$-\overline{5544}$
1·05	·988,3379	1220	$-\overline{9068}$	$-\overline{6922}$	$-\overline{4783}$	·987,2651	0525	$-\overline{8406}$	$-\overline{6294}$	$-\overline{4188}$
1·06	·986,2089	$-\overline{9996}$	$-\overline{7910}$	$-\overline{5830}$	$-\overline{3757}$	·985,1690	$-\overline{9630}$	$-\overline{7577}$	$-\overline{5530}$	$-\overline{3489}$
1·07	·984,1455	$-\overline{9428}$	$-\overline{7407}$	$-\overline{5392}$	$-\overline{3384}$	·983,1382	$-\overline{9387}$	$-\overline{7398}$	$-\overline{5415}$	$-\overline{3439}$
1·08	·982,1469	$-\overline{9506}$	$-\overline{7549}$	$-\overline{5599}$	$-\overline{3655}$	·981,1717	$-\overline{9785}$	$-\overline{7860}$	$-\overline{5941}$	$-\overline{4029}$
1·09	·980,2123	0223	$-\overline{8329}$	$-\overline{6442}$	$-\overline{4561}$	·979,2686	0818	$-\overline{8956}$	$-\overline{7100}$	$-\overline{5250}$
1·10	·978,3407	1570	$-\overline{9738}$	$-\overline{7914}$	$-\overline{6095}$	·977,4283	2476	0676	$-\overline{8882}$	$-\overline{7095}$
1·11	·976,5313	3538	1768	0005	$-\overline{8248}$	·975,6497	4753	3014	1281	$-\overline{9555}$
1·12	·974,7834	6120	4411	2709	1013	·973,9323	7638	5960	4288	2622
1·13	·973,0962	$-\overline{9308}$	$-\overline{7659}$	$-\overline{6017}$	$-\overline{4381}$	·972,2751	1126	$-\overline{9508}$	$-\overline{7896}$	$-\overline{6289}$
1·14	·971,4689	3094	1505	$-\overline{9922}$	$-\overline{8345}$	·970,6774	5209	3650	2096	0549
1·15	·969,9007	7471	5941	4417	2898	·969,1386	$-\overline{9879}$	$-\overline{8378}$	$-\overline{6883}$	$-\overline{5393}$
1·16	·968,3910	2432	0960	$-\overline{9493}$	$-\overline{8033}$	·967,6578	5129	3686	2248	0816
1·17	·966,9390	7969	6554	5145	3742	·966,2344	0952	$-\overline{9566}$	$-\overline{8185}$	$-\overline{6810}$
1·18	·965,5440	4076	2718	1366	0019	·964,8677	7341	6011	4687	3368
1·19	·964,2054	0746	$-\overline{9444}$	$-\overline{8147}$	$-\overline{6856}$	·963,5570	4290	3016	1747	0483
1·20	·962,9225	7973	6725	5484	4248	·962,3017	1792	0573	$-\overline{9358}$	$-\overline{8150}$
1·21	·961,6946	5748	4556	3369	2188	·961,1011	$-\overline{9841}$	$-\overline{8675}$	$-\overline{7515}$	$-\overline{6361}$
1·22	·960,5212	4068	2930	1796	0669	·959,9546	8430	7318	6212	5111
1·23	·959,4015	2925	1840	0760	$-\overline{9685}$	·958,8616	7553	6494	5441	4393
1·24	·958,3350	2313	1280	0253	$-\overline{9232}$	·957,8215	7204	6198	5197	4201
1·25	·957,3211	2226	1246	0271	$-\overline{9301}$	·956,8337	7377	6423	5474	4530
1·26	·956,3592	2658	1730	0806	$-\overline{9888}$	·955,8975	8067	7165	6267	5374
1·27	·955,4487	3604	2727	1855	0988	·955,0126	$-\overline{9268}$	$-\overline{8416}$	$-\overline{7570}$	$-\overline{6728}$
1·28	·954,5891	5059	4232	3410	2593	·954,1782	0975	0173	$-\overline{9376}$	$-\overline{8585}$
1·29	·953,7798	7016	6239	5467	4700	·953,3938	3181	2429	1682	0940
1·30	·953,0203	$-\overline{9470}$	$-\overline{8743}$	$-\overline{8021}$	$-\overline{7303}$	·952,6590	5883	5180	4482	3789
1·31	·952,3100	2417	1739	1065	0396	·951,9732	9073	8419	7770	7125
1·32	·951,6485	5850	5220	4595	3975	·951,3359	2748	2142	1541	0944
1·33	·951,0353	$-\overline{9766}$	$-\overline{9184}$	$-\overline{8606}$	$-\overline{8034}$	·950,7466	6903	6344	5791	5242
1·34	·950,4698	4158	3624	3094	2568	·950,2048	1532	1021	0514	0012
1·35	·949,9515	9023	8535	8052	7573	·949,7100	6630	6166	5706	5251
1·36	·949,4800	4355	3913	3477	3044	·949,2617	2194	1776	1362	0953
1·37	·949,0549	0149	$-\overline{9754}$	$-\overline{9363}$	$-\overline{8977}$	·948,8595	8218	7846	7478	7115
1·38	·948,6756	6402	6052	5707	5366	·948,5030	4698	4371	4049	3731
1·39	·948,3417	3108	2803	2503	2208	·948,1916	1630	1348	1070	0797
1·40	·948,0528	0263	0003	$-\overline{9748}$	$-\overline{9497}$	·947,9250	9008	8770	8537	8308
1·41	·947,8084	7864	7648	7437	7230	·947,7027	6829	6636	6446	6261
1·42	·947,6081	5905	5733	5565	5402	·947,5243	5089	4939	4793	4652
1·43	·947,4515	4382	4254	4130	4010	·947,3894	3783	3676	3574	3476
1·44	·947,3382	3292	3207	3125	3049	·947,2976	2908	2844	2784	2728
1·45	·947,2677	2630	2587	2549	2514	·947,2484	2459	2437	2419	2406
1·46	·947,2397	2393	2392	2396	2404	·947,2416	2432	2452	2477	2506
1·47	·947,2539	2576	2617	2662	2712	·947,2766	2824	2886	2952	3022
1·48	·947,3097	3175	3258	3345	3436	·947,3531	3630	3734	3841	3953
1.49	·947,4068	4188	4312	4440	4572	947,4708	4848	4992	5141	5293
1.50	·947,5449	5610	5774	5943	6116	·947,6292	6473	6658	6847	7040

A horizontal bar means that the third figure of the mantissa has changed, a negative sign that it must be *lowered* one unit.

Differences :—Negative down to rule										
0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*p*
2503	2496	2489	2482	2475	2468	2460	2454	2446	2440	*1·00*
2432	2425	2419	2411	2404	2398	2390	2383	2377	2369	*1·01*
2363	2355	2349	2342	2335	2328	2321	2314	2307	2301	*1·02*
2293	2287	2280	2273	2267	2259	2253	2246	2239	2233	*1·03*
2226	2219	2212	2206	2198	2193	2185	2179	2172	2165	*1·04*
2159	2152	2146	2139	2132	2126	2119	2112	2106	2099	*1·05*
2093	2086	2080	2073	2067	2060	2053	2047	2041	2034	*1·06*
2027	2021	2015	2008	2002	1995	1989	1983	1976	1970	*1·07*
1963	1957	1950	1944	1938	1932	1925	1919	1912	1906	*1·08*
1900	1894	1887	1881	1875	1868	1862	1856	1850	1843	*1·09*
1837	1832	1824	1819	1812	1807	1800	1794	1787	1782	*1·10*
1775	1770	1763	1757	1751	1744	1739	1733	1726	1721	*1·11*
1714	1709	1702	1696	1690	1685	1678	1672	1666	1660	*1·12*
1654	1649	1642	1636	1630	1625	1618	1612	1607	1600	*1·13*
1595	1589	1583	1577	1571	1565	1559	1554	1547	1542	*1·14*
1536	1530	1524	1519	1512	1507	1501	1495	1490	1483	*1·15*
1478	1472	1467	1460	1455	1449	1443	1438	1432	1426	*1·16*
1421	1415	1409	1403	1398	1392	1386	1381	1375	1370	*1·17*
1364	1358	1352	1347	1342	1336	1330	1324	1319	1314	*1·18*
1308	1302	1297	1291	1286	1280	1274	1269	1264	1258	*1·19*
1252	1248	1241	1236	1231	1225	1219	1215	1208	1204	*1·20*
1198	1192	1187	1181	1177	1170	1166	1160	1154	1149	*1·21*
1144	1138	1134	1127	1123	1116	1112	1106	1101	1096	*1·22*
1090	1085	1080	1075	1069	1063	1059	1053	1048	1043	*1·23*
1037	1033	1027	1021	1017	1011	1006	1001	996	990	*1·24*
985	980	975	970	964	960	954	949	944	938	*1·25*
934	928	924	918	913	908	902	898	893	887	*1·26*
883	877	872	867	862	858	852	846	842	837	*1·27*
832	827	822	817	811	807	802	797	791	787	*1·28*
782	777	772	767	762	757	752	747	742	737	*1·29*
733	727	722	718	713	707	703	698	693	689	*1·30*
683	678	674	669	664	659	654	649	645	640	*1·31*
635	630	625	620	616	611	606	601	597	591	*1·32*
587	582	578	572	568	563	559	553	549	544	*1·33*
540	534	530	526	520	516	511	507	502	497	*1·34*
492	488	483	479	473	470	464	460	455	451	*1·35*
445	442	436	433	427	423	418	414	409	404	*1·36*
400	395	391	386	382	377	372	368	363	359	*1·37*
354	350	345	341	336	332	327	322	318	314	*1·38*
309	305	300	295	292	286	282	278	273	269	*1·39*
265	260	255	251	247	242	238	233	229	224	*1·40*
220	216	211	207	203	198	193	190	185	180	*1·41*
176	172	168	163	159	154	150	146	141	137	*1·42*
133	128	124	120	116	111	107	102	98	94	*1·43*
90	85	82	76	73	68	64	60	56	51	*1·44*
47	43	−38	−35	−30	−25	−22	−18	−13	− 9	*1·45*
− 4	− 1									
*		+ 4	+ 8	+12	+16	+20	+25	+29	+33	*1·46*
+37	+41	45	50	54	58	62	66	70	75	*1·47*
78	83	87	91	95	99	104	107	112	115	*1·48*
120	124	128	132	136	140	144	149	152	156	*1·49*
161	164	169	173	176	181	185	189	193	197	*1·50*

* Differences change sign at horizontal rule.

TABLE XXXI. *The Γ-Function.*

	Log Γ(*p*), Negative Characteristic, $\bar{1}$									
p	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
1·51	·947,7237	7437	7642	7851	8064	·947,8281	8502	8727	8956	9189
1·52	·947,9426	9667	9912	+$\bar{0}$161	+$\bar{0}$414	·948,0671	0932	1196	1465	1738
1·53	·948,2015	2295	2580	2868	3161	·948,3457	3758	4062	4370	4682
1·54	·948,4998	5318	5642	5970	6302	·948,6638	6977	7321	7668	8019
1·55	·948,8374	8733	9096	9463	9834	·949,0208	0587	0969	1355	1745
1·56	·949,2139	2537	2938	3344	3753	·949,4166	4583	5004	5429	5857
1·57	·949,6289	6725	7165	7609	8056	·949,8508	8963	9422	9885	+$\bar{0}$351
1·58	·950,0822	1296	1774	2255	2741	·950,3230	3723	4220	4720	5225
1·59	·950,5733	6245	6760	7280	7803	·950,8330	8860	9395	9933	+$\bar{0}$475
1·60	·951,1020	1569	2122	2679	3240	·951,3804	4372	4943	5519	6098
1·61	·951,6680	7267	7857	8451	9048	·951,9649	+$\bar{0}$254	+$\bar{0}$862	+$\bar{1}$475	+$\bar{2}$091
1·62	·952,2710	3333	3960	4591	5225	·952,5863	6504	7149	7798	8451
1·63	·952,9107	9766	+$\bar{0}$430	+$\bar{1}$097	+$\bar{1}$767	·953,2442	3120	3801	4486	5175
1·64	·953,5867	6563	7263	7966	8673	·953,9383	+$\bar{0}$097	+$\bar{0}$815	+$\bar{1}$536	+$\bar{2}$260
1·65	·954,2989	3721	4456	5195	5938	·954,6684	7434	8187	8944	9704
1·66	·955,0468	1236	2007	2782	3560	·955,4342	5127	5916	6708	7504
1·67	·955,8303	9106	9913	+$\bar{0}$723	+$\bar{1}$536	·956,2353	3174	3998	4825	5656
1·68	·956,6491	7329	8170	9015	9864	·957,0716	1571	2430	3293	4159
1·69	·957,5028	5901	6777	7657	8540	·957,9427	+$\bar{0}$317	+$\bar{1}$211	+$\bar{2}$108	+$\bar{3}$008
1·70	·958,3912	4820	5731	6645	7563	·958,8484	9409	+$\bar{0}$337	+$\bar{1}$268	+$\bar{2}$203
1·71	·959,3141	4083	5028	5977	6929	·959,7884	8843	9805	+$\bar{0}$771	+$\bar{1}$740
1·72	·960,2712	3688	4667	5650	6636	·960,7625	8618	9614	+$\bar{0}$613	+$\bar{1}$616
1·73	·961,2622	3632	4645	5661	6681	·961,7704	8730	9760	+$\bar{0}$793	+$\bar{1}$830
1·74	·962,2869	3912	4959	6009	7062	·962,8118	9178	+$\bar{0}$241	+$\bar{1}$308	+$\bar{2}$378
1·75	·963,3451	4527	5607	6690	7776	·963,8866	9959	+$\bar{1}$055	+$\bar{2}$155	+$\bar{3}$258
1·76	·964,4364	5473	6586	7702	8821	·964,9944	+$\bar{1}$070	+$\bar{2}$199	+$\bar{3}$331	+$\bar{4}$467
1·77	·965,5606	6749	7894	9043	+$\bar{0}$195	·966,1350	2509	3671	4836	6004
1·78	·966,7176	8351	9529	+$\bar{0}$710	+$\bar{1}$895	·967,3082	4274	5468	6665	7866
1·79	·967,9070	+$\bar{0}$277	+$\bar{1}$488	+$\bar{2}$701	+$\bar{3}$918	·968,5138	6361	7588	8818	+$\bar{0}$051
1·80	·969,1287	2526	3768	5014	6263	·969,7515	8770	+$\bar{0}$029	+$\bar{1}$291	+$\bar{2}$555
1·81	·970,3823	5095	6369	7646	8927	·971,0211	1498	2788	4082	5378
1·82	·971,6678	7981	9287	+$\bar{0}$596	+$\bar{1}$908	·972,3224	4542	5864	7189	8517
1·83	·972,9848	+$\bar{1}$182	+$\bar{2}$520	+$\bar{3}$860	+$\bar{5}$204	·973,6551	7900	9254	+$\bar{0}$610	+$\bar{1}$969
1·84	·974,3331	4697	6065	7437	8812	·975,0190	1571	2955	4342	5733
1·85	·975,7126	8522	9922	+$\bar{1}$325	+$\bar{2}$730	·976,4139	5551	6966	8384	9805
1·86	·977,1230	2657	4087	5521	6957	·977,8397	9839	+$\bar{1}$285	+$\bar{2}$734	+$\bar{4}$186
1·87	·978,5640	7098	8559	+$\bar{0}$023	+$\bar{1}$490	·979,2960	4433	5909	7389	8871
1·88	·980,0356	1844	3335	4830	6327	·980,7827	9331	+$\bar{0}$837	+$\bar{2}$346	+$\bar{3}$859
1·89	·981,5374	6893	8414	9939	+$\bar{1}$466	·982,2996	4530	6066	7606	9148
1·90	·983,0693	2242	3793	5348	6905	·983,8465	+$\bar{0}$028	+$\bar{1}$595	+$\bar{3}$164	+$\bar{4}$736
1·91	·984,6311	7890	9471	+$\bar{1}$055	+$\bar{2}$642	·985,4232	5825	7421	9020	+$\bar{0}$621
1·92	·986,2226	3834	5445	7058	8675	·987,0294	1917	3542	5170	6802
1·93	·987,8436	+$\bar{0}$073	+$\bar{1}$713	+$\bar{3}$356	+$\bar{5}$002	·988,6651	8302	9957	+$\bar{1}$614	+$\bar{3}$275
1.94	·989,4938	6605	8274	9946	+$\bar{1}$621	·990,3299	4980	6663	8350	+$\bar{0}$039
1·95	·991,1732	3427	5125	6826	8530	·992,0237	1947	3659	5375	7093
1·96	·992,8815	+$\bar{0}$539	+$\bar{2}$266	+$\bar{3}$995	+$\bar{5}$728	·993,7464	9202	+$\bar{0}$943	+$\bar{2}$688	+$\bar{4}$435
1·97	·994,6185	7937	9693	+$\bar{1}$451	+$\bar{3}$213	·995,4977	6744	8513	+$\bar{0}$286	+$\bar{2}$062
1·98	·996,3840	5621	7405	9192	+$\bar{0}$982	·997,2774	4569	6368	8169	9972
1·99	·998,1779	3588	5401	7216	9034	·999,0854	2678	4504	6333	8165

A horizontal bar means that the third figure of the mantissa has changed, a positive sign that it must be *raised* one unit.

Differences :—on this page, Positive										
0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*p*
200	205	209	213	217	221	225	229	233	237	*1·51*
241	245	249	253	257	261	264	269	273	277	*1·52*
280	285	288	293	296	301	304	308	312	316	*1·53*
320	324	328	332	336	339	344	347	351	355	*1·54*
359	363	367	371	374	379	382	386	390	394	*1·55*
398	401	406	409	413	417	421	425	428	432	*1·56*
436	440	444	447	452	455	459	463	466	471	*1·57*
474	478	481	486	489	493	497	500	505	508	*1·58*
512	515	520	523	527	530	535	538	542	545	*1·59*
549	553	557	561	564	568	571	576	579	582	*1·60*
587	590	594	597	601	605	608	613	616	619	*1·61*
623	627	631	634	638	641	645	649	653	656	*1·62*
659	664	667	670	675	678	681	685	689	692	*1·63*
696	700	703	707	710	714	718	721	724	729	*1·64*
732	735	739	743	746	750	753	757	760	764	*1·65*
768	771	775	778	782	785	789	792	796	799	*1·66*
803	807	810	813	817	821	824	827	831	835	*1·67*
838	841	845	849	852	855	859	863	866	869	*1·68*
873	876	880	883	887	890	894	897	900	904	*1·69*
908	911	914	918	921	925	928	931	935	938	*1·70*
942	945	949	952	955	959	962	966	969	972	*1·71*
976	979	983	986	989	993	996	999	1003	1006	*1·72*
1010	1013	1016	1020	1023	1026	1030	1033	1037	1039	*1·73*
1043	1047	1050	1053	1056	1060	1063	1067	1070	1073	*1·74*
1076	1080	1083	1086	1090	1093	1096	1100	1103	1106	*1·75*
1109	1113	1116	1119	1123	1126	1129	1132	1136	1139	*1·76*
1143	1145	1149	1152	1155	1159	1162	1165	1168	1172	*1·77*
1175	1178	1181	1185	1187	1192	1194	1197	1201	1204	*1·78*
1207	1211	1213	1217	1220	1223	1227	1230	1233	1236	*1·79*
1239	1242	1246	1249	1252	1255	1259	1262	1264	1268	*1·80*
1272	1274	1277	1281	1284	1287	1290	1294	1296	1300	*1·81*
1303	1306	1309	1312	1316	1318	1322	1325	1328	1331	*1·82*
1334	1338	1340	1344	1347	1349	1354	1356	1359	1362	*1·83*
1366	1368	1372	1375	1378	1381	1384	1387	1391	1393	*1·84*
1396	1400	1403	1405	1409	1412	1415	1418	1421	1425	*1·85*
1427	1430	1434	1436	1440	1442	1446	1449	1452	1454	*1·86*
1458	1461	1464	1467	1470	1473	1476	1480	1482	1485	*1·87*
1488	1491	1495	1497	1500	1504	1506	1509	1513	1515	*1·88*
1519	1521	1525	1527	1530	1534	1536	1540	1542	1545	*1·89*
1549	1551	1555	1557	1560	1563	1567	1569	1572	1575	*1·90*
1579	1581	1584	1587	1590	1593	1596	1599	1601	1605	*1·91*
1608	1611	1613	1617	1619	1623	1625	1628	1632	1634	*1·92*
1637	1640	1643	1646	1649	1651	1655	1657	1661	1663	*1·93*
1667	1669	1672	1675	1678	1681	1683	1687	1689	1693	*1·94*
1695	1698	1701	1704	1707	1710	1712	1716	1718	1722	*1·95*
1724	1727	1729	1733	1736	1738	1741	1745	1747	1750	*1·96*
1752	1756	1758	1762	1764	1767	1769	1773	1776	1778	*1·97*
1781	1784	1787	1790	1792	1795	1799	1801	1803	1807	*1·98*
1809	1813	1815	1818	1820	1824	1826	1829	1832	1835	*1·99*

TABLE XXXII. *Subtense from Arc and Chord*

Table to pass from measured index $\beta = 100\,(arc - chord)/chord$ *of a curve to the index and may be closely represented by a common catenary. Suggested use: to pass*

Values of α for given values of β as argument.

β	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9
13	23·1	23·2	23·2	23·3	23·4	23·5	23·6	23·7	23·8	23·9
14	24·0	24·1	24·2	24·3	24·4	24·5	24·6	24·7	24·7	24·8
15	24·9	25·0	25·1	25·2	25·3	25·4	25·5	25·6	25·6	25·7
16	25·8	25·9	26·0	26·1	26·2	26·3	26·4	26·4	26·5	26·6
17	26·7	26·8	26·9	27·0	27·0	27·1	27·2	27·3	27·4	27·5
18	27·6	27·7	27·7	27·8	27·9	28·0	28·1	28·2	28·3	28·3
19	28·4	28·5	28·6	28·7	28·7	28·8	28·9	29·0	29·1	29·2
20	29·2	29·3	29·4	29·5	29·6	29·6	29·7	29·8	29·9	30·0
21	30·0	30·1	30·2	30·3	30·4	30·4	30·5	30·6	30·7	30·8
22	30·8	30·9	31·0	31·1	31·2	31·2	31·3	31·4	31·5	31·6
23	31·6	31·7	31·8	31·9	31·9	32·0	32·1	32·2	32·3	32·3
24	32·4	32·5	32·6	32·6	32·7	32·8	32·9	32·9	33·0	33·1
25	33·2	33·3	33·3	33·4	33·5	33·6	33·6	33·7	33·8	33·9
26	33·9	34·0	34·1	34·2	34·2	34·3	34·4	34·5	34·5	34·6
27	34·7	34·8	34·8	34·9	35·0	35·1	35·1	35·2	35·3	35·3
28	35·4	35·5	35·6	35·6	35·7	35·8	35·9	35·9	36·0	36·1
29	36·2	36·2	36·3	36·4	36·4	36·5	36·6	36·7	36·7	36·8
30	36·9	36·9	37·0	37·1	37·2	37·2	37·3	37·4	37·5	37·5
31	37·6	37·7	37·7	37·8	37·9	38·0	38·0	38·1	38·2	38·2
32	38·3	38·4	38·4	38·5	38·6	38·7	38·7	38·8	38·9	38·9
33	39·0	39·1	39·2	39·2	39·3	39·4	39·4	39·5	39·6	39·6
34	39·7	39·8	39·8	39·9	40·0	40·1	40·1	40·2	40·3	40·3
35	40·4	40·5	40·5	40·6	40·7	40·7	40·8	40·9	41·0	41·0
36	41·1	41·2	41·2	41·3	41·4	41·4	41·5	41·6	41·6	41·7
37	41·8	41·8	41·9	42·0	42·0	42·1	42·2	42·2	42·3	42·4
38	42·4	42·5	42·6	42·6	42·7	42·8	42·9	42·9	43·0	43·1
39	43·1	43·2	43·3	43·3	43·4	43·5	43·5	43·6	43·7	43·7
40	43·8	43·9	43·9	44·0	44·1	44·1	44·2	44·3	44·3	44·4
41	44·5	44·5	44·6	44·6	44·7	44·8	44·8	44·9	45·0	45·0
42	45·1	45·2	45·2	45·3	45·4	45·4	45·5	45·6	45·6	45·7
43	45·8	45·8	45·9	46·0	46·0	46·1	46·2	46·2	46·3	46·4
44	46·4	46·5	46·5	46·6	46·7	46·7	46·8	46·9	46·9	47·0
45	47·1	47·1	47·2	47·3	47·3	47·4	47·5	47·5	47·6	47·6
46	47·7	47·8	47·8	47·9	48·0	48·0	48·1	48·2	48·2	48·3
47	48·4	48·4	48·5	48·5	48·6	48·7	48·7	48·8	48·9	48·9
48	49·0	49·1	49·1	49·2	49·2	49·3	49·4	49·4	49·5	49·6
49	49·6	49·7	49·8	49·8	49·9	49·9	50·0	50·1	50·1	50·2
50	50·3	50·3	50·4	50·5	50·5	50·6	50·6	50·7	50·8	50·8
51	50·9	51·0	51·0	51·1	51·1	51·2	51·3	51·3	51·4	51·5
52	51·5	51·6	51·6	51·7	51·8	51·8	51·9	52·0	52·0	52·1
53	52·1	52·2	52·3	52·3	52·4	52·5	52·5	52·6	52·6	52·7
54	52·8	52·8	52·9	53·0	53·0	53·1	53·1	53·2	53·3	53·3
55	53·4	53·4	53·5	53·6	53·6	53·7	53·8	53·8	53·9	53·9
56	54·0	54·1	54·1	54·2	54·3	54·3	54·4	54·4	54·5	54·6
57	54·6	54·7	54·7	54·8	54·9	54·9	55·0	55·0	55·1	55·2
58	55·2	55·3	55·4	55·4	55·5	55·5	55·6	55·7	55·7	55·8
59	55·8	55·9	56·0	56·0	56·1	56·1	56·2	56·3	56·3	56·4
60	56·5	56·5	56·6	56·6	56·7	56·8	56·8	56·9	56·9	57·0
61	57·1	57·1	57 2	57·2	57·3	57·4	57·4	57·5	57·5	57·6
62	57·7	57·7	57·8	57·8	57·9	58·0	58·0	58·1	58·1	58·2
63	58·3	58·3	58·4	58·4	58·5	58·6	58·6	58·7	58·7	58·8
64	58·9	58·9	59·0	59·0	59·1	59·2	59·2	59·3	59·3	59·4

in the case of the Common Catenary.

$\alpha = 100$ *subtense/chord, on the assumption that the curve is symmetrical about the subtense from callipers and tape measurements of the nasal bridge to the ratio of "rise" to "span."*

Values of α for given values of β as argument.

β	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9
65	59·5	59·5	59·6	59·6	59·7	59·8	59·8	59·9	59·9	60·0
66	60·1	60·1	60·2	60·2	60·3	60·4	60·4	60·5	60·5	60·6
67	60·7	60·7	60·8	60·8	60·9	61·0	61·0	61·1	61·1	61·2
68	61·3	61·3	61·4	61·4	61·5	61·6	61·6	61·7	61·7	61·8
69	61·9	61·9	62·0	62·0	62·1	62·1	62·2	62·3	62·3	62·4
70	62·4	62·5	62·6	62·6	62·7	62·7	62·8	62·9	62·9	63·0
71	63·0	63·1	63·1	63·2	63·3	63·3	63·4	63·4	63·5	63·6
72	63·6	63·7	63·7	63·8	63·9	63·9	64·0	64·0	64·1	64·1
73	64·2	64·3	64·3	64·4	64·4	64·5	64·6	64·6	64·7	64·7
74	64·8	64·9	64·9	65·0	65·0	65·1	65·1	65·2	65·3	65·3
75	65·4	65·4	65·5	65·6	65·6	65·7	65·7	65·8	65·8	65·9
76	66·0	66·0	66·1	66·2	66·2	66·3	66·3	66·4	66·4	66·5
77	66·5	66·6	66·7	66·7	66·8	66·8	66·9	66·9	67·0	67·1
78	67·1	67·2	67·2	67·3	67·4	67·4	67·5	67·5	67·6	67·6
79	67·7	67·8	67·8	67·9	67·9	68·0	68·0	68·1	68·2	68·2
80	68·3	68·3	68·4	68·5	68·5	68·6	68·6	68·7	68·7	68·8
81	68·9	68·9	69·0	69·0	69·1	69·1	69·2	69·3	69·3	69·4
82	69·4	69·5	69·5	69·6	69·7	69·7	69·8	69·8	69·9	70·0
83	70·0	70·1	70·1	70·2	70·2	70·3	70·4	70·4	70·5	70·5
84	70·6	70·6	70·7	70·8	70·8	70·9	70·9	71·0	71·0	71·1
85	71·2	71·2	71·3	71·3	71·4	71·4	71·5	71·6	71·6	71·7
86	71·7	71·8	71·8	71·9	72·0	72·0	72·1	72·1	72·2	72·2
87	72·3	72·4	72·4	72·5	72·5	72·6	72·6	72·7	72·8	72·8
88	72·9	72·9	73·0	73·0	73·1	73·2	73·2	73·3	73·3	73·4
89	73·4	73·5	73·6	73·6	73·7	73·7	73·8	73·8	73·9	73·9
90	74·0	74·1	74·1	74·2	74·2	74·3	74·3	74·4	74·5	74·5
91	74·6	74·6	74·7	74·7	74·8	74·9	74·9	75·0	75·0	75·1
92	75·1	75·2	75·3	75·3	75·4	75·4	75·5	75·5	75·6	75·6
93	75·7	75·8	75·8	75·9	75·9	76·0	76·0	76·1	76·2	76·2
94	76·3	76·3	76·4	76·4	76·5	76·6	76·6	76·7	76·7	76·8
95	76·8	76·9	76·9	77·0	77·1	77·1	77·2	77·2	77·3	77·3
96	77·4	77·5	77·5	77·6	77·6	77·7	77·7	77·8	77·8	77·9
97	78·0	78·0	78·1	78·1	78·2	78·2	78·3	78·3	78·4	78·5
98	78·5	78·6	78·6	78·7	78·7	78·8	78·9	78·9	79·0	79·0
99	79·1	79·1	79·2	79·2	79·3	79·4	79·4	79·5	79·5	79·6
100	79·6	79·7	79·8	79·8	79·9	79·9	80·0	80·0	80·1	80·1
101	80·2	80·3	80·3	80·4	80·4	80·5	80·5	80·6	80·6	80·7
102	80·8	80·8	80·9	80·9	81·0	81·0	81·1	81·1	81·2	81·3
103	81·3	81·4	81·4	81·5	81·5	81·6	81·6	81·7	81·8	81·3
104	81·9	81·9	82·0	82·0	82·1	82·1	82·2	82·3	82·3	82·4
105	82·4	82·5	82·5	82·6	82·6	82·7	82·8	82·8	82·9	82·9
106	83·0	83·0	83·1	83·1	83·2	83·3	83·3	83·4	83·4	83·5
107	83·5	83·6	83·6	83·7	83·8	83·8	83·9	83·9	84·0	84·0
108	84·1	84·1	84·2	84·3	84·3	84·4	84·4	84·5	84·5	84·6
109	84·6	84·7	84·8	84·8	84·9	84·9	85·0	85·0	85·1	85·1
110	85·2	85·3	85·3	85·4	85·4	85·5	85·5	85·6	85·6	85·7
111	85·8	85·8	85·9	85·9	86·0	86·0	86·1	86·1	86·2	86·2
112	86·3	86·4	86·4	86·5	86·5	86·6	86·6	86·7	86·7	86·8
113	86·9	86·9	87·0	87·0	87·1	87·1	87·2	87·2	87·3	87·4
114	87·4	87·5	87·5	87·6	87·6	87·7	87·7	87·8	87·8	87·9
115	88·0	88·0	88·1	88·1	88·2	88·2	88·3	88·3	88·4	88·5
116	88·5	88·6	88·6	88·7	88·7	88·8	88·8	88·9	88·9	89·0

TABLE XXXIII, A *and* B. *Supplementary Tables of Subtense from Arc and Chord.*

TABLE XXXIII.

Supplementary Tables for Subtense Index α as calculated from the arcual value β on the Catenary Hypothesis.

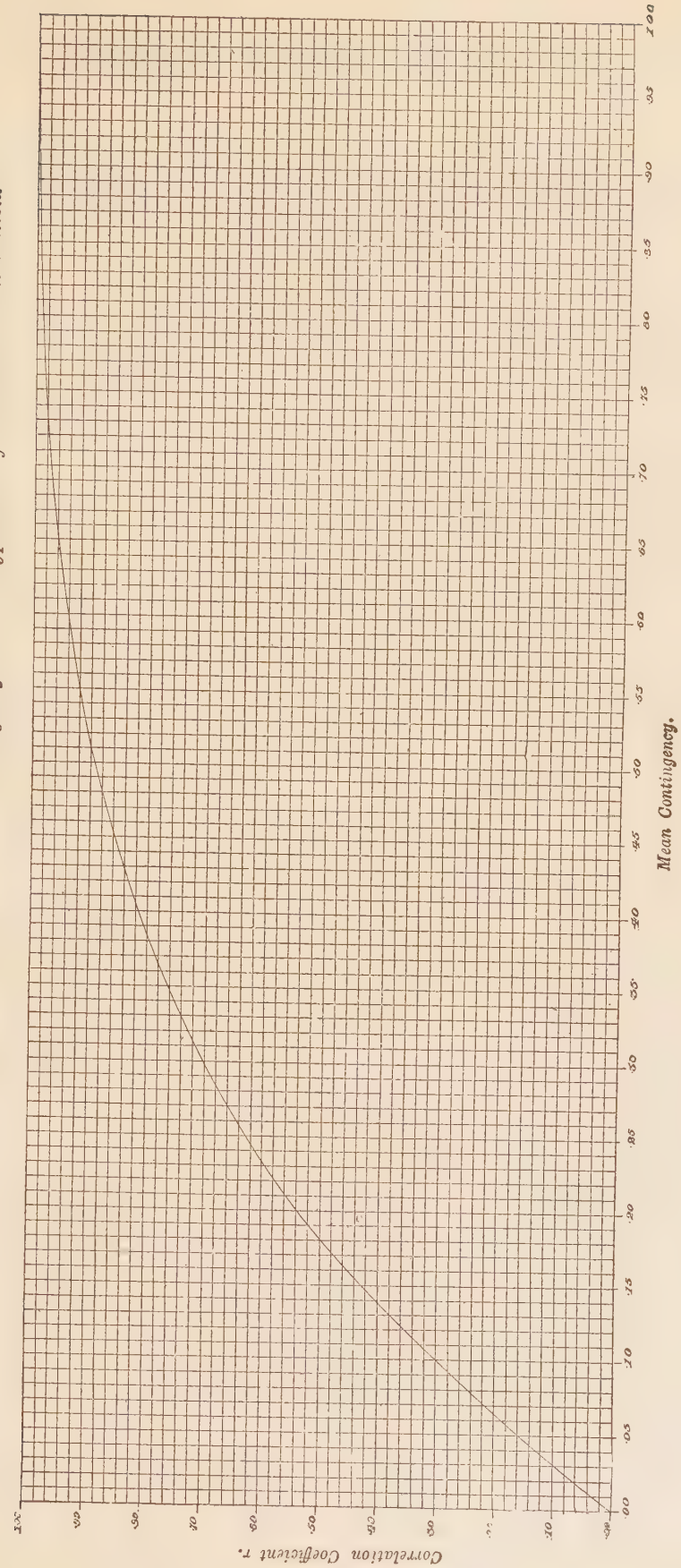
(A) *Values of α for low β.*

β	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9
6	15·3	15·4	15·6	15·7	15·8	16·0	16·1	16·2	16·3	16·5
7	16·6	16·7	16·8	17·0	17·1	17·2	17·3	17·4	17·6	17·7
8	17·8	17·9	18·0	18·1	18·3	18·4	18·5	18·6	18·7	18·8
9	18·9	19·1	19·2	19·3	19·4	19·5	19·6	19·7	19·8	19·9
10	20·0	20·1	20·2	20·3	20·4	20·6	20·7	20·8	20·9	21·0
11	21·1	21·2	21·3	21·4	21·5	21·6	21·7	21·8	21·9	22·0
12	22·1	22·2	22·3	22·4	22·5	22·6	22·7	22·8	22·9	23·0

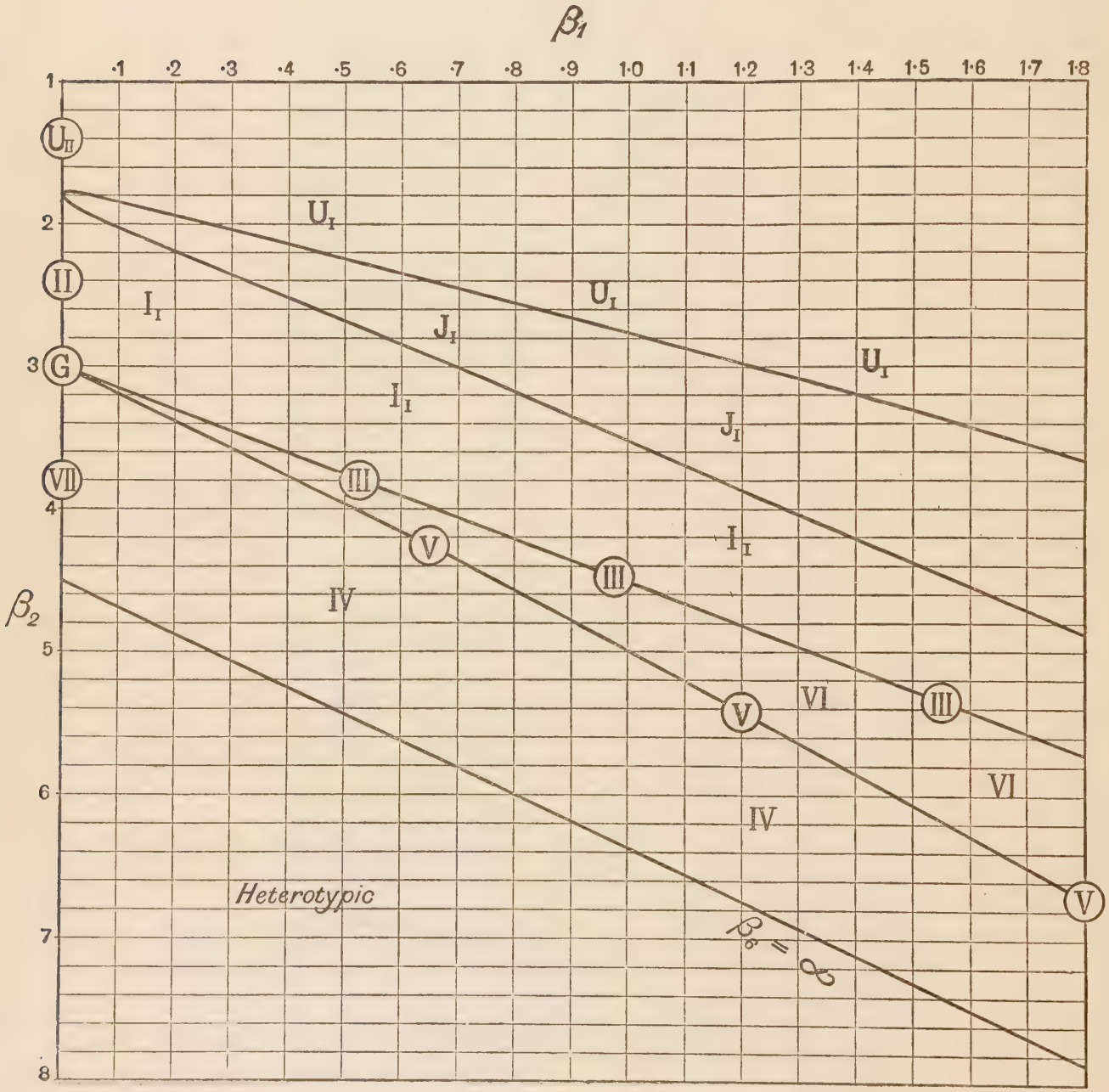
(B) *Values of α for high β.*

β	α	β	α	β	α	β	α	β	α	β	α
101	80·2	*126*	94·0	*151*	107·5	*176*	120·8	*201*	133·9	*226*	147·0
102	80·8	*127*	94·5	*152*	108·0	*177*	121·3	*202*	134·5	*227*	147·5
103	81·3	*128*	95·1	*153*	108·5	*178*	121·8	*203*	135·0	*228*	148·0
104	81·9	*129*	95·6	*154*	109·1	*179*	122·4	*204*	135·5	*229*	148·6
105	82·4	*130*	96·2	*155*	109·6	*180*	122·9	*205*	136·0	*230*	149·1
106	83·0	*131*	96·7	*156*	110·2	*181*	123·4	*206*	136·6	*231*	149·6
107	83·5	*132*	97·2	*157*	110·7	*182*	124·0	*207*	137·1	*232*	150·1
108	84·1	*133*	97·8	*158*	111·2	*183*	124·5	*208*	137·6	*233*	150·6
109	84·6	*134*	98·3	*159*	111·8	*184*	125·0	*209*	138·1	*234*	151·2
110	85·2	*135*	98·9	*160*	112·3	*185*	125·5	*210*	138·7	*235*	151·7
111	85·8	*136*	99·4	*161*	112·8	*186*	126·1	*211*	139·2	*236*	152·2
112	86·3	*137*	99·9	*162*	113·4	*187*	126·6	*212*	139·7	*237*	152·7
113	86·9	*138*	100·5	*163*	113·9	*188*	127·1	*213*	140·2	*238*	153·2
114	87·4	*139*	101·0	*164*	114·4	*189*	127·7	*214*	140·8	*239*	153·8
115	88·0	*140*	101·6	*165*	114·9	*190*	128·2	*215*	141·3	*240*	154·3
116	88·5	*141*	102·1	*166*	115·5	*191*	128·7	*216*	141·8	*241*	154·8
117	89·1	*142*	102·7	*167*	116·0	*192*	129·2	*217*	142·3	*242*	155·3
118	89·6	*143*	103·2	*168*	116·5	*193*	129·8	*218*	142·8	*243*	155·8
119	90·1	*144*	103·7	*169*	117·1	*194*	130·3	*219*	143·4	*244*	156·3
120	90·7	*145*	104·3	*170*	117·6	*195*	130·8	*220*	143·9	*245*	156·9
121	91·2	*146*	104·8	*171*	118·1	*196*	131·3	*221*	144·4	*246*	157·4
122	91·8	*147*	105·3	*172*	118·7	*197*	131·8	*222*	144·9	*247*	157·9
123	92·3	*148*	105·9	*173*	119·2	*198*	132·4	*223*	145·4	*248*	158·4
124	92·9	*149*	106·4	*174*	119·7	*199*	132·9	*224*	146·0	*249*	159·0
125	93·4	*150*	106·9	*175*	120·2	*200*	133·4	*225*	146·5	*250*	159·5

XXXIV. *Diagram to find Correlation r from Mean Contingency on the Hypothesis of a Normal Distribution.*



XXXV. *Diagram to determine the type of a Frequency Distribution from a knowledge of the Constants β_1 and β_2. Customary Values of β_1 and β_2.*



XXXVI. *Diagram showing Distribution of Frequency Types for High Values for β_1 and β_2.*

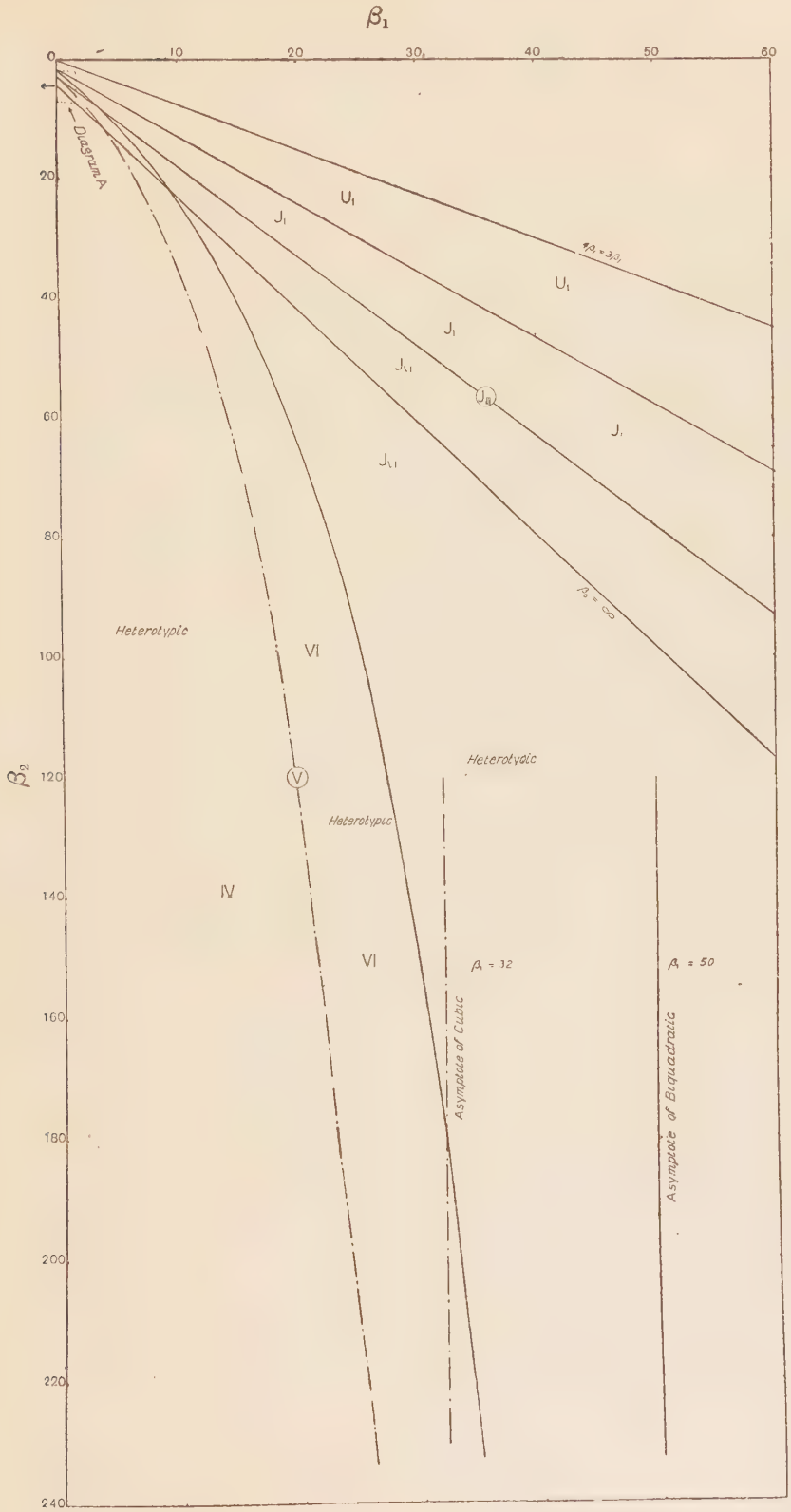


TABLE XXXVII. *To find the Probable Error of* β_1.

Values of $\sqrt{N}\,\Sigma_\beta$.

β_1

β_2	0·00	0·05	0·10	0·15	0·20	0·25	0·30	0·35	0·40	0·45	0·50	0·55	0·60	0·65	0·70	0·75
2·0	0·00	0·58	0·93	1·15	1·37	1·57	1·77	1·97	2·17	2·38	2·58	2·80	3·02	3·24	3·46	3·71
2·1	0·00	0·59	0·95	1·12	1·30	1·50	1·70	1·90	2·10	2·29	2·48	2·69	2·91	3·12	3·34	3·57
2·2	0·00	0·60	0·97	1·13	1·30	1·48	1·67	1·86	2·05	2·22	2·41	2·61	2·81	3·01	3·22	3·44
2·3	0·00	0·62	0·99	1·15	1·32	1·49	1·67	1·84	2·02	2·19	2·36	2·55	2·74	2·93	3·12	3·32
2·4	0·00	0·64	1·02	1·20	1·37	1·54	1·70	1·86	2·02	2·18	2·34	2·51	2·68	2·85	3·03	3·22
2·5	0·00	0·66	1·05	1·26	1·45	1·61	1·76	1·91	2·05	2·19	2·33	2·49	2·65	2·81	2·97	3·14
2·6	0·00	0·69	1·10	1·34	1·54	1·72	1·86	2·00	2·13	2·25	2·37	2·50	2·64	2·78	2·92	3·08
2·7	0·00	0·73	1·15	1·42	1·64	1·83	1·96	2·09	2·22	2·32	2·42	2·53	2·65	2·77	2·90	3·05
2·8	0·00	0·77	1·22	1·51	1·75	1·94	2·07	2·20	2·32	2·41	2·50	2·60	2·70	2·80	2·93	3·05
2·9	0·00	0·81	1·30	1·61	1·87	2·06	2·20	2·33	2·44	2·53	2·62	2·70	2·79	2·89	2·99	3·09
3·0	0·00	0·87	1·40	1·73	2·01	2·20	2·34	2·47	2·57	2·67	2·76	2·84	2·92	3·00	3·09	3·18
3·1	0·00	0·94	1·53	1·86	2·17	2·35	2·51	2·64	2·75	2·85	2·94	3·00	3·08	3·15	3·23	3·30
3·2	0·00	1·02	1·67	2·02	2·33	2·52	2·71	2·84	2·95	3·05	3·14	3·22	3·27	3·33	3·40	3·46
3·3	0·00	1·12	1·82	2·20	2·50	2·71	2·92	3·06	3·18	3·28	3·37	3·44	3·50	3·55	3·60	3·65
3·4	0·00	1·24	1·99	2·38	2·68	2·93	3·14	3·30	3·43	3·53	3·63	3·70	3·75	3·79	3·83	3·87
3·5	0·00	1·37	2·16	2·57	2·89	3·17	3·39	3·56	3·69	3·81	3·91	3·98	4·02	4·06	4·10	4·12
3·6	0·00	1·50	2·33	2·78	3·11	3·43	3·65	3·84	3·99	4·12	4·22	4·29	4·33	4·37	4·40	4·41
3·7	0·00	1·64	2·50	2·99	3·36	3·70	3·93	4·14	4·31	4·44	4·54	4·61	4·66	4·70	4·72	4·74
3·8	0·00	1·78	2·67	3·20	3·62	3·97	4·23	4·46	4·64	4·77	4·87	4·95	5·00	5·05	5·07	5·09
3·9	0·00	1·93	2·86	3·43	3·89	4·25	4·54	4·79	4·97	5·11	5·23	5·32	5·38	5·43	5·46	5·48
4·0	0·00	2·10	3·07	3·69	4·17	4·55	4·87	5·13	5·32	5·48	5·62	5·72	5·79	5·84	5·88	5·89
4·1	—	—	3·29	3·87	4·47	4·87	5·21	5·49	5·69	5·87	6·03	6·15	6·23	6·28	6·32	6·33
4·2	—	—	3·53	4·19	4·79	5·21	5·58	5·88	6·10	6·30	6·46	6·60	6·69	6·75	6·80	6·81
4·3	—	—	3·78	4·52	5·13	5·58	5·97	6·29	6·54	6·75	6·93	7·07	7·18	7·25	7·29	7·31
4·4	—	—	4·05	4·85	5·49	5·98	6·40	6·74	7·01	7·24	7·42	7·57	7·68	7·76	7·80	7·83
4·5	—	—	4·33	5·18	5·88	6·42	6·87	7·23	7·52	7·75	7·95	8·10	8·21	8·29	8·34	8·37
4·6	—	—	—	—	—	—	7·37	7·76	8·07	8·30	8·51	8·66	8·76	8·85	8·91	8·95
4·7	—	—	—	—	—	—	7·90	8·31	8·64	8·90	9·11	9·25	9·35	9·44	9·50	9·54
4·8	—	—	—	—	—	—	8·46	8·88	9·24	9·54	9·75	9·89	9·99	10·08	10·14	10·18
4·9	—	—	—	—	—	—	9·05	9·47	9·86	10·21	10·42	10·58	10·69	10·78	10·84	10·80
5·0	—	—	—	—	—	—	9·66	10·08	10·50	10·90	11·19	11·33	11·44	11·53	11·60	11·64
5·1	—	—	—	—	—	—	—	—	—	—	—	—	12·26	12·36	12·42	12·43
5·2	—	—	—	—	—	—	—	—	—	—	—	—	13·10	13·26	13·29	13·29
5·3	—	—	—	—	—	—	—	—	—	—	—	—	13·98	14·15	14·18	14·18
5·4	—	—	—	—	—	—	—	—	—	—	—	—	14·91	15·05	15·10	15·11
5·5	—	—	—	—	—	—	—	—	—	—	—	—	15·90	15·98	16·05	16·07
5·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

TABLE XXXVII.—(*continued*).

Values of $\sqrt{N}\Sigma_{\beta_1}$.

β_1

0·80	*0·85*	*0·90*	*0·95*	*1·00*	*1·05*	*1·10*	*1·15*	*1·20*	*1·25*	*1·30*	*1·35*	*1·40*	*1·45*	*1·50*	
3·96	4·21	4·47	4·73	5·00	5·27	5·55	5·83	6·12	6·41	6·71	7·01	7·31	7·62	7·94	*2·0*
3·80	4·03	4·27	4·53	4·80	5·07	5·34	5·62	5·90	6·18	6·48	6·77	7·07	7·37	7·69	*2·1*
3·66	3·88	4·11	4·36	4·63	4·88	5·15	5·42	5·69	5·96	6·25	6·54	6·84	7·14	7·45	*2·2*
3·52	3·74	3·96	4·20	4·46	4·71	4·96	5·22	5·48	5·75	6·02	6·31	6·61	6·91	7·21	*2·3*
3·41	3·62	3·83	4·05	4·29	4·54	4·78	5·03	5·28	5·55	5·82	6·10	6·38	6·68	6·97	*2·4*
3·32	3·51	3·71	3·92	4·15	4·38	4·61	4·85	5·10	5·36	5·62	5·89	6·16	6·45	6·74	*2·5*
3·25	3·42	3·60	3·80	4·01	4·23	4·45	4·68	4·92	5·17	5·43	5·68	5·94	6·22	6·51	*2·6*
3·20	3·35	3·51	3·69	3·89	4·10	4·32	4·54	4·76	5·00	5·24	5·48	5·73	6·00	6·28	*2·7*
3·18	3·32	3·47	3·63	3·80	4·00	4·21	4·41	4·62	4·84	5·07	5·30	5·53	5·79	6·06	*2·8*
3·19	3·32	3·45	3·60	3·75	3·92	4·11	4·30	4·49	4·70	4·91	5·12	5·34	5·59	5·85	*2·9*
3·27	3·38	3·49	3·61	3·74	3·87	4·03	4·21	4·39	4·58	4·78	4·98	5·19	5·42	5·68	*3·0*
3·38	3·47	3·57	3·67	3·77	3·89	4·02	4·16	4·31	4·48	4·66	4·85	5·05	5·28	5·53	*3·1*
3·53	3·60	3·68	3·76	3·84	3·93	4·03	4·15	4·28	4·43	4·59	4·75	4·92	5·15	5·40	*3·2*
3·70	3·75	3·81	3·88	3·95	4·02	4·10	4·19	4·28	4·42	4·56	4·69	4·84	5·04	5·28	*3·3*
3·90	3·93	3·97	4·03	4·08	4·14	4·20	4·26	4·34	4·45	4·56	4·66	4·78	4·97	5·18	*3·4*
4·14	4·17	4·19	4·22	4·26	4·30	4·34	4·39	4·45	4·52	4·60	4·68	4·79	4·93	5·12	*3·5*
4·42	4·44	4·45	4·47	4·49	4·51	4·54	4·57	4·62	4·67	4·74	4·78	4·81	4·95	5·09	*3·6*
4·74	4·75	4·76	4·76	4·77	4·78	4·79	4·81	4·84	4·87	4·90	4·92	4·95	5·04	5·13	*3·7*
5·10	5·10	5·09	5·08	5·08	5·07	5·07	5·06	5·06	5·08	5·09	5·11	5·14	5·18	5·22	*3·8*
5·49	5·48	5·46	5·44	5·42	5·40	5·37	5·35	5·33	5·32	5·32	5·33	5·34	5·35	5·37	*3·9*
5·89	5·88	5·86	5·83	5·80	5·76	5·72	5·69	5·65	5·62	5·60	5·58	5·57	5·57	5·59	*4·0*
6·33	6·32	6·30	6·26	6·21	6·16	6·11	6·06	6·02	5·98	5·94	5·91	5·88	5·86	5·86	*4·1*
6·80	6·79	6·76	6·71	6·65	6·60	6·54	6·48	6·42	6·36	6·31	6·27	6·24	6·21	6·18	*4·2*
7·30	7·28	7·25	7·19	7·13	7·07	7·01	6·93	6·87	6·80	6·74	6·67	6·62	6·57	6·53	*4·3*
7·83	7·80	7·76	7·71	7·65	7·58	7·51	7·44	7·37	7·28	7·20	7·12	7·05	6·98	6·92	*4·4*
8·38	8·36	8·32	8·28	8·21	8·14	8·07	7·99	7·90	7·81	7·71	7·61	7·51	7·42	7·34	*4·5*
8·96	8·95	8·91	8·86	8·79	8·72	8·64	8·55	8·45	8·35	8·24	8·13	8·00	7·90	7·80	*4·6*
9·57	9·57	9·53	9·47	9·40	9·33	9·24	9·14	9·04	8·93	8·82	8·69	8·55	8·42	8·31	*4·7*
10·20	10·23	10·16	10·10	10·05	9·97	9·88	9·77	9·67	9·55	9·42	9·28	9·14	9·00	8·87	*4·8*
10·91	10·92	10·87	10·80	10·74	10·66	10·57	10·44	10·32	10·18	10·04	9·90	9·76	9·63	9·50	*4·9*
11·66	11·65	11·61	11·55	11·48	11·39	11·29	11·17	11·04	10·90	10·77	10·62	10·46	10·30	10·14	*5·0*
12·45	12·43	12·38	12·32	12·24	12·14	12·03	11·91	11·78	11·64	11·50	11·34	11·15	10·96	10·82	*5·1*
13·28	13·25	13·20	13·13	13·04	12·92	12·80	12·67	12·54	12·40	12·24	12·07	11·88	11·72	11·54	*5·2*
14·16	14·12	14·07	13·98	13·87	13·76	13·63	13·47	13·35	13·20	13·02	12·84	12·66	12·48	12·30	*5·3*
15·09	15·06	15·00	14·90	14·78	14·65	14·51	14·36	14·22	14·05	13·81	13·67	13·48	13·29	13·11	*5·4*
16·06	16·02	15·96	15·87	15·76	15·63	15·49	15·33	15·17	15·00	14·81	14·61	14·40	14·18	13·97	*5·5*
—	—	17·02	16·91	16·79	16·67	16·51	16·34	16·18	15·95	15·70	15·50	15·30	15·07	14·84	*5·6*
—	—	18·14	17·99	17·88	17·75	17·58	17·40	17·23	16·94	16·70	16·47	16·26	16·04	15·77	*5·7*
—	—	19·34	19·13	19·02	18·87	18·69	18·48	18·26	17·98	17·74	17·50	17·26	17·01	16·76	*5·8*
—	—	20·57	20·36	20·20	20·03	19·84	19·62	19·39	19·11	18·84	18·59	18·32	18·05	17·78	*5·9*
—	—	21·86	21·65	21·45	21·25	21·03	20·79	20·54	20·29	20·02	19·76	19·47	19·18	18·90	*6·0*
—	—	—	—	—	—	22·36	22·18	21·92	21·61	21·31	20·97	20·61	20·30	20·13	*6·1*
—	—	—	—	—	—	23·77	23·61	23·32	23·00	22·63	22·22	21·82	21·50	21·29	*6·2*
—	—	—	—	—	—	25·33	25·09	24·74	24·38	24·00	23·55	23·13	22·78	22·50	*6·3*
—	—	—	—	—	—	26·95	26·64	26·27	25·86	25·43	26·00	24·52	24·12	23·82	*6·4*
—	—	—	—	—	—	28·61	28·18	27·73	27·30	26·89	26·46	26·06	25·65	25·24	*6·5*
—	—	—	—	—	—	—	—	—	—	—	—	27·67	27·21	26·75	*6·6*
—	—	—	—	—	—	—	—	—	—	—	—	29·40	28·90	28·35	*6·7*
—	—	—	—	—	—	—	—	—	—	—	—	31·15	30·61	29·94	*6·8*
—	—	—	—	—	—	—	—	—	—	—	—	33·02	32·41	31·72	*6·9*
—	—	—	—	—	—	—	—	—	—	—	—	34·89	34·16	33·59	*7·0*
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	

β_2

TABLE XXXVIII. *To find Probable Error of* β_2.

Values of $\sqrt{N}\Sigma_{\beta_2}$.

β_2 \ β_1	0·00	0·05	0·10	0·15	0·20	0·25	0·30	0·35	0·40	0·45	0·50	0·55	0·60	0·65	0·70	0·75
2·0	1·41	1·60	1·74	1·93	2·11	2·28	2·44	2·60	2·77	2·94	3·12	3·29	3·46	3·65	3·86	4·05
2·1	1·57	1·76	1·89	2·00	2·10	2·20	2·35	2·51	2·68	2·86	3·05	3·24	3·44	3·64	3·84	4·04
2·2	1·75	1·94	2·07	2·16	2·20	2·28	2·40	2·53	2·68	2·85	3·04	3·23	3·43	3·63	3·83	4·03
2·3	1·95	2·16	2·28	2·35	2·42	2·49	2·58	2·67	2·78	2·92	3·09	3·27	3·45	3·64	3·84	4·03
2·4	2·18	2·39	2·53	2·60	2·72	2·82	2·90	2·96	3·02	3·10	3·22	3·35	3·50	3·68	3·86	4·03
2·5	2·46	2·68	2·83	2·97	3·09	3·19	3·27	3·31	3·32	3·36	3·47	3·53	3·63	3·75	3·88	4·03
2·6	2·78	3·03	3·24	3·38	3·52	3·60	3·66	3·69	3·69	3·70	3·75	3·78	3·83	3·87	3·95	4·07
2·7	3·17	3·48	3·71	3·87	3·98	4·03	4·08	4·12	4·11	4·09	4·07	4·06	4·06	4·06	4·07	4·15
2·8	3·64	4·02	4·26	4·42	4·52	4·58	4·60	4·59	4·57	4·52	4·44	4·39	4·34	4·32	4·31	4·34
2·9	4·22	4·65	4·94	5·11	5·20	5·22	5·18	5·13	5·07	4·99	4·90	4·80	4·70	4·63	4·60	4·61
3·0	4·90	5·48	5·76	5·89	5·95	5·93	5·86	5·76	5·65	5·53	5·41	5·30	5·20	5·12	5·05	5·00
3·1	5·75	6·41	6·72	6·88	6·90	6·82	6·70	6·54	6·38	6·22	6·07	5·92	5·79	5·69	5·60	5·53
3·2	6·77	7·55	7·90	8·00	7·97	7·83	7·63	7·42	7·21	7·03	6·86	6·70	6·53	6·39	6·26	6·14
3·3	8·00	8·83	9·22	9·30	9·22	9·02	8·80	8·53	8·29	8·05	7·83	7·60	7·38	7·20	7·02	6·84
3·4	9·37	10·28	10·68	10·76	10·67	10·46	10·20	9·91	9·62	9·31	9·01	8·73	8·44	8·18	7·92	7·66
3·5	10·85	11·75	12·31	12·52	12·46	12·25	11·95	11·60	11·24	10·86	10·45	10·03	9·63	9·26	8·90	8·54
3·6	12·67	13·74	14·40	14·78	14·53	14·21	13·80	13·38	12·95	12·55	12·10	11·60	11·06	10·54	10·02	9·55
3·7	14·78	15·98	16·78	17·09	16·93	16·53	16·05	15·58	15·09	14·61	14·08	13·49	12·74	12·02	11·36	10·80
3·8	17·50	18·83	19·83	20·03	19·78	19·36	18·76	18·22	17·64	16·98	16·25	15·30	14·42	13·60	12·88	12·27
3·9	20·80	22·50	23·68	23·81	23·34	22·67	21·98	21·14	20·29	19·45	18·58	17·54	16·50	15·54	14·77	14·06
4·0	24·74	26·83	28·47	28·05	27·24	26·29	25·25	24·18	23·03	22·02	21·01	20·01	19·04	18·12	17·23	16·36
4·1	—	—	35·00	34·17	32·88	31·36	29·77	28·13	26·60	25·12	23·82	22·64	21·54	20·53	19·56	18·62
4·2	—	—	43·3	41·4	39·2	37·2	35·2	33·2	31·2	29·2	27·4	26·0	24·7	23·4	22·3	21·3
4·3	—	—	55·3	51·6	48·0	44·6	41·2	38·6	36·2	33·8	31·8	30·1	28·5	26·9	25·6	24·3
4·4	—	—	72·7	66·0	59·	54·1	49·5	45·7	42·1	39·2	36·8	34·8	32·9	31·0	29·2	27·7
4·5	—	—	96·5	82·7	72·7	65·3	59·8	54·7	50·8	47·2	44·0	41·0	38·5	36·2	34·1	32·0
4·6	—	—	—	—	—	—	75·0	68·0	62·2	56·9	52·2	48·2	45·1	42·2	39·6	37·2
4·7	—	—	—	—	—	—	101·3	87·2	76·8	68·3	62·0	56·9	52·7	49·1	45·9	42·8
4·8	—	—	—	—	—	—	140·0	115·2	96·2	82·6	72·7	66·1	60·9	56·7	52·9	49·3
4·9	—	—	—	—	—	—	204·5	150·8	122·3	102·5	89·1	80·2	72·4	66·6	61·4	56·7
5·0	—	—	—	—	—	—	325·7	206·0	154·2	126·8	110·1	96·9	86·6	78·1	71·2	65·6
5·1	—	—	—	—	—	—	—	—	—	—	—	—	103·6	94·4	85·9	78·0
5·2	—	—	—	—	—	—	—	—	—	—	—	—	130·4	116·4	104·0	91·0
5·3	—	—	—	—	—	—	—	—	—	—	—	—	175·2	144·8	124·4	109·6
5·4	—	—	—	—	—	—	—	—	—	—	—	—	224·4	178·0	151·0	132·6
5·5	—	—	—	—	—	—	—	—	—	—	—	—	340·8	246·0	195·3	163·2
5·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

TABLE XXXVIII.—(*continued*).

Values of $\sqrt{N}\Sigma_{\beta_2}$.

β_1.

0·80	*0·85*	*0·90*	*0·95*	*1·00*	*1·05*	*1·10*	*1·15*	*1·20*	*1·25*	*1·30*	*1·35*	*1·40*	*1·45*	*1·50*	
4·24	4·43	4·62	4·81	5·00	5·19	5·38	5·56	5·75	5·94	6·12	6·30	6·49	6·67	6·84	*2·0*
4·23	4·41	4·59	4·77	4·96	5·15	5·34	5·53	5·72	5·90	6·08	6·27	6·47	6·65	6·83	*2·1*
4·22	4·39	4·56	4·74	4·93	5·12	5·31	5·50	5·69	5·87	6·05	6·24	6·44	6·63	6·82	*2·2*
4·20	4·36	4·53	4·71	4·90	5·08	5·27	5·46	5·65	5·84	6·02	6·21	6·41	6·61	6·80	*2·3*
4·19	4·35	4·51	4·69	4·87	5·05	5·23	5·42	5·61	5·80	5·99	6·18	6·38	6·58	6·78	*2·4*
4·18	4·34	4·50	4·67	4·85	5·03	5·21	5·39	5·58	5·77	5·96	6·15	6·35	6·54	6·74	*2·5*
4·20	4·35	4·50	4·67	4·84	5·01	5·20	5·36	5·54	5·72	5·91	6·11	6·30	6·49	6·68	*2·6*
4·26	4·38	4·52	4·68	4·84	5·01	5·18	5·34	5·51	5·67	5·85	6·05	6·25	6·44	6·62	*2·7*
4·40	4·50	4·60	4·72	4·86	5·03	5·19	5·34	5·49	5·65	5·83	6·02	6·21	6·39	6·58	*2·8*
4·63	4·67	4·73	4·82	4·93	5·05	5·20	5·35	5·50	5·66	5·82	6·00	6·18	6·36	6·54	*2·9*
4·98	4·97	4·99	5·03	5·10	5·18	5·28	5·39	5·52	5·66	5·82	5·98	6·15	6·33	6·51	*3·0*
5·47	5·42	5·38	5·34	5·36	5·37	5·41	5·48	5·58	5·70	5·83	5·97	6·14	6·32	6·52	*3·1*
6·03	5·92	5·83	5·75	5·67	5·62	5·60	5·62	5·68	5·78	5·90	6·03	6·18	6·34	6·52	*3·2*
6·67	6·51	6·35	6·22	6·09	6·00	5·90	5·94	5·92	5·95	6·01	6·12	6·25	6·37	6·53	*3·3*
7·41	7·17	6·95	6·77	6·61	6·48	6·29	6·26	6·24	6·22	6·22	6·26	6·34	6·47	6·61	*3·4*
8·22	7·92	7·64	7·38	7·17	6·99	6·84	6·72	6·63	6·57	6·54	6·53	6·56	6·61	6·71	*3·5*
9·14	8·80	8·51	8·23	7·98	7·70	7·53	7·40	7·22	7·09	6·99	6·98	6·95	6·92	6·93	*3·6*
10·34	9·94	9·58	9·25	8·96	8·66	8·36	8·14	7·90	7·75	7·61	7·51	7·42	7·34	7·23	*3·7*
11·77	11·29	10·82	10·37	9·98	9·62	9·31	9·03	8·73	8·51	8·29	8·11	7·94	7·78	7·60	*3·8*
13·42	12·85	12·31	11·79	11·30	10·86	10·41	10·02	9·64	9·34	9·03	8·77	8·52	8.30	8·10	*3·9*
15·58	14·84	14·10	13·42	12·79	12·20	11·64	11·13	10·65	10·21	9·83	9·51	9·20	8·92	8·67	*4·0*
17·72	16·85	16·01	15·21	14·44	13·70	13·00	12·34	11·73	11·17	10·67	10·24	9·87	9·58	9·32	*4·1*
20·2	19·2	18·3	17·3	16·4	15·5	14·7	14·0	13·3	12·6	12·0	11·5	11·0	10·5	10·3	*4·2*
23·1	22·0	20·9	19·8	18·7	17·6	16·7	15·8	15·0	14·2	13·5	12·8	12·3	11·8	11·3	*4·3*
26·3	25·0	23·8	22·5	21·3	20·1	19·0	18·0	17·1	16·1	15·3	14·6	13·9	13·2	12·6	*4·4*
30·1	28·4	26·8	25·3	23·9	22·6	21·4	20·3	19·3	18·3	17·3	16·4	15·6	14·8	14·1	*4·5*
34·7	32·5	30·5	28·8	27·3	25·6	24·2	22·9	21·7	20·6	19·5	18·4	17·5	16·7	16·1	*4·6*
40·0	37·4	35·0	32·8	30·9	29·2	27·6	26·1	24·7	23·3	22·0	20·9	19·8	18·8	18·1	*4·7*
46·1	43·1	40·3	37·7	35·3	33·2	31·4	29·7	28·0	26·4	25·0	23·6	22·3	21·2	20·3	*4·8*
52·4	48·8	46·8	43·1	40·2	37·8	35·6	33·6	31·6	29·8	28·1	26·6	25·1	23·8	22·7	*4·9*
60·6	56·1	52·3	48·8	45·5	42·6	40·0	37·6	35·4	33·4	31·5	29·8	28·1	26·6	25·2	*5·0*
71·2	65·1	60·6	56·5	52·6	49·1	45·8	43·1	40·5	38·0	35·6	33·6	31·7	30·0	28·4	*5·1*
83·0	76·4	70·5	65·4	60·6	56·3	52·5	49·3	46·3	43·4	40·4	38·0	35·6	33·6	31·7	*5·2*
98·8	89·6	81·9	75·6	70·2	65·0	60·2	56·2	52·5	48·9	45·5	42·6	39·9	37·4	35·2	*5·3*
118·4	105·2	96·0	87·6	80·4	74·0	68·3	63·4	58·8	54·7	51·0	47·7	44·5	41·5	38·9	*5·4*
141·4	124·0	111·2	99·6	91·2	84·0	77·4	71·2	65·7	61·2	56·9	52·9	49·4	46·2	43·5	*5·5*
—	—	131·2	117·4	105·2	96·0	87·3	79·3	72·8	67·8	63·2	58·6	54·8	51·4	48·8	*5·6*
—	—	160·0	142·4	126·4	113·4	102·2	93·0	84·4	77·3	71·1	65·6	60·8	57·2	54·7	*5·7*
—	—	199·2	175·8	154·8	134·2	119·6	107·0	97·2	88·4	80·6	74·4	69·4	64·9	61·5	*5·8*
—	—	266·0	221·6	192·8	163·6	142·8	128·0	114·6	104·0	94·6	86·0	79·6	74·4	70·2	*5·9*
—	—	378·1	284·0	231·5	198·2	171·6	151·5	136·2	123·8	112·8	103·4	94·8	87·5	81·4	*6·0*
—	—	—	—	—	—	206·3	186·3	167·5	150·0	134·2	121·5	111·0	101·8	92·8	*6·1*
—	—	—	—	—	—	264	232	205	280	160	141	128	116	107	*6·2*
—	—	—	—	—	—	350	297	251	216	188	164	148	132	122	*6·3*
—	—	—	—	—	—	510	376	308	263	225	196	172	152	138	*6·4*
—	—	—	—	—	—	889	524	387	313	264	229	200	177	161	*6·5*
—	—	—	—	—	—	—	—	—	—	—	—	237	204	184	*6·6*
—	—	—	—	—	—	—	—	—	—	—	—	286	249	220	*6·7*
—	—	—	—	—	—	—	—	—	—	—	—	363	305	268	*6·8*
—	—	—	—	—	—	—	—	—	—	—	—	485	392	333	*6·9*
—	—	—	—	—	—	—	—	—	—	—	—	747	510	416	*7·0*

β_2

TABLE XXXIX. *To find the correlation in errors of β_1 and β_2.*

Values of $R_{\beta_1\beta_2}$.

β_2 \ β_1	0·00	0·05	0·10	0·15	0·20	0·25	0·30	0·35	0·40	0·45	0·50	0·55	0·60	0·65	0·70	0·75
2·0	0·00	·570	·706	·770	·823	·863	·894	·917	·935	·949	·960	·968	·976	·983	·989	·992
2·1	0·00	·557	·685	·755	·798	·838	·870	·895	·914	·936	·948	·959	·969	·977	·983	·986
2·2	0·00	·551	·672	·728	·771	·814	·847	·874	·896	·919	·934	·948	·960	·968	·975	·980
2·3	0·00	·550	·663	·719	·765	·799	·829	·859	·880	·900	·919	·935	·948	·957	·965	·971
2·4	0·00	·551	·660	·712	·752	·787	·814	·843	·867	·886	·905	·920	·936	·945	·954	·962
2·5	0·00	·554	·659	·706	·745	·776	·805	·834	·858	·878	·893	·908	·924	·933	·941	·949
2·6	0·00	·557	·662	·706	·742	·773	·799	·825	·851	·871	·883	·898	·913	·921	·930	·938
2·7	0·00	·557	·668	·710	·744	·773	·800	·825	·846	·863	·876	·889	·902	·911	·920	·928
2·8	0·00	·556	·674	·716	·750	·779	·803	·826	·842	·858	·871	·883	·893	·901	·910	·919
2·9	0·00	·550	·680	·724	·760	·787	·810	·830	·844	·857	·868	·878	·887	·895	·903	·912
3·0	0·00	·542	·684	·738	·774	·796	·816	·835	·847	·857	·867	·875	·883	·890	·898	·906
3·1	0·00	·534	·687	·744	·781	·808	·825	·840	·850	·858	·867	·874	·882	·889	·897	·903
3·2	0·00	·524	·688	·746	·786	·811	·830	·842	·852	·860	·868	·875	·882	·889	·896	·902
3·3	0·00	·512	·688	·747	·788	·814	·832	·845	·855	·863	·870	·876	·882	·888	·895	·901
3·4	0·00	·501	·686	·748	·790	·816	·833	·848	·858	·865	·872	·878	·883	·889	·895	·900
3·5	0·00	·490	·681	·747	·790	·815	·833	·849	·860	·867	·873	·879	·884	·890	·895	·900
3·6	0·00	·477	·676	·745	·788	·813	·832	·850	·860	·867	·874	·880	·886	·891	·896	·900
3·7	0·00	·462	·670	·741	·784	·810	·831	·848	·859	·867	·874	·881	·887	·892	·897	·901
3·8	0·00	·450	·662	·736	·779	·803	·828	·845	·858	·866	·874	·882	·888	·893	·898	·901
3·9	0·00	·438	·654	·720	·770	·796	·822	·841	·856	·866	·875	·882	·889	·894	·899	·903
4·0	0·00	·422	·645	·713	·760	·788	·816	·837	·853	·865	·873	·881	·888	·894	·899	·903
4·1	—	—	·630	·702	·748	·780	·807	·830	·849	·862	·871	·880	·887	·892	·897	·901
4·2	—	—	·608	·682	·733	·770	·793	·822	·842	·857	·867	·877	·884	·890	·894	·899
4·3	—	—	·580	·658	·712	·753	·784	·811	·832	·848	·860	·871	·878	·885	·890	·897
4·4	—	—	·540	·628	·688	·732	·770	·796	·819	·837	·851	·863	·872	·880	·887	·894
4·5	—	—	·481	·590	·657	·709	·749	·780	·804	·824	·841	·853	·865	·874	·882	·890
4·6	—	—	—	—	—	—	·716	·754	·784	·808	·828	·842	·856	·868	·877	·886
4·7	—	—	—	—	—	—	·674	·723	·759	·788	·812	·830	·846	·860	·870	·879
4·8	—	—	—	—	—	—	·615	·681	·727	·761	·791	·815	·834	·849	·861	·872
4·9	—	—	—	—	—	—	·532	·620	·680	·728	·766	·795	·818	·835	·850	·862
5·0	—	—	—	—	—	—	·362	·534	·628	·687	·731	·767	·798	·822	·837	·851
5·1	—	—	—	—	—	—	—	—	—	—	—	—	·768	·799	·820	·837
5·2	—	—	—	—	—	—	—	—	—	—	—	—	·730	·768	·799	·820
5·3	—	—	—	—	—	—	—	—	—	—	—	—	·679	·729	·769	·799
5·4	—	—	—	—	—	—	—	—	—	—	—	—	·608	·686	·736	·774
5·5	—	—	—	—	—	—	—	—	—	—	—	—	·496	·601	·674	·724
5·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

TABLE XXXIX.—(*continued*).

Values of $R_{\beta_1\beta_2}$.

β_1

0·80	*0·85*	*0·90*	*0·95*	*1·00*	*1·05*	*1·10*	*1·15*	*1·20*	*1·25*	*1·30*	*1·35*	*1·40*	*1·45*	*1·50*	
·993	·995	·997	·999	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	*2·0*
·989	·991	·994	·996	·998	·998	·999	·999	·999	1·000	1·000	1·000	1·000	1·000	1·000	*2·1*
·983	·986	·989	·992	·995	·996	·997	·998	·998	·999	1·000	1·000	1·000	1·000	1·000	*2·2*
·976	·980	·984	·988	·992	·993	·994	·995	·997	·998	·999	·999	·999	1·000	1·000	*2·3*
·968	·973	·978	·983	·987	·989	·991	·993	·995	·996	·998	·998	·999	·999	1·000	*2·4*
·958	·965	·972	·977	·982	·985	·988	·990	·992	·994	·996	·997	·998	·999	1·000	*2·5*
·947	·956	·964	·970	·976	·980	·984	·986	·988	·991	·993	·995	·997	·998	·999	*2·6*
·937	·947	·957	·963	·968	·973	·977	·980	·983	·986	·989	·992	·995	·997	·998	*2·7*
·928	·939	·949	·955	·960	·965	·970	·974	·978	·981	·985	·989	·992	·994	·996	*2·8*
·921	·932	·942	·947	·952	·957	·963	·968	·972	·976	·980	·984	·988	·990	·992	*2·9*
·915	·923	·931	·937	·943	·948	·954	·960	·966	·971	·975	·979	·983	·986	·988	*3·0*
·909	·915	·922	·929	·936	·942	·947	·953	·959	·965	·970	·974	·978	·981	·984	*3·1*
·907	·912	·918	·924	·930	·936	·941	·946	·952	·958	·963	·968	·973	·977	·980	*3·2*
·906	·909	·914	·919	·925	·930	·935	·940	·946	·951	·956	·961	·966	·971	·975	*3·3*
·905	·908	·912	·916	·920	·925	·930	·935	·940	·945	·950	·954	·958	·964	·974	*3·4*
·904	·907	·910	·914	·918	·922	·926	·931	·936	·940	·944	·948	·952	·958	·965	*3·5*
·904	·907	·910	·914	·918	·921	·924	·928	·932	·935	·938	·942	·946	·952	·959	*3·6*
·905	·907	·910	·914	·917	·920	·923	·927	·930	·933	·935	·937	·940	·946	·953	*3·7*
·905	·908	·911	·914	·917	·920	·922	·925	·928	·930	·932	·934	·936	·941	·948	*3·8*
·906	·909	·911	·914	·917	·919	·921	·924	·927	·929	·931	·933	·935	·939	·944	*3·9*
·906	·909	·912	·914	·917	·919	·921	·923	·926	·928	·930	·932	·934	·936	·940	*4·0*
·905	·908	·911	·914	·917	·919	·921	·923	·925	·927	·930	·931	·932	·933	·934	*4·1*
·905	·907	·910	·913	·916	·919	·921	·923	·924	·926	·929	·929	·930	·930	·929	*4·2*
·903	·906	·910	·913	·916	·918	·920	·922	·924	·926	·929	·928	·928	·927	·924	*4·3*
·900	·904	·908	·912	·916	·918	·920	·922	·923	·926	·928	·927	·927	·925	·922	*4·4*
·897	·902	·906	·910	·915	·918	·920	·922	·923	·926	·928	·927	·926	·923	·920	*4·5*
·893	·898	·903	·908	·913	·916	·919	·920	·922	·925	·927	·926	·925	·923	·920	*4·6*
·887	·894	·900	·905	·910	·913	·917	·919	·921	·924	·926	·925	·925	·923	·922	*4·7*
·881	·890	·896	·901	·906	·910	·914	·917	·920	·923	·925	·926	·926	·925	·925	*4·8*
·874	·884	·890	·895	·901	·907	·911	·915	·919	·922	·925	·926	·927	·927	·928	*4·9*
·863	·875	·883	·889	·896	·903	·908	·913	·918	·922	·925	·927	·928	·930	·932	*5·0*
·851	·864	·875	·882	·890	·898	·905	·911	·917	·922	·925	·928	·931	·933	·936	*5·1*
·837	·852	·866	·875	·884	·892	·901	·909	·916	·921	·924	·928	·933	·937	·941	*5·2*
·820	·839	·853	·865	·876	·885	·895	·904	·913	·918	·923	·929	·935	·940	·945	*5·3*
·798	·818	·837	·853	·867	·877	·888	·898	·908	·915	·921	·928	·935	·941	·947	*5·4*
·764	·792	·817	·837	·854	·867	·880	·890	·900	·910	·918	·925	·933	·940	·947	*5·5*
—	—	·789	·815	·835	·852	·868	·880	·890	·904	·911	·917	·926	·935	·944	*5·6*
—	—	·750	·786	·811	·835	·854	·869	·880	·892	·901	·909	·917	·927	·938	*5·7*
—	—	·701	·748	·783	·811	·835	·852	·866	·879	·890	·897	·905	·915	·928	*5·8*
—	—	·640	·700	·748	·781	·810	·828	·846	·861	·875	·883	·892	·901	·913	*5·9*
—	—	·544	·639	·703	·746	·778	·802	·825	·842	·857	·867	·879	·886	·893	*6·0*
—	—	—	—	—	—	·741	·769	·796	·820	·837	·852	·866	·872	·873	*6·1*
—	—	—	—	—	—	·691	·727	·762	·792	·815	·836	·852	·858	·856	*6·2*
—	—	—	—	—	—	·628	·678	·724	·761	·790	·818	·838	·845	·842	*6·3*
—	—	—	—	—	—	·526	·606	·675	·724	·763	·793	·818	·831	·834	*6·4*
—	—	—	—	—	—	·354	·526	·619	·680	·726	·761	·791	·814	·831	*6·5*
—	—	—	—	—	—	—	—	—	—	—	—	·761	·790	·832	*6·6*
—	—	—	—	—	—	—	—	—	—	—	—	·721	·760	·837	*6·7*
—	—	—	—	—	—	—	—	—	—	—	—	·670	·727	·845	*6·8*
—	—	—	—	—	—	—	—	—	—	—	—	·600	·683	·857	*6·9*
—	—	—	—	—	—	—	—	—	—	—	—	·468	·602	·876	*7·0*

β_2

TABLE XL. *To find the Probable Error of the distance from Mean to Mode.*

Values of $\frac{\sqrt{N}}{\sigma}\Sigma_d$.

β_2 \ β_1	0·00	0·05	0·10	0·15	0·20	0·25	0·30	0·35	0·40	0·45	0·50	0·55	0·60	0·65	0·70	0·75	0·80
2·0	3·54	—	—	—	—	—	—	—	—	—	3·03	2·44	2·10	1·80	1·58	1·42	1·30
2·1	2·15	4·36	—	—	—	—	—	—	—	—	—	—	—	3·10	2·53	2·16	1·88
2·2	1·87	2·75	9·65	—	—	—	—	—	—	—	—	—	—	—	3·91	3·17	2·60
2·3	1·64	1·86	3·00	—	—	—	—	—	—	—	—	—	—	—	—	—	3·78
2·4	1·46	1·58	2·07	—	—	—	—	—	—	—	—	—	—	—	—	—	—
2·5	1·35	1·46	1·67	2·08	2·87	4·04	5·21	—	—	—	—	—	—	—	—	—	—
2·6	1·28	1·37	1·58	1·98	2·60	3·42	4·43	6·72	—	—	—	—	—	—	—	—	—
2·7	1·25	1·30	1·50	1·83	2·34	2·98	3·75	5·06	7·48	—	—	—	—	—	—	—	—
2·8	1·23	1·28	1·43	1·71	2·11	2·60	3·17	4·12	5·28	7·45	—	—	—	—	—	—	—
2·9	1·22	1·27	1·38	1·60	1·90	2·27	2·69	3·20	3·84	4·78	6·65	—	—	—	—	—	—
3·0	1·23	1·26	1·34	1·51	1·73	1·98	2·29	2·63	3·06	3·58	4·28	5·18	6·43	8·24	10·89	—	—
3·1	1·25	1·27	1·32	1·44	1·58	1·76	2·00	2·23	2·54	2·94	3·42	3·93	4·52	5·50	6·76	8·66	—
3·2	1·27	1·28	1·30	1·38	1·48	1·60	1·75	1·92	2·13	2·37	2·72	3·12	3·54	4·21	5·07	6·22	8·00
3·3	1·29	1·29	1·28	1·32	1·39	1·47	1·55	1·68	1·83	2·03	2·27	2·57	2·90	3·39	4·04	4·66	5·53
3·4	1·30	1·29	1·28	1·29	1·31	1·37	1·45	1·54	1·63	1·79	2·00	2·24	2·51	2·88	3·36	3·86	4·50
3·5	1·31	1·30	1·29	1·27	1·25	1·30	1·37	1·45	1·54	1·66	1·83	2·03	2·26	2·55	2·89	3·18	3·61
3·6	1·32	1·31	1·30	1·26	1·22	1·26	1·32	1·40	1·50	1·61	1·74	1·89	2·08	2·31	2·56	2·86	3·24
3·7	1·31	1·31	1·31	1·26	1·22	1·25	1·30	1·37	1·46	1·57	1·69	1·82	1·97	2·14	2·34	2·62	2·95
3·8	1·30	1·31	1·32	1·28	1·25	1·27	1·32	1·38	1·46	1·55	1·65	1·76	1·88	2·03	2·20	2·43	2·69
3·9	1·29	1·33	1·35	1·33	1·30	1·32	1·36	1·41	1·48	1·56	1·64	1·73	1·84	1·96	2·11	2·27	2·49
4·0	1·27	1·37	1·40	1·39	1·39	1·40	1·42	1·46	1·51	1·58	1·65	1·73	1·83	1·94	2·06	2·19	2·36
4·1	—	—	1·47	1·48	1·50	1·51	1·53	1·55	1·57	1·61	1·66	1·75	1·85	1·94	2·03	2·13	2·26
4·2	—	—	1·58	1·62	1·64	1·65	1·65	1·65	1·65	1·66	1·70	1·77	1·85	1·94	2·03	2·12	2·22
4·3	—	—	1·75	1·77	1·78	1·79	1·78	1·76	1·75	1·75	1·78	1·83	1·90	1·97	2·05	2·13	2·23
4·4	—	—	1·98	1·97	1·95	1·94	1·93	1·90	1·88	1·89	1·92	1·96	2·01	2·07	2·13	2·20	2·29
4·5	—	—	2·27	2·20	2·15	2·11	2·10	2·09	2·09	2·10	2·12	2·15	2·18	2·22	2·26	2·32	2·38
4·6	—	—	—	—	—	—	2·44	2·40	2·38	2·36	2·34	2·36	2·38	2·40	2·42	2·45	2·48
4·7	—	—	—	—	—	—	2·93	2·85	2·78	2·71	2·67	2·65	2·64	2·63	2·62	2·61	2·61
4·8	—	—	—	—	—	—	3·74	3·52	3·33	3·16	3·07	3·00	2·95	2·90	2·85	2·81	2·80
4·9	—	—	—	—	—	—	5·44	4·64	4·16	3·87	3·63	3·45	3·32	3·24	3·11	3·05	3·03
5·0	—	—	—	—	—	—	10·66	6·83	5·53	4·84	4·37	4·04	3·79	3·62	3·47	3·37	3·31
5·1	—	—	—	—	—	—	—	—	—	—	—	—	4·46	4·21	3·99	3·85	3·74
5·2	—	—	—	—	—	—	—	—	—	—	—	—	5·38	5·05	4·73	4·47	4·24
5·3	—	—	—	—	—	—	—	—	—	—	—	—	6·84	6·19	5·66	5·27	4·92
5·4	—	—	—	—	—	—	—	—	—	—	—	—	9·24	7·96	7·00	6·24	5·74
5·5	—	—	—	—	—	—	—	—	—	—	—	—	14·81	10·89	8·87	7·64	6·81
5·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

TABLE XL—(*continued*).

Values of $\frac{\sqrt{N}}{\sigma}\Sigma_d$.

β_1

0·85	*0·90*	*0·95*	*1·00*	*1·05*	*1·10*	*1·15*	*1·20*	*1·25*	*1·30*	*1·35*	*1·40*	*1·45*	*1·50*		
1·20	1·13	1·07	1·02	·97	·92	·87	·83	·80	·76	·72	·68	·64	·60	*2·0*	
1·64	1·49	1·38	1·29	1·21	1·14	1·08	1·03	·99	·94	·90	·86	·82	·78	*2·1*	
2·22	1·97	1·80	1·66	1·54	1·45	1·38	1·32	1·26	1·20	1·14	1·08	1·02	·96	*2·2*	
3·10	2·63	2·46	2·15	1·98	1·85	1·75	1·67	1·58	1·50	1·41	1·32	1·24	1·16	*2·3*	
—	3·72	3·20	2·81	2·56	2·36	2·22	2·09	1·96	1·84	1·73	1·62	1·51	1·39	*2·4*	
—	—	—	3·94	3·40	3·08	2·87	2·66	2·47	2·29	2·12	1·96	1·80	1·64	*2·5*	
—	—	—	—	—	4·32	3·82	3·48	3·14	2·86	2·60	2·34	2·10	1·91	*2·6*	
—	—	—	—	—	—	—	4·82	4·20	3·66	3·18	2·78	2·46	2·19	*2·7*	
—	—	—	—	—	—	—	—	5·73	4·82	3·88	3·28	2·87	2·49	*2·8*	
—	—	—	—	—	—	—	—	—	6·63	4·81	3·94	3·31	2·80	*2·9*	
—	—	—	—	—	—	—	—	—	—	6·15	4·80	3·79	3·12	*3·0*	
—	—	—	—	—	—	—	—	—	—	—	—	4·29	3·48	*3·1*	
—	—	—	—	—	—	—	—	—	—	—	—	—	3·88	*3·2*	
6·80	—	—	—	—	—	—	—	—	—	—	—	—	—	*3·3*	
5·38	6·55	—	—	—	—	—	—	—	—	—	—	—	—	*3·4*	
4·21	4·95	6·00	7·33	9·36	12·16	—	—	—	—	—	—	—	—	*3·5*	
3·74	4·34	5·12	6·03	7·17	8·80	11·52	—	—	—	—	—	—	—	*3·6*	
3·34	3·80	4·32	5·01	5·78	6·65	8·28	11·12	—	—	—	—	—	—	*3·7*	
3·00	3·35	3·74	4·23	4·72	5·41	6·36	8·00	10·22	—	—	—	—	—	*3·8*	
2·74	3·00	3·32	3·66	4·04	4·50	5·18	6·16	7·43	9·32	—	—	—	—	*3·9*	
2·55	2·77	3·02	3·29	3·60	3·98	4·50	5·12	5·92	6·91	8·00	9·23	11·08	—	*4·0*	
2·42	2·60	2·79	3·00	3·27	3·61	4·06	4·61	5·20	5·90	6·80	7·86	9·48	11·58	*4·1*	
2·35	2·50	2·67	2·86	3·09	3·38	3·71	4·13	4·60	5·15	5·86	6·72	7·76	9·10	*4·2*	
2·35	2·48	2·62	2·77	2·96	3·21	3·49	3·82	4·18	4·59	5·14	5·82	6·66	7·59	*4·3*	β_2
2·39	2·50	2·61	2·73	2·89	3·10	3·32	3·55	3·83	4·16	4·60	5·16	5·82	6·60	*4·4*	
2·45	2·53	2·63	2·74	2·87	3·02	3·20	3·39	3·61	3·87	4·21	4·66	5·18	5·86	*4·5*	
2·52	2·60	2·69	2·79	2·89	3·01	3·15	3·31	3·48	3·67	3·95	4·31	4·77	5·36	*4·6*	
2·64	2·69	2·77	2·85	2·93	3·02	3·13	3·25	3·39	3·55	3·78	4·10	4·50	4·99	*4·7*	
2·81	2·82	2·86	2·93	3·00	3·08	3·17	3·27	3·38	3·50	3·68	3·94	4·26	4·66	*4·8*	
3·02	3·03	3·04	3·08	3·12	3·16	3·22	3·30	3·39	3·50	3·63	3·81	4·07	4·35	*4·9*	
3·28	3·26	3·25	3·26	3·28	3·31	3·34	3·39	3·44	3·51	3·61	3·73	3·90	4·09	*5·0*	
3·64	3·55	3·51	3·49	3·47	3·47	3·48	3·49	3·52	3·56	3·61	3·68	3·77	3·90	*5·1*	
4·04	3·90	3·81	3·74	3·68	3·65	3·63	3·62	3·61	3·62	3·63	3·65	3·69	3·76	*5·2*	
4·60	4·35	4·18	4·05	3·95	3·88	3·81	3·75	3·72	3·70	3·69	3·68	3·69	3·70	*5·3*	
5·33	4·98	4·71	4·52	4·37	4·24	4·12	4·01	3·92	3·85	3·79	3·75	3·73	3·72	*5·4*	
6·21	5·74	5·36	5·08	4·86	4·66	4·48	4·32	4·20	4·09	3·99	3·92	3·87	3·82	*5·5*	
—	6·69	6·27	5·83	5·49	5·19	4·94	4·73	4·56	4·42	4·30	4·18	4·11	4·04	*5·6*	
—	8·11	7·48	6·82	6·32	5·90	5·55	5·27	5·03	4·84	4·68	4·54	4·43	4·35	*5·7*	
—	10·18	9·11	8·12	7·45	6·85	6·35	5·95	5·62	5·37	5·16	5·00	4·87	4·76	*5·8*	
—	13·53	11·44	9·84	8·71	7·94	7·32	6·82	6·45	6·13	5·85	5·61	5·43	5·29	*5·9*	
—	19·95	14·26	11·92	10·48	9·38	8·55	7·89	7·38	6·95	6·62	6·33	6·10	5·90	*6·0*	
—	—	—	—	—	11·64	10·26	9·31	8·62	8·03	7·53	7·15	6·87	6·60	*6·1*	
—	—	—	—	—	14·83	12·55	11·19	10·24	9·40	8·64	8·08	7·68	7·36	*6·2*	
—	—	—	—	—	19·65	15·85	13·69	12·21	11·01	10·02	9·19	8·65	8·22	*6·3*	
—	—	—	—	—	28·03	20·85	17·09	14·56	12·84	11·45	10·45	9·69	9·11	*6·4*	
—	—	—	—	—	47·99	28·04	21·30	17·44	15·07	13·20	11·90	10·83	10·07	*6·5*	
—	—	—	—	—	—	—	—	—	—	—	14·2	12·9	12·4	*6·6*	
—	—	—	—	—	—	—	—	—	—	—	17·2	15·6	14·7	*6·7*	
—	—	—	—	—	—	—	—	—	—	—	21·8	19·6	18·3	*6·8*	
—	—	—	—	—	—	—	—	—	—	—	29·4	26·0	24·1	*6·9*	
—	—	—	—	—	—	—	—	—	—	—	43·0	37·8	34·9	*7·0*	

TABLE XLI. *To find the Probable Error of the Skewness sk.*

Values of $\sqrt{N}\Sigma_{sk}$.

β_2 \ β_1	0·00	0·05	0·10	0·15	0·20	0·25	0·30	0·35	0·40	0·45	0·50	0·55	0·60	0·65	0·70	0·75
2·0	3·54	—	—	—	—	—	—	—	—	—	3·41	2·80	2·43	2·12	1·89	1·72
2·1	2·15	4·20	—	—	—	—	—	—	—	—	—	—	—	3·67	3·02	2·58
2·2	1·87	2·63	9·50	—	—	—	—	—	—	—	—	—	—	—	5·57	4·10
2·3	1·64	1·78	2·88	—	—	—	—	—	—	—	—	—	—	—	—	—
2·4	1·46	1·49	2·02	—	—	—	—	—	—	—	—	—	—	—	—	—
2·5	1·35	1·41	1·62	2·02	2·80	4·06	5·08	—	—	—	—	—	—	—	—	—
2·6	1·28	1·30	1·43	1·75	2·18	2·82	3·65	4·96	—	—	—	—	—	—	—	—
2·7	1·25	1·25	1·31	1·52	1·84	2·29	2·85	3·68	4·84	—	—	—	—	—	—	—
2·8	1·23	1·22	1·24	1·36	1·59	1·89	2·21	2·70	3·36	4·36	—	—	—	—	—	—
2·9	1·22	1·20	1·20	1·28	1·43	1·63	1·86	2·19	2·60	3·33	4·30	—	—	—	—	—
3·0	1·23	1·21	1·20	1·25	1·34	1·48	1·62	1·84	2·12	2·49	3·00	3·76	4·62	6·02	8·12	—
3·1	1·25	1·22	1·21	1·23	1·27	1·36	1·50	1·67	1·88	2·16	2·50	3·06	3·76	4·72	6·10	8·08
3·2	1·27	1·23	1·22	1·22	1·23	1·29	1·40	1·53	1·70	1·90	2·17	2·58	3·17	3·97	4·87	5·83
3·3	1·29	1·25	1·23	1·21	1·21	1·24	1·33	1·44	1·58	1·74	1·94	2·27	2·73	3·28	3·88	4·53
3·4	1·30	1·27	1·24	1·21	1·20	1·23	1·29	1·38	1·49	1·61	1·78	2·04	2·36	2·75	3·18	3·63
3·5	1·31	1·29	1·25	1·21	1·20	1·22	1·28	1·35	1·43	1·54	1·68	1·87	2·09	2·37	2·68	2·98
3·6	1·32	1·30	1·26	1·22	1·20	1·22	1·26	1·32	1·40	1·50	1·61	1·75	1·91	2·13	2·39	2·65
3·7	1·31	1·31	1·28	1·24	1·20	1·23	1·27	1·32	1·39	1·47	1·56	1·67	1·80	1·98	2·19	2·40
3·8	1·30	1·32	1·30	1·27	1·23	1·25	1·28	1·33	1·38	1·46	1·54	1·63	1·75	1·88	2·04	2·22
3·9	1·29	1·34	1·33	1·30	1·28	1·28	1·30	1·34	1·39	1·45	1·53	1·61	1·71	1·82	1·96	2·12
4·0	1·27	1·36	1·38	1·36	1·35	1·35	1·36	1·39	1·43	1·48	1·56	1·63	1·72	1·81	1·92	2·04
4·1	—	—	1·46	1·45	1·45	1·44	1·44	1·46	1·50	1·54	1·60	1·67	1·74	1·82	1·91	2·03
4·2	—	—	1·58	1·57	1·57	1·56	1·55	1·56	1·59	1·62	1·67	1·72	1·78	1·85	1·93	2·02
4·3	—	—	1·75	1·74	1·73	1·71	1·70	1·69	1·70	1·72	1·76	1·81	1·86	1·92	1·98	2·06
4·4	—	—	1·95	1·94	1·92	1·90	1·88	1·85	1·86	1·87	1·90	1·93	1·97	2·01	2·06	2·12
4·5	—	—	2·26	2·19	2·13	2·09	2·07	2·06	2·05	2·06	2·07	2·08	2·11	2·15	2·19	2·23
4·6	—	—	—	—	—	—	2·48	2·42	2·37	2·35	2·33	2·31	2·31	2·32	2·33	2·35
4·7	—	—	—	—	—	—	3·11	2·93	2·82	2·73	2·65	2·60	2·57	2·54	2·52	2·50
4·8	—	—	—	—	—	—	3·78	3·53	3·35	3·21	3·08	2·97	2·89	2·82	2·76	2·71
4·9	—	—	—	—	—	—	5·48	4·66	4·17	3·87	3·63	3·44	3·30	3·17	3·07	3·01
5·0	—	—	—	—	—	—	11·12	6·96	5·52	4·82	4·36	4·02	3·77	3·58	3·45	3·37
5·1	—	—	—	—	—	—	—	—	—	—	—	—	4·45	4·16	4·00	3·87
5·2	—	—	—	—	—	—	—	—	—	—	—	—	5·38	5·02	4·72	4·49
5·3	—	—	—	—	—	—	—	—	—	—	—	—	6·84	6·18	5·66	5·26
5·4	—	—	—	—	—	—	—	—	—	—	—	—	9·24	7·76	6·80	6·22
5·5	—	—	—	—	—	—	—	—	—	—	—	—	14·80	10·67	8·87	7·71
5·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

TABLE XLI—(*continued*).

Values of $\sqrt{N}\Sigma_{sk}$.

β_1

0·80	*0·85*	*0·90*	*0·95*	*1·00*	*1·05*	*1·10*	*1·15*	*1·20*	*1·25*	*1·30*	*1·35*	*1·40*	*1·45*	*1·50*	
1·59	1·48	1·39	1·30	1·24	1·19	1·14	1·10	1·06	1·02	·99	·95	·91	·87	·83	*2·0*
2·20	1·95	1·80	1·68	1·58	1·52	1·47	1·42	1·37	1·32	1·26	1·20	1·15	1·10	1·05	*2·1*
3·22	2·65	2·29	2·08	1·98	1·91	1·84	1·78	1·72	1·66	1·59	1·52	1·45	1·38	1·31	*2·2*
5·23	3·80	3·04	2·75	2·53	2·40	2·30	2·21	2·12	2·03	1·94	1·85	1·76	1·67	1·58	*2·3*
—	—	4·29	3·64	3·31	3·12	2·94	2·78	2·63	2·49	2·36	2·23	2·10	1·98	1·86	*2·4*
—	—	—	—	4·77	4·27	3·84	3·55	3·29	3·06	2·85	2·66	2·49	2·32	2·15	*2·5*
—	—	—	—	—	—	5·72	5·00	4·39	3·94	3·54	3·20	2·93	2·67	2·44	*2·6*
—	—	—	—	—	—	—	—	6·25	5·20	4·46	3·88	3·42	3·08	2·74	*2·7*
—	—	—	—	—	—	—	—	—	7·05	5·68	4·78	4·03	3·54	3·05	*2·8*
—	—	—	—	—	—	—	—	—	—	7·45	6·00	4·85	4·11	3·37	*2·9*
—	—	—	—	—	—	—	—	—	—	—	7·80	6·05	4·77	3·70	*3·0*
—	—	—	—	—	—	—	—	—	—	—	—	—	5·57	4·10	*3·1*
7·00	—	—	—	—	—	—	—	—	—	—	—	—	—	4·58	*3·2*
5·30	6·24	—	—	—	—	—	—	—	—	—	—	—	—	—	*3·3*
4·16	4·71	5·60	—	—	—	—	—	—	—	—	—	—	—	—	*3·4*
3·38	3·89	4·59	5·58	6·85	8·38	11·48	—	—	—	—	—	—	—	—	*3·5*
2·95	3·38	3·99	4·66	5·50	6·48	7·55	9·92	—	—	—	—	—	—	—	*3·6*
2·67	3·05	3·49	3·97	4·52	5·22	6·00	7·36	9·42	—	—	—	—	—	—	*3·7*
2·44	2·75	3·08	3·43	3·80	4·25	4·78	5·64	7·08	9·02	—	—	—	—	—	*3·8*
2·29	2·53	2·79	3·07	3·38	3·70	4·15	4·77	5·62	6·89	8·76	—	—	—	—	*3·9*
2·20	2·38	2·58	2·82	3·06	3·33	3·69	4·14	4·77	5·50	6·42	7·40	8·57	10·12	—	*4·0*
2·16	2·31	2·47	2·65	2·85	3·09	3·34	3·72	4·15	4·61	5·14	5·84	6·80	8·44	11·00	*4·1*
2·14	2·26	2·40	2·55	2·71	2·88	3·10	3·37	3·67	4·03	4·44	5·04	5·94	7·12	8·67	*4·2*
2·16	2·27	2·38	2·50	2·65	2·80	2·96	3·14	3·37	3·65	4·01	4·55	5·28	6·18	7·21	*4·3*
2·20	2·30	2·41	2·52	2·64	2·76	2·89	3·04	3·23	3·48	3·78	4·22	4·78	5·44	6·26	*4·4*
2·29	2·35	2·44	2·53	2·63	2·75	2·88	3·02	3·20	3·40	3·65	3·97	4·40	4·91	5·56	*4·5*
2·39	2·43	2·48	2·56	2·66	2·77	2·89	3·01	3·16	3·34	3·55	3·79	4·16	4·55	5·10	*4·6*
2·52	2·54	2·58	2·63	2·71	2·80	2·90	3·01	3·14	3·29	3·47	3·67	3·96	4·28	4·72	*4·7*
2·70	2·72	2·74	2·78	2·83	2·89	2·97	3·06	3·16	3·28	3·41	3·58	3·80	4·06	4·42	*4·8*
2·98	2·97	2·96	2·97	3·00	3·04	3·08	3·14	3·21	3·30	3·40	3·53	3·68	3·88	4·16	*4·9*
3·31	3·25	3·21	3·20	3·21	3·22	3·24	3·26	3·31	3·36	3·43	3·51	3·60	3·73	3·96	*5·0*
3·75	3·64	3·55	3·49	3·45	3·42	3·41	3·41	3·43	3·44	3·46	3·51	3·57	3·65	3·78	*5·1*
4·28	4·10	3·96	3·85	3·76	3·68	3·63	3·60	3·57	3·55	3·53	3·53	3·55	3·60	3·67	*5·2*
4·93	4·69	4·48	4·29	4·13	4·02	3·92	3·84	3·76	3·68	3·63	3·60	3·58	3·59	3·62	*5·3*
5·78	5·42	5·09	4·80	4·56	4·40	4·26	4·13	4·00	3·89	3·80	3·73	3·68	3·65	3·63	*5·4*
6·94	6·32	5·82	5·40	5·07	4·84	4·64	4·46	4·30	4·17	4·06	3·95	3·87	3·80	3·75	*5·5*
—	—	6·75	6·22	5·79	5·46	5·19	4·97	4·77	4·60	4·44	4·30	4·19	4·10	4·01	*5·6*
—	—	8·15	7·30	6·73	6·26	5·91	5·61	5·34	5·10	4·90	4·73	4·59	4·47	4·38	*5·7*
—	—	10·20	8·76	7·98	7·26	6·76	6·34	5·99	5·68	5·44	5·24	5·06	4·91	4·78	*5·8*
—	—	13·53	10·83	9·66	8·71	7·90	7·28	6·78	6·40	6·10	5·84	5·62	5·44	5·28	*5·9*
—	—	19·96	14·30	12·02	10·51	9·39	8·56	7·90	7·39	6·96	6·61	6·33	6·10	5·89	*6·0*
—	—	—	—	—	—	11·64	10·26	9·31	8·56	8·03	7·53	7·15	6·82	6·54	*6·1*
—	—	—	—	—	—	14·83	12·55	11·19	10·13	9·30	8·64	8·08	7·63	7·24	*6·2*
—	—	—	—	—	—	19·65	15·85	13·69	12·21	11·01	10·02	9·19	8·55	8·01	*6·3*
—	—	—	—	—	—	28·03	20·85	17·09	14·56	12·84	11·45	10·45	9·60	8·92	*6·4*
—	—	—	—	—	—	47·99	28·04	21·30	17·44	15·07	13·20	11·90	10·83	10·07	*6·5*
—	—	—	—	—	—	—	—	—	—	—	—	14·2	12·9	12·4	*6·6*
—	—	—	—	—	—	—	—	—	—	—	—	17·2	15·6	14·7	*6·7*
—	—	—	—	—	—	—	—	—	—	—	—	21·8	19·6	18·3	*6·8*
—	—	—	—	—	—	—	—	—	—	—	—	29·4	26·0	24·1	*6·9*
—	—	—	—	—	—	—	—	—	—	—	—	43·0	37·8	34·9	*7·0*

β_2

TABLE XLII. *To give values of β_3, β_4, β_5 and β_6 in terms*

TABLE XLII (*a*).

Values of β_3.

β_1 \ β_2	2·0	2·5	3·0	3·5	4·0	4·5	5·0	5·5	6·0	6·5	7·0
0·0	0	0	0	0	0						
0·1	0·48493	0·68971	0·94286	1·25688	1·64906	2·14375					
0·2	0·91585	1·32958	1·78182	2·37049	3·10000	4·01176					
0·3	1·29873	1·85270	2·53043	3·36094	4·38305	5·65000	7·2368				
0·4	1·63902	2·34286	3·20000	4·24478	5·52277	7·09474	9·0462				
0·5	1·94118	2·78126	3·80000	5·03582	6·53846	8·37500	10·6364				
0·6	2·20909	3·17350	4·33846	5·80178	7·44706	9·51429	12·0414	15·1585			
0·7	2·44615	3·52441	4·82222	6·38287	8·26202	10·53182	13·2885	16·6624			
0·8	2·65532	3·83820	5·25714	6·95698	8·99462	11·44375	14·4000	17·9932			
0·9	2·83917	4·12064	5·64828	7·47438	9·65454	12·26250	15·3940	19·1758	23·7791		
1·0	3·00000	4·36842	6·00000	7·94121	10·24999	13·00000	16·2857	20·2308	25·0000		
1·1	3·13980	4·59081	6·31613	8·36246	10·78796	13·66538	17·0877	21·1750	26·0857	32·0328	
1·2	3·26038	4·78812	6·60000	8·74286	11·27443	14·26667	17·8105	22·0225	27·0546	33·1082	
1·3	3·36330	4·96250	6·85454	9·08619	11·71461	14·81071	18·4633	22·7851	27·9217	34·0635	
1·4	3·45000	5·11589	7·08235	9·39582	12·11304	15·30345	19·0536	23·4727	28·7000	34·9164	42·3613
1·5	3·52174	5·25000	7·28571	9·67501	12·47368	15·75000	19·5864	24·0937	29·4000	35·6786	43·1538

TABLE XLII (*b*).

Values of β_4.

β_1 \ β_2	2·0	2·5	3·0	3·5	4·0	4·5	5·0	5·5	6·0	6·5	7·0
0·0	5·00000	8·92856	15·0000	23·7288	31·0000						
0·1	5·27356	9·41054	15·7973	25·7430	41·7660	69·3682					
0·2	5·44361	9·75086	16·2648	26·4018	42·5000	69·4796					
0·3	5·53293	9·86224	16·4907	26·6520	42·5613	68·5776	114·4732				
0·4	5·55998	9·91072	16·5385	26·6144	42·1807	67·0888	109·4534				
0·5	5·53802	9·87751	16·4545	26·3742	41·5076	65·2679	104·4652				
0·6	5·47791	9·81734	16·2732	26·1077	40·6453	63·2707	99·6442	162·125			
0·7	5·38824	9·63895	16·0200	25·5026	39·6623	61·1946	95·0525	151·253			
0·8	5·27513	9·45991	15·7143	24·9478	38·6061	59·1016	90·7143	141·707			
0·9	5·14437	9·25645	15·3706	24·3462	37·5099	57·0279	86·6331	133·240	210·995		
1·0	5·00000	9·02746	15·0000	23·7495	36·3971	55·0000	82·8022	125·664	195·000		
1·1	4·84537	8·78075	14·6111	23·0744	35·2835	53·0316	79·2091	118·839	181·299	286·374	
1·2	4·68319	8·53522	14·2105	22·4107	34·1811	51·1309	75·8392	112·653	169·394	261·436	
1·3	4·51562	8·27700	13·8032	21·7535	33·0971	49·3447	72·6772	107·016	158·930	240·845	
1·4	4·34440	8·01454	13·3931	21·1002	32·0367	47·5471	69·7076	101·850	149·643	223·304	343·147
1·5	4·17097	7·75000	12·9832	20·4546	31·0037	45·9038	66·9117	97·112	141·333	208·129	313·704

of β_1 and β_2 on the assumption that the Frequency falls into one or other of Pearson's Types.

TABLE XLII (*c*).

Values of β_5.

β_1 \ β_2	2·0	2·5	3·0	3·5	4·0	4·5	5·0	5·5	6·0	6·5	7·0
0·0	0	0	0	0	0						
0·1	1·99086	4·39480	9·3207	19·9714	45·9387	128·529					
0·2	3·59438	8·03374	16·5960	34·7825	76·6000	193·361					
0·3	4·86677	10·68765	22·2196	45·7142	97·2263	228·104	668·284				
0·4	5·85929	12·85477	26·5187	53·7090	111·0237	246·506	650·398				
0·5	6·51704	14·51540	29·7545	59·4655	120·0543	255·295	614·633				
0·6	7·17383	15·7892	32·1362	63·9266	125·6629	258·147	581·205	1618·635			
0·7	7·56616	16·6546	33·8306	66·2045	128·8283	257·225	550·107	1368·373			
0·8	7·81963	17·2668	34·9714	67·8804	130·2010	253·872	521·257	1196·612			
0·9	7·95777	17·6667	35·6658	68·7533	130·2587	248·937	495·375	1068·877	2769·42		
1·0	8·00000	17·8291	36·0000	69·0644	129·3434	243·000	469·637	968·318	2280·00		
1·1	7·96281	17·8472	36·0437	68·7730	127·7158	236·441	446·547	886·541	1945·69	5313·80	
1·2	7·86015	17·7503	35·8535	68·1357	125·5684	229·524	425·062	818·040	1700·98	4135·56	
1·3	7·70375	17·5396	35·4754	67·2181	123·0362	222·562	405·663	759·486	1512·94	3388·18	
1·4	7·50358	17·2423	34·9467	66·0678	120·5142	214·828	386·347	708·620	1363·20	2870·08	7265·31
1·5	7·26808	16·8768	34·2983	64·7210	117·2460	207·227	368·843	663·926	1240·65	2488·62	5719·68

TABLE XLII (*d*).

Values of β_6.

β_1 \ β_2	2·0	2·5	3·0	3·5	4·0	4·5	5·0	5·5	6·0	6·5	7·0
0·0	14·0000	39·0649	105·000	290·678	868·015						
0·1	16·4616	45·7741	124·835	355·508	1243·832	10228·33					
0·2	17·7296	50·2472	132·998	369·894	1190·700	6204·69					
0·3	18·1764	51·0927	134·215	361·909	1089·739	4485·38	107697·95				
0·4	18·0667	50·2458	131·337	344·886	977·506	3471·87	25413·18				
0·5	17·5474	48·7896	126·107	323·447	877·884	2792·19	13737·63				
0·6	16·8560	46·8558	119·601	303·252	784·431	2303·07	9048·43	119230·33			
0·7	15·9787	44·3106	112·492	277·658	701·500	1934·79	6534·78	40994·77			
0·8	15·0148	41·7081	105·200	255·716	628·450	1648·52	5045·80	22660·09			
0·9	14·0113	39·0906	97·984	235·072	564·277	1420·51	4024·45	14836·90	137288·7		
1·0	13·0000	36·4119	91·000	216·137	507·894	1235·50	3286·65	10612·25	57584·9		
1·1	12·0030	33·7916	84·339	198·263	456·575	1083·04	2741·39	8135·91	33078·5	797653·2	
1·2	11·0354	31·3418	78·047	181·987	414·455	955·78	2322·13	6314·06	21891·8	155693·9	
1·3	10·1070	28·9775	72·146	167·142	375·834	848·97	1994·05	5108·55	15690·2	75009·7	
1·4	9·2240	26·7355	66·637	153·582	342·057	755·79	1726·18	4219·50	11846·9	44891·9	565740
1·5	8·3899	24·6268	61·512	141·477	310·976	676·32	1508·92	3544·82	9281·2	30280·3	180793

TABLE XLIII.

Probable Error of Criterion κ_2. *Values of* $\sqrt{N}\,\Sigma_{\kappa_2}$ *for values of* β_1, β_2

β_2 \ β_1	·00	·05	·10	·15	·20	·25	·30	·35	·40	·45	·50	·55	·60	·65	·70
2·0	·000	·242	·332	·399	·454	·498	·545	·582	·631	·671	·716	·758	·806	·854	·899
2·1	·000	·271	·367	·430	·483	·521	·557	·600	·639	·678	·717	·760	·800	·843	·890
2·2	·000	·310	·415	·480	·527	·565	·597	·626	·656	·691	·728	·767	·809	·845	·890
2·3	·000	·355	·477	·550	·596	·635	·660	·678	·700	·725	·753	·790	·826	·858	·895
2·4	·000	·417	·560	·642	·691	·722	·748	·759	·770	·787	·806	·830	·857	·884	·914
2·5	·000	·500	·697	·771	·816	·841	·855	·860	·867	·873	·881	·892	·909	·928	·947
2·6	·000	·660	·840	·946	1·01	1·03	1·04	1·02	1·01	1·00	1·00	1·00	1·00	1·00	1·00
2·7	·000	1·04	1·30	1·35	1·34	1·33	1·30	1·27	1·24	1·19	1·15	1·12	1·10	1·09	1·08
2·8	·000	1·83	1·97	1·98	1·93	1·84	1·74	1·64	1·55	1·47	1·40	1·33	1·27	1·23	1·20
2·9	·000	3·51	3·71	3·42	3·00	2·66	2·42	2·22	2·05	1·89	1·73	1·61	1·51	1·45	1·39
3·0	—	18·8	9·89	6·94	5·30	4·30	3·62	3·10	2·73	2·43	2·18	2·00	1·84	1·71	1·61
3·1	·000	—	62·0	20·5	8·47	6·36	5·22	4·42	3·78	3·34	2·96	2·66	2·41	2·20	2·01
3·2	·000	7·82	91·7	—	49·7	20·2	12·2	8·48	6·44	5·09	4·28	3·71	3·21	2·86	2·58
3·3	·000	2·99	11·7	70·2	—	142	32·4	15·4	10·8	8·56	6·88	5·62	4·66	3·98	3·46
3·4	·000	1·82	4·80	13·8	55·5	—	344	182	28·9	16·8	11·8	8·69	6·88	5·60	4·76
3·5	·000	1·43	2·89	6·43	15·8	50·6	380	—	127	46·5	25·1	16·1	11·5	8·55	6·91
3·6	·000	1·17	2·18	4·08	8·00	17·2	46·5	215	—	277	76·4	35·4	22·4	15·8	11·5
3·7	·000	1·04	1·79	3·08	5·18	9·36	24·6	44·3	155	—	767	133	54·6	28·5	19·2
3·8	·000	·979	1·54	2·54	4·09	6·33	14·3	26·2	44·0	126	855	—	230	80·0	40·8
3·9	·000	·920	1·41	2·20	3·32	4·91	8·84	13·1	20·5	38·2	105	448	—	—	125
4·0	·000	·869	1·35	2·00	2·83	4·08	5·98	9·08	13·7	22·3	41·4	98·0	296	—	—
4·1	—	—	1·30	1·94	2·60	3·61	5·08	7·25	10·4	15·8	24·4	40·5	78·7	216	—
4·2	—	—	1·33	1·94	2·58	3·43	4·54	6·09	8·49	11·8	16·8	25·3	39·4	67·2	169
4·3	—	—	1·44	2·01	2·63	3·37	4·20	5·41	7·19	9·52	12·8	17·8	26·2	45·2	72·2
4·4	—	—	1·58	2·15	2·74	3·39	4·20	5·18	6·58	8·56	10·9	14·6	20·2	29·0	40·4
4·5	—	—	1·81	2·32	2·94	3·59	4·36	5·29	6·45	7·76	9·85	12·4	16·1	21·1	29·9
4·6	—	—	—	—	—	—	4·90	5·63	6·60	7·84	9·20	11·1	13·6	17·2	24·2
4·7	—	—	—	—	—	—	5·87	6·46	7·11	7·97	8·96	10·2	12·2	15·3	20·0
4·8	—	—	—	—	—	—	7·43	7·61	7·94	8·41	9·16	10·1	12·0	14·3	17·5
4·9	—	—	—	—	—	—	10·1	9·45	9·08	9·36	9·80	10·6	12·0	13·7	15·9
5·0	—	—	—	—	—	—	15·1	11·3	10·4	10·4	10·8	11·5	12·4	13·6	15·4
5·1	—	—	—	—	—	—	—	—	—	—	—	—	13·5	14·0	15·3
5·2	—	—	—	—	—	—	—	—	—	—	—	—	15·3	15·5	16·0
5·3	—	—	—	—	—	—	—	—	—	—	—	—	18·3	17·8	17·7
5·4	—	—	—	—	—	—	—	—	—	—	—	—	22·8	20·6	20·0
5·5	—	—	—	—	—	—	—	—	—	—	—	—	30·6	26·0	23·3
5·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

TABLE XLIII—(*continued*).

Probable Error of Criterion κ_2. *Values of* $\sqrt{N}\Sigma_{\kappa_2}$ *for values of* β_1, β_2.

β_1																
·75	·80	·85	·90	·95	1·00	1·05	1·10	1·15	1·20	1·25	1·30	1·35	1·40	1·45	1·50	
·949	1·00	1·06	1·12	1·18	1·25	1·31	1·39	1·46	1·56	1·64	1·75	1·86	1·97	2·11	2·25	*2·0*
·937	·992	1·05	1·10	1·16	1·22	1·28	1·35	1·42	1·49	1·58	1·67	1·77	1·88	2·00	2·12	*2·1*
·936	·987	1·04	1·09	1·14	1·19	1·25	1·31	1·37	1·43	1·51	1·59	1·69	1·79	1·89	1·99	*2·2*
·939	·982	1·04	1·08	1·12	1·17	1·22	1·27	1·32	1·38	1·45	1·53	1·62	1·70	1·79	1·87	*2·3*
·950	·990	1·03	1·07	1·11	1·15	1·19	1·24	1·29	1·34	1·40	1·47	1·55	1·62	1·69	1·77	*2·4*
·972	·998	1·03	1·07	1·10	1·14	1·18	1·22	1·27	1·32	1·37	1·43	1·49	1·55	1·61	1·68	*2·5*
1·01	1·03	1·06	1·09	1·12	1·15	1·18	1·21	1·26	1·31	1·36	1·41	1·46	1·51	1·57	1·63	*2·6*
1·07	1·08	1·09	1·11	1·14	1·17	1·19	1·21	1·25	1·30	1·35	1·39	1·43	1·48	1·53	1·59	*2·7*
1·17	1·15	1·16	1·17	1·18	1·19	1·21	1·23	1·27	1·31	1·35	1·39	1·43	1·47	1·52	1·58	*2·8*
1·33	1·28	1·26	1·26	1·25	1·25	1·25	1·27	1·29	1·32	1·35	1·39	1·43	1·47	1·52	1·57	*2·9*
1·53	1·47	1·42	1·38	1·36	1·34	1·34	1·34	1·35	1·36	1·38	1·40	1·43	1·46	1·51	1·56	*3·0*
1·85	1·72	1·62	1·56	1·52	1·49	1·46	1·44	1·43	1·43	1·43	1·44	1·45	1·47	1·51	1·57	*3·1*
2·34	2·13	1·96	1·83	1·75	1·68	1·62	1·57	1·54	1·52	1·51	1·50	1·50	1·51	1·54	1·58	*3·2*
3·03	2·69	2·44	2·23	2·06	1·94	1·85	1·77	1·72	1·67	1·63	1·60	1·59	1·58	1·59	1·61	*3·3*
4·08	3·50	3·06	2·74	2·49	2·29	2·14	2·02	1·92	1·84	1·78	1·73	1·70	1·68	1·68	1·69	*3·4*
5·72	4·75	4·12	3·56	3·15	2·83	2·59	2·38	2·22	2·09	1·99	1·91	1·85	1·81	1·79	1·79	*3·5*
8·85	6·90	5·71	4·79	4·15	3·68	3·22	2·90	2·67	2·49	2·31	2·20	2·09	2·00	1·96	1·92	*3·6*
14·2	10·4	8·22	6·69	5·70	4·81	4·17	3·67	3·31	3·05	2·81	2·62	2·45	2·29	2·19	2·10	*3·7*
24·9	18·1	13·8	10·1	8·16	6·71	5·76	4·92	4·28	3·84	3·48	3·16	2·91	2·68	2·50	2·36	*3·8*
65·5	36·5	21·7	15·8	12·2	10·0	8·17	6·75	5·75	5·00	4·43	3·93	3·52	3·18	2·91	2·72	*3·9*
242	86·6	48·5	30·0	20·9	15·4	12·0	9·61	7·86	6·60	5·67	4·90	4·34	3·88	3·47	3·19	*4·0*
—	374	127	62·9	38·3	26·0	18·3	14·4	11·5	9·15	7·53	6·45	5·66	4·98	4·38	3·80	*4·1*
—	—	—	200	91·0	51·6	32·7	23·4	17·4	13·4	10·6	8·70	7·50	6·48	5·52	4·61	*4·2*
144	478	—	—	314	112	70·0	42·9	29·1	21·3	16·1	12·7	10·3	8·55	7·24	6·25	*4·3*
62·8	135	—	—	—	580	192	93·9	55·8	37·1	26·5	19·8	15·4	12·1	9·74	8·26	*4·4*
42·8	68·3	119	280	—	—	—	286	126	72·8	46·4	32·2	23·9	18·3	14·6	11·8	*4·5*
32·3	44·7	62·7	99·6	240	742	—	—	—	181	91·0	58·9	41·0	30·0	22·3	16·9	*4·6*
26·2	33·8	46·1	68·0	105	240	532	—	—	—	260	128	76·7	50·4	36·0	26·6	*4·7*
21·6	27·1	35·0	47·3	66·8	104	182	413	—	—	—	403	172	99·3	63·0	44·8	*4·8*
18·9	22·6	26·9	33·7	44·0	61·5	84·2	115	337	—	—	—	—	249	140	80·8	*4·9*
17·8	20·7	24·6	30·1	37·8	48·9	66·0	97·0	157	286	—	—	—	—	—	172	*5·0*
17·2	20·0	23·0	27·1	32·5	40·6	51·5	69·2	99·8	147	253	559	—	—	—	—	*5·1*
17·3	19·3	21·7	24·8	29·5	35·4	43·1	54·7	70·6	94·6	138	216	—	—	—	—	*5·2*
18·0	19·7	21·4	23·9	27·1	31·5	37·0	44·2	54·5	69·3	93·2	132	205	380	—	—	*5·3*
19·6	20·5	21·6	23·3	25·6	28·6	32·8	38·0	44·8	54·0	68·6	94·0	130	185	—	—	*5·4*
22·3	22·0	22·5	23·6	25·2	27·5	30·5	34·3	39·4	45·4	54·7	67·3	86·5	116	169	275	*5·5*
—	—	—	24·5	25·8	27·6	29·8	32·2	35·6	39·6	44·8	51·4	63·2	83·0	118	168	*5·6*
—	—	—	26·8	27·8	28·8	30·0	31·5	33·8	36·5	39·8	44·4	53·1	67·0	85·2	116	*5·7*
—	—	—	30·7	30·8	31·0	31·3	32·0	33·4	35·3	38·2	42·2	48·4	56·6	69·9	87·9	*5·8*
—	—	—	38·4	36·0	34·4	33·5	33·1	34·1	36·0	38·7	42·2	47·0	53·1	61·9	74·8	*5·9*
—	—	—	50·4	42·5	38·3	36·8	36·4	36·5	37·9	40·0	43·0	46·6	51·3	57·8	66·2	*6·0*
—	—	—	—	—	—	—	41·0	41·0	41·6	42·7	44·6	47·2	50·3	55·0	61·5	*6·1*
—	—	—	—	—	—	—	48·2	47·0	46·1	46·0	46·7	48·0	50·0	53·5	58·5	*6·2*
—	—	—	—	—	—	—	57·3	54·0	51·6	49·8	49·1	49·4	50·0	52·5	56·8	*6·3*
—	—	—	—	—	—	—	79·4	66·6	58·6	54·2	51·8	51·2	50·8	52·5	55·5	*6·4*
—	—	—	—	—	—	—	128	84·0	66·9	59·9	55·9	53·9	53·1	53·6	55·4	*6·5*
—	—	—	—	—	—	—	—	—	—	—	—	—	57·4	57·0	56·7	*6·6*
—	—	—	—	—	—	—	—	—	—	—	—	—	63·5	61·5	59·2	*6·7*
—	—	—	—	—	—	—	—	—	—	—	—	—	73·6	67·8	63·5	*6·8*
—	—	—	—	—	—	—	—	—	—	—	—	—	97·2	77·7	70·5	*6·9*
—	—	—	—	—	—	—	—	—	—	—	—	—	134	96·0	82·0	*7·0*

β_2

TABLE XLIV. *To find probable Frequency Type.*

Values of $1{\cdot}177\sqrt{N}\Sigma_1$ *for given values of* β_1, β_2 (*Semi-Minor Axis of Probability Ellipse*).

β_2 \ β_1	0	·05	·1	·15	·2	·25	·3	·35	·4	·45	·5	·55	·6	·65	·7	·75
2·0	0·0	0·5	0·7	0·8	0·8	0·8	0·8	0·8	0·7	0·7	0·7	0·6	0·6	0·5	0·5	0·4
2·1	0·0	0·5	0·7	0·8	0·9	0·9	0·8	0·8	0·8	0·8	0·7	0·7	0·7	0·6	0·6	0·5
2·2	0·0	0·5	0·8	0·9	0·9	0·9	0·9	0·9	0·9	0·8	0·8	0·8	0·7	0·7	0·7	0·6
2·3	0·0	0·5	0·8	0·9	1·0	1·0	1·0	1·0	1·0	0·9	0·9	0·9	0·8	0·8	0·8	0·7
2·4	0·0	0·6	0·9	1·0	1·0	1·0	1·0	1·0	1·0	1·0	1·0	0·9	0·9	0·9	0·9	0·8
2·5	0·0	0·6	0·9	1·0	1·1	1·1	1·1	1·1	1·1	1·1	1·0	1·0	1·0	1·0	1·0	0·9
2·6	0·0	0·6	1·0	1·1	1·1	1·2	1·2	1·2	1·1	1·1	1·1	1·1	1·1	1·0	1·0	1·0
2·7	0·0	0·7	1·0	1·2	1·2	1·3	1·3	1·3	1·2	1·2	1·2	1·2	1·1	1·1	1·1	1·1
2·8	0·0	0·7	1·1	1·3	1·3	1·3	1·3	1·3	1·3	1·3	1·3	1·3	1·2	1·2	1·2	1·2
2·9	0·0	0·8	1·2	1·3	1·3	1·4	1·4	1·4	1·4	1·4	1·4	1·4	1·3	1·3	1·3	1·3
3·0	0·0	0·8	1·2	1·4	1·4	1·5	1·5	1·5	1·5	1·5	1·5	1·5	1·4	1·4	1·4	1·4
3·1	0·0	0·9	1·3	1·5	1·5	1·6	1·6	1·6	1·6	1·6	1·6	1·6	1·6	1·5	1·5	1·5
3·2	0·0	1·0	1·4	1·6	1·6	1·7	1·7	1·7	1·7	1·7	1·7	1·7	1·7	1·6	1·6	1·6
3·3	·00	1·1	1·6	1·8	1·8	1·9	1·9	1·9	1·8	1·8	1·8	1·8	1·8	1·7	1·7	1·7
3·4	·00	1·2	1·7	1·9	1·9	2·0	2·0	2·0	2·0	2·0	1·9	1·9	1·9	1·9	1·9	1·8
3·5	·00	1·4	1·8	2·0	2·1	2·1	2·1	2·1	2·1	2·1	2·1	2·1	2·1	2·0	2·0	1·9
3·6	·00	1·5	2·0	2·1	2·2	2·3	2·3	2·3	2·3	2·3	2·3	2·2	2·2	2·1	2·1	2·1
3·7	·00	1·6	2·1	2·3	2·4	2·5	2·5	2·5	2·5	2·5	2·4	2·4	2·4	2·3	2·3	2·3
3·8	·00	1·8	2·3	2·5	2·6	2·7	2·7	2·7	2·7	2·7	2·6	2·6	2·6	2·5	2·5	2·4
3·9	·00	2·0	2·5	2·7	2·9	3·0	3·0	3·0	3·0	2·9	2·9	2·8	2·8	2·7	2·7	2·6
4·0	·00	2·2	2·8	3·0	3·2	3·3	3·3	3·3	3·3	3·2	3·1	3·1	3·0	2·9	2·9	2·8
4·1	—	—	3·0	3·3	3·5	3·6	3·6	3·6	3·6	3·5	3·4	3·4	3·3	3·2	3·2	3·1
4·2	—	—	3·3	3·6	3·8	3·9	4·0	4·0	3·9	3·8	3·8	3·7	3·6	3·5	3·5	3·4
4·3	—	—	3·6	4·0	4·2	4·3	4·4	4·4	4·3	4·2	4·1	4·0	3·9	3·8	3·8	3·7
4·4	—	—	4·0	4·4	4·7	4·8	4·8	4·8	4·7	4·6	4·5	4·4	4·3	4·2	4·1	4·0
4·5	—	—	4·4	4·9	5·2	5·3	5·3	5·3	5·2	5·1	5·0	4·9	4·7	4·6	4·5	4·4
4·6	—	—	—	—	—	—	5·9	5·9	5·8	5·7	5·6	5·4	5·2	5·1	5·0	4·8
4·7	—	—	—	—	—	—	6·7	6·7	6·6	6·4	6·2	6·0	5·8	5·7	5·6	5·3
4·8	—	—	—	—	—	—	7·6	7·6	7·4	7·2	6·9	6·6	6·4	6·3	6·2	5·9
4·9	—	—	—	—	—	—	8·5	8·5	8·4	8·1	7·8	7·5	7·2	7·0	6·8	6·5
5·0	—	—	—	—	—	—	11	10	9·7	9·3	8·9	8·5	8·1	7·7	7·4	7·1
5·1	—	—	—	—	—	—	—	—	—	—	—	—	9·3	8·6	8·1	7·7
5·2	—	—	—	—	—	—	—	—	—	—	—	—	11	9·5	8·9	8·6
5·3	—	—	—	—	—	—	—	—	—	—	—	—	12	11	10	9·6
5·4	—	—	—	—	—	—	—	—	—	—	—	—	14	13	12	11
5·5	—	—	—	—	—	—	—	—	—	—	—	—	16	15	14	13
5·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

TABLE XLIV—(*continued*).

Values of $1{\cdot}177\sqrt{N}\,\Sigma_1$ *for given values of* β_1, β_2 (*Semi-Minor Axis of Probability Ellipse*).

β_1

·8	·85	·9	·95	1·0	1·05	1·1	1·15	1·2	1·25	1·3	1·35	1·4	1·45	1·5	β_2
0·4	0·3	0·3	0·2	0·1	0·1	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	2·0
0·5	0·5	0·4	0·3	0·2	0·2	0·2	0·1	0·1	0·1	0·0	0·0	0·0	0·0	0·0	2·1
0·6	0·6	0·5	0·5	0·4	0·4	0·3	0·3	0·3	0·2	0·1	0·0	0·0	0·0	0·0	2·2
0·7	0·7	0·6	0·6	0·5	0·5	0·4	0·4	0·4	0·3	0·2	0·1	0·1	0·0	0·0	2·3
0·8	0·8	0·7	0·7	0·6	0·6	0·5	0·5	0·4	0·4	0·3	0·2	0·2	0·1	0·0	2·4
0·9	0·8	0·8	0·7	0·7	0·7	0·6	0·6	0·5	0·5	0·4	0·3	0·3	0·2	0·1	2·5
1·0	0·9	0·9	0·8	0·8	0·8	0·7	0·7	0·7	0·6	0·6	0·5	0·4	0·3	0·2	2·6
1·1	1·0	1·0	0·9	0·9	0·9	0·8	0·8	0·8	0·7	0·7	0·6	0·5	0·4	0·3	2·7
1·1	1·1	1·1	1·0	1·0	1·0	0·9	0·9	0·9	0·8	0·8	0·7	0·6	0·5	0·5	2·8
1·2	1·2	1·2	1·1	1·1	1·0	1·0	1·0	1·0	1·0	0·9	0·8	0·7	0·7	0·7	2·9
1·3	1·3	1·3	1·2	1·2	1·2	1·1	1·1	1·1	1·0	1·0	0·9	0·9	0·8	0·8	3·0
1·4	1·4	1·4	1·3	1·3	1·3	1·2	1·2	1·2	1·1	1·1	1·0	1·0	0·9	0·9	3·1
1·5	1·5	1·5	1·4	1·4	1·4	1·3	1·3	1·2	1·2	1·2	1·1	1·1	1·0	1·0	3·2
1·7	1·6	1·6	1·5	1·4	1·4	1·3	1·3	1·3	1·2	1·2	1·2	1·1	1·1	1·0	3·3
1·8	1·7	1·7	1·6	1·6	1·6	1·5	1·5	1·4	1·4	1·3	1·3	1·2	1·2	1·1	3·4
1·9	1·9	1·8	1·8	1·7	1·7	1·6	1·6	1·5	1·5	1·4	1·4	1·3	1·3	1·2	3·5
2·0	2·0	2·0	1·9	1·8	1·8	1·7	1·7	1·6	1·6	1·5	1·5	1·4	1·4	1·4	3·6
2·2	2·2	2·1	2·1	2·0	2·0	1·9	1·8	1·7	1·7	1·6	1·6	1·6	1·5	1·5	3·7
2·4	2·3	2·2	2·2	2·1	2·1	2·0	2·0	1·9	1·9	1·8	1·7	1·7	1·6	1·6	3·8
2·6	2·5	2·4	2·4	2·3	2·3	2·2	2·1	2·0	2·0	1·9	1·9	1·8	1·7	1·7	3·9
2·8	2·7	2·6	2·6	2·5	2·5	2·4	2·3	2·2	2·2	2·1	2·1	2·0	2·0	1·9	4·0
3·0	2·9	2·8	2·8	2·7	2·7	2·6	2·5	2·4	2·4	2·3	2·3	2·2	2·2	2·1	4·1
3·2	3·2	3·1	3·0	2·9	2·9	2·8	2·7	2·6	2·6	2·5	2·4	2·3	2·3	2·2	4·2
3·5	3·5	3·4	3·3	3·2	3·1	3·0	2·9	2·8	2·8	2·7	2·6	2·5	2·4	2·4	4·3
3·9	3·8	3·7	3·6	3·5	3·4	3·3	3·1	3·0	3·0	2·9	2·8	2·7	2·7	2·6	4·4
4·2	4·1	4·0	3·9	3·7	3·6	3·5	3·4	3·3	3·2	3·1	3·1	3·0	3·0	2·9	4·5
4·6	4·5	4·4	4·2	4·1	3·9	3·8	3·7	3·5	3·4	3·3	3·3	3·2	3·2	3·1	4·6
5·1	4·9	4·8	4·6	4·4	4·3	4·2	4·0	3·8	3·7	3·6	3·5	3·4	3·4	3·3	4·7
5·6	5·4	5·2	5·0	4·8	4·7	4·6	4·4	4·2	4·0	3·9	3·8	3·7	3·6	3·5	4·8
6·2	6·0	5·7	5·5	5·3	5·1	5·0	4·8	4·6	4·4	4·2	4·1	4·0	3·9	3·8	4·9
6·8	6·6	6·3	6·1	5·8	5·6	5·4	5·2	5·0	4·8	4·6	4·4	4·3	4·2	4·1	5·0
7·4	7·2	7·0	6·7	6·4	6·2	5·9	5·7	5·4	5·1	4·9	4·7	4·6	4·4	4·3	5·1
8·3	8·0	7·7	7·4	7·1	6·8	6·5	6·2	5·8	5·5	5·2	5·0	4·9	4·7	4·5	5·2
9·2	8·9	8·6	8·2	7·8	7·5	7·1	6·7	6·3	5·9	5·6	5·4	5·2	5·0	4·7	5·3
10	10	9·6	9·1	8·6	8·2	7·8	7·3	6·9	6·5	6·1	5·8	5·6	5·3	5·0	5·4
12	11	11	10	9·5	9·0	8·5	8·0	7·5	7·1	6·7	6·4	6·1	5·8	5·5	5·5
—	—	12	11	10	10	9·5	8·9	8·4	7·9	7·4	7·0	6·6	6·3	6·0	5·6
—	—	14	13	12	11	10	10	9·4	8·8	8·3	7·8	7·4	7·0	6·7	5·7
—	—	16	14	13	12	12	11	10	10	9·4	8·8	8·3	7·8	7·4	5·8
—	—	18	16	15	14	14	13	12	11	10	10	9·5	8·9	8·4	5·9
—	—	22	20	18	16	15	14	13	12	12	11	11	10	9·5	6·0
—	—	—	—	—	—	18	16	15	14	13	13	12	12	11	6·1
—	—	—	—	—	—	21	19	17	16	15	14	13	13	12	6·2
—	—	—	—	—	—	24	21	19	18	17	16	15	14	13	6·3
—	—	—	—	—	—	27	24	22	20	19	18	17	16	15	6·4
—	—	—	—	—	—	32	28	25	23	21	20	19	18	16	6·5
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	6·6
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	6·7
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	6·8
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	6·9
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	7·0

TABLE XLV. *To find probable Frequency Type.*

Values of $1{\cdot}177\sqrt{N}\,\Sigma_2$ *for values of* β_1, β_2 (*Semi-Major Axis of Probability Ellipse*).

β_2 \ β_1	0	·05	·1	·15	·2	·25	·3	·35	·4	·45	·5	·55	·6	·65	·7	·75
2·0	1·6	1·9	2·2	2·5	2·8	3·1	3·4	3·7	4·1	4·4	4·7	5·0	5·4	5·7	6·1	6·4
2·1	1·7	2·1	2·4	2·7	2·9	3·2	3·5	3·8	4·1	4·4	4·6	4·9	5·3	5·6	6·0	6·3
2·2	1·9	2·3	2·6	2·9	3·1	3·4	3·7	3·9	4·1	4·4	4·6	4·9	5·2	5·5	5·9	6·2
2·3	2·1	2·5	2·8	3·1	3·3	3·5	3·8	4·0	4·2	4·4	4·6	4·9	5·2	5·5	5·8	6·1
2·4	2·4	2·8	3·1	3·3	3·5	3·7	3·9	4·1	4·3	4·5	4·7	4·9	5·1	5·4	5·7	6·0
2·5	2·8	3·3	3·5	3·7	3·8	4·0	4·1	4·3	4·5	4·7	4·8	5·0	5·2	5·4	5·7	5·9
2·6	3·3	3·8	4·0	4·2	4·3	4·5	4·6	4·7	4·8	4·9	5·0	5·1	5·3	5·5	5·7	6·0
2·7	3·8	4·4	4·6	4·7	4·9	5·0	5·1	5·2	5·3	5·4	5·4	5·4	5·5	5·6	5·8	6·1
2·8	4·4	5·0	5·3	5·4	5·6	5·7	5·8	5·9	5·9	5·9	5·8	5·8	5·8	5·9	6·0	6·2
2·9	5·1	5·7	6·0	6·2	6·3	6·4	6·5	6·6	6·5	6·5	6·4	6·3	6·3	6·3	6·4	6·5
3·0	5·8	6·5	6·9	7·1	7·2	7·3	7·3	7·3	7·2	7·1	7·0	6·9	6·8	6·8	6·8	6·8
3·1	6·7	7·5	8·0	8·3	8·4	8·4	8·4	8·3	8·2	8·0	7·9	7·8	7·7	7·6	7·5	7·4
3·2	7·8	8·7	9·2	9·6	9·8	9·8	9·7	9·5	9·4	9·2	9·0	8·8	8·6	8·4	8·3	8·1
3·3	9·2	10	11	11	11	11	11	11	11	11	10	10	9·6	9·4	9·2	8·9
3·4	11	11	12	12	13	13	13	12	12	12	11	11	11	11	10	10
3·5	13	13	14	14	15	15	14	14	14	14	13	12	12	12	11	11
3·6	15	16	17	17	17	17	16	16	16	16	15	14	14	14	13	13
3·7	18	19	20	20	20	20	19	19	18	18	17	17	16	16	15	14
3·8	21	23	24	24	24	23	22	22	21	21	20	19	18	18	17	16
3·9	25	27	28	28	28	27	26	25	24	24	23	22	21	20	19	18
4·0	29	32	34	33	32	31	30	29	28	27	26	24	23	22	21	20
4·1	—	—	43	41	39	37	36	34	32	31	30	28	26	25	24	23
4·2	—	—	55	51	48	45	42	40	38	36	34	32	30	29	27	26
4·3	—	—	69	62	57	53	50	48	45	43	40	37	35	33	31	29
4·4	—	—	87	76	69	64	60	56	53	50	46	43	40	38	36	34
4·5	—	—	113	98	86	78	71	66	62	57	53	49	46	43	41	39
4·6	—	—	—	—	—	—	90	79	73	67	61	57	53	50	48	45
4·7	—	—	—	—	—	—	110	99	90	80	72	67	62	58	55	52
4·8	—	—	—	—	—	—	140	124	110	97	86	79	73	68	64	60
4·9	—	—	—	—	—	—	200	160	135	119	105	95	86	79	74	69
5·0	—	—	—	—	—	—	384	234	182	153	130	115	102	92	85	79
5·1	—	—	—	—	—	—	—	—	—	—	—	—	124	110	101	93
5·2	—	—	—	—	—	—	—	—	—	—	—	—	148	134	120	109
5·3	—	—	—	—	—	—	—	—	—	—	—	—	188	166	146	129
5·4	—	—	—	—	—	—	—	—	—	—	—	—	245	203	177	154
5·5	—	—	—	—	—	—	—	—	—	—	—	—	405	297	230	194
5·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

TABLE XLV—(*continued*).

Values of $1{\cdot}177\sqrt{N}\,\Sigma_2$ *for values of* β_1, β_2 (*Semi-Major Axis of Probability Ellipse*).

β_1															
·8	·85	·9	·95	1·0	1·05	1·1	1·15	1·2	1·25	1·3	1·35	1·4	1·45	1·5	
6·8	7·2	7·6	7·9	8·3	8·7	9·1	9·5	9·9	10	11	11	11	12	12	2·0
6·7	7·0	7·4	7·8	8·1	8·5	8·9	9·2	9·6	10	11	11	11	11	12	2·1
6·5	6·8	7·2	7·6	8·0	8·3	8·6	9·0	9·4	9·8	10	10	11	11	12	2·2
6·4	6·7	7·1	7·4	7·8	8·1	8·5	8·8	9·2	9·6	9·9	10	11	11	12	2·3
6·3	6·6	6·9	7·3	7·6	8·0	8·3	8·7	9·0	9·4	9·8	10	10	11	11	2·4
6·2	6·5	6·8	7·1	7·5	7·8	8·2	8·5	8·9	9·2	9·6	10	10	11	11	2·5
6·2	6·5	6·8	7·1	7·4	7·7	8·0	8·3	8·7	9·0	9·4	9·9	10	11	11	2·6
6·3	6·5	6·8	7·0	7·3	7·6	7·9	8·2	8·5	8·8	9·2	9·7	10	10	11	2·7
6·4	6·6	6·8	7·0	7·2	7·5	7·7	8·0	8·3	8·7	9·1	9·5	9·8	10	10	2·8
6·6	6·7	6·8	7·0	7·2	7·4	7·6	7·9	8·2	8·6	8·9	9·3	9·6	10	10	2·9
6·9	7·0	7·1	7·2	7·3	7·5	7·7	7·9	8·2	8·5	8·8	9·1	9·4	9·7	10	3·0
7·3	7·3	7·3	7·4	7·5	7·6	7·8	8·0	8·2	8·5	8·7	9·0	9·3	9·6	9·9	3·1
7·9	7·9	7·8	7·8	7·8	7·9	8·1	8·2	8·3	8·5	8·7	8·9	9·2	9·5	9·8	3·2
8·7	8·5	8·4	8·3	8·3	8·3	8·3	8·4	8·5	8·7	8·9	9·1	9·3	9·5	9·7	3·3
9·5	9·3	9·1	9·0	8·8	8·8	8·8	8·8	8·9	9·0	9·1	9·2	9·4	9·5	9·7	3·4
11	10	10	9·8	9·6	9·5	9·4	9·3	9·3	9·3	9·3	9·4	9·5	9·6	9·8	3·5
12	11	11	11	10	10	10	10	9·9	9·9	9·8	9·7	9·7	9·8	9·9	3·6
13	12	12	12	11	11	11	11	11	10	10	10	10	10	10	3·7
15	14	14	13	13	13	12	12	12	11	11	10	10	10	11	3·8
17	16	16	15	14	14	14	13	13	12	12	11	11	11	11	3·9
19	18	18	17	16	16	15	14	14	13	13	12	12	12	12	4·0
22	21	20	19	18	17	17	16	15	15	14	14	13	13	13	4·1
24	23	22	21	20	19	18	18	17	16	15	15	14	14	14	4·2
28	26	25	23	22	21	20	20	19	18	17	16	15	15	15	4·3
32	30	29	27	25	24	23	22	21	20	19	18	17	17	17	4·4
37	35	33	31	29	27	26	25	24	23	22	21	20	19	18	4·5
43	40	37	35	33	31	29	28	27	25	24	23	22	21	20	4·6
49	45	42	39	37	35	33	32	30	28	27	26	24	23	22	4·7
56	52	48	45	42	40	38	36	34	32	31	29	27	26	25	4·8
64	59	55	51	48	46	42	40	38	36	35	33	31	29	28	4·9
73	68	63	59	55	52	49	46	43	41	39	37	35	33	32	5·0
85	78	72	67	63	59	55	52	49	46	43	41	39	37	36	5·1
99	90	82	77	72	67	63	58	55	52	49	46	43	41	40	5·2
114	104	95	88	82	76	71	66	62	58	55	51	48	46	44	5·3
136	123	112	102	95	87	80	75	70	65	61	57	54	51	49	5·4
167	147	132	119	108	100	92	85	79	74	69	64	60	57	54	5·5
—	—	160	141	126	115	105	96	89	83	78	73	68	64	60	5·6
—	—	206	169	148	132	120	110	102	95	88	82	76	72	67	5·7
—	—	258	206	175	150	136	126	116	108	100	93	87	81	75	5·8
—	—	318	255	215	190	168	150	136	125	115	107	99	92	85	5·9
—	—	446	332	273	228	200	178	161	147	134	123	113	104	98	6·0
—	—	—	—	—	—	264	215	190	171	157	144	130	120	112	6·1
—	—	—	—	—	—	345	268	230	207	184	167	150	138	127	6·2
—	—	—	—	—	—	480	364	294	250	215	194	174	159	144	6·3
—	—	—	—	—	—	680	477	370	299	252	224	201	181	165	6·4
—	—	—	—	—	—	1047	680	456	368	312	268	237	212	191	6·5
—	—	—	—	—	—	—	—	—	—	—	—	280	248	223	6·6
—	—	—	—	—	—	—	—	—	—	—	—	338	297	266	6·7
—	—	—	—	—	—	—	—	—	—	—	—	412	362	320	6·8
—	—	—	—	—	—	—	—	—	—	—	—	525	446	390	6·9
—	—	—	—	—	—	—	—	—	—	—	—	809	584	491	7·0

β_2

TABLE XLVI. *To find probable Frequency Type.*

Angle between Major-Axis and Axis of β_2 (Probability Ellipse) measured in degrees.

β_2 \ β_1	0	·05	·1	·15	·2	·25	·3	·35	·4	·45	·5	·55	·6	·65	·7	·75
2·0	0	12	23	28	31	33	35	36	37	38	39	40	41	41	42	42
2·1	0	11	21	25	28	30	32	34	35	37	38	39	40	40	41	41
2·2	0	10	19	23	26	28	30	32	33	35	36	38	39	39	40	40
2·3	0	10	18	22	25	27	28	30	32	34	35	37	38	38	39	39
2·4	0	9	17	20	23	25	26	29	31	33	34	35	36	37	38	38
2·5	0	8	15	18	21	23	25	27	29	31	33	34	35	36	37	37
2·6	0	7	14	17	20	22	24	26	28	30	31	33	34	35	35	36
2·7	0	7	13	16	19	21	23	25	26	28	29	31	32	33	34	35
2·8	0	6	12	15	17	19	21	23	25	27	28	30	31	32	33	33
2·9	0	6	11	14	16	18	20	22	23	25	26	28	29	30	31	32
3·0	0	5	10	13	15	17	19	21	22	24	25	27	28	29	30	31
3·1	0	5	9	12	14	16	18	20	21	23	24	26	27	28	29	30
3·2	0	5	9	12	14	16	17	19	20	21	22	24	25	26	27	28
3·3	0	4	8	11	13	15	16	18	19	20	21	22	24	25	26	27
3·4	0	4	8	10	12	14	15	17	18	19	20	21	22	23	24	25
3·5	0	3	7	9	11	13	14	15	17	18	19	20	21	22	23	24
3·6	0	3	6	8	10	12	13	14	15	16	18	19	20	21	22	23
3·7	0	3	5	7	9	11	12	13	14	15	16	17	18	19	20	21
3·8	0	3	5	7	9	10	11	12	13	14	15	16	17	18	19	20
3·9	0	2	4	6	8	9	10	11	12	13	14	15	16	17	18	19
4·0	0	2	4	6	7	8	9	10	11	12	13	14	15	16	17	18
4·1	—	—	3	5	6	7	8	9	10	11	12	13	14	15	16	17
4·2	—	—	3	4	5	6	7	8	9	10	11	12	13	14	15	16
4·3	—	—	2	4	5	5	6	7	9	10	10	11	12	13	14	15
4·4	—	—	2	3	4	5	6	7	8	9	9	10	11	12	13	14
4·5	—	—	1	2	3	4	5	6	7	8	8	9	10	11	12	13
4·6	—	—	—	—	—	—	4	5	6	7	7	8	9	10	11	12
4·7	—	—	—	—	—	—	3	4	5	6	6	7	8	9	10	11
4·8	—	—	—	—	—	—	3	4	4	5	5	6	7	8	9	10
4·9	—	—	—	—	—	—	2	3	3	4	5	6	6	7	8	9
5·0	—	—	—	—	—	—	1	2	2	3	4	5	6	6	7	8
5·1	—	—	—	—	—	—	—	—	—	—	—	—	5	5	6	7
5·2	—	—	—	—	—	—	—	—	—	—	—	—	4	5	6	7
5·3	—	—	—	—	—	—	—	—	—	—	—	—	3	4	5	5
5·4	—	—	—	—	—	—	—	—	—	—	—	—	2	3	4	5
5·5	—	—	—	—	—	—	—	—	—	—	—	—	1	2	3	4
5·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

TABLE XLVI—(*continued*).

Angle between Major-Axis and Axis of β_2 *(Probability Ellipse) measured in degrees.*

β_1															
·8	·85	·9	·95	1·0	1·05	1·1	1·15	1·2	1·25	1·3	1·35	1·4	1·45	1·5	
43	43	44	44	45	45	46	46	46	46	47	47	48	48	49	2·0
42	42	43	44	44	44	45	45	45	46	46	47	47	48	48	2·1
41	41	42	42	43	43	44	44	45	45	46	46	46	47	47	2·2
40	40	41	41	42	42	43	43	44	44	45	45	46	46	47	2·3
39	39	40	40	41	41	42	42	43	43	44	44	45	45	46	2·4
38	38	39	39	40	40	41	41	42	42	43	43	44	44	45	2·5
37	37	38	39	39	39	40	40	41	41	42	42	43	43	44	2·6
36	36	37	38	38	39	39	40	41	41	42	42	43	43	44	2·7
34	35	36	36	37	38	38	39	40	40	41	41	42	42	43	2·8
33	34	35	35	36	37	38	38	39	39	40	40	41	41	42	2·9
32	33	34	34	35	36	37	37	38	38	39	39	40	40	41	3·0
30	31	32	33	34	35	35	36	37	38	38	39	39	40	40	3·1
29	30	31	32	33	34	34	35	36	37	38	38	39	39	39	3·2
28	29	30	31	32	32	33	34	35	36	37	37	38	38	38	3·3
26	27	28	29	30	31	32	33	34	35	36	36	37	38	38	3·4
25	26	27	28	29	30	31	32	33	34	35	35	36	36	37	3·5
24	25	26	27	28	29	30	31	32	33	34	34	35	35	36	3·6
22	23	24	25	26	27	28	29	31	32	33	33	34	34	35	3·7
21	22	23	24	25	26	27	28	29	30	31	32	33	33	34	3·8
20	21	22	23	24	25	26	27	28	29	30	31	32	32	33	3·9
19	20	21	22	23	24	25	26	27	28	29	30	31	31	32	4·0
18	19	20	21	22	23	24	25	26	27	28	29	30	30	31	4·1
17	18	19	19	20	21	22	23	25	26	27	28	29	30	30	4·2
16	17	18	19	19	20	21	22	23	24	25	26	27	28	29	4·3
15	16	17	18	18	19	20	21	22	23	24	25	26	27	28	4·4
14	15	16	16	17	18	19	20	21	22	23	24	25	25	26	4·5
13	14	15	15	16	17	18	19	20	21	22	23	24	25	25	4·6
12	13	14	14	15	16	17	18	19	20	21	22	23	24	24	4·7
11	12	13	14	15	15	16	17	18	19	20	21	22	23	23	4·8
10	11	12	13	14	14	15	16	17	18	19	20	21	21	22	4·9
9	10	11	12	13	13	14	15	16	17	18	19	20	20	21	5·0
8	9	10	11	12	13	13	15	16	16	17	18	19	20	20	5·1
7	8	9	10	11	12	13	14	15	15	16	17	18	19	19	5·2
6	7	8	9	10	11	12	13	14	14	15	16	17	18	18	5·3
6	7	8	8	9	10	11	12	13	13	14	15	16	17	17	5·4
5	6	7	7	8	9	10	11	12	12	13	14	15	16	17	5·5
—	—	6	7	7	8	9	10	11	12	13	13	14	15	16	5·6
—	—	5	6	6	7	8	9	10	11	12	12	13	14	15	5·7
—	—	4	5	5	6	7	8	9	10	11	11	12	13	14	5·8
—	—	3	4	5	5	6	7	8	9	10	10	11	12	13	5·9
—	—	2	3	4	4	5	6	7	8	9	9	10	11	12	6·0
—	—	—	—	—	—	—	—	6	7	8	9	9	10	11	6·1
—	—	—	—	—	—	—	—	5	6	7	8	8	9	10	6·2
—	—	—	—	—	—	—	—	4	5	6	6	7	8	9	6·3
—	—	—	—	—	—	—	—	3	4	5	6	6	7	8	6·4
—	—	—	—	—	—	—	—	2	3	4	5	6	6	7	6·5
—	—	—	—	—	—	—	—	—	—	—	—	5	6	7	6·6
—	—	—	—	—	—	—	—	—	—	—	—	4	5	6	6·7
—	—	—	—	—	—	—	—	—	—	—	—	3	4	5	6·8
—	—	—	—	—	—	—	—	—	—	—	—	2	3	4	6·9
—	—	—	—	—	—	—	—	—	—	—	—	1	3	4	7·0

β_2

Diagram XLVII determining the probability of a given Type of Frequency.

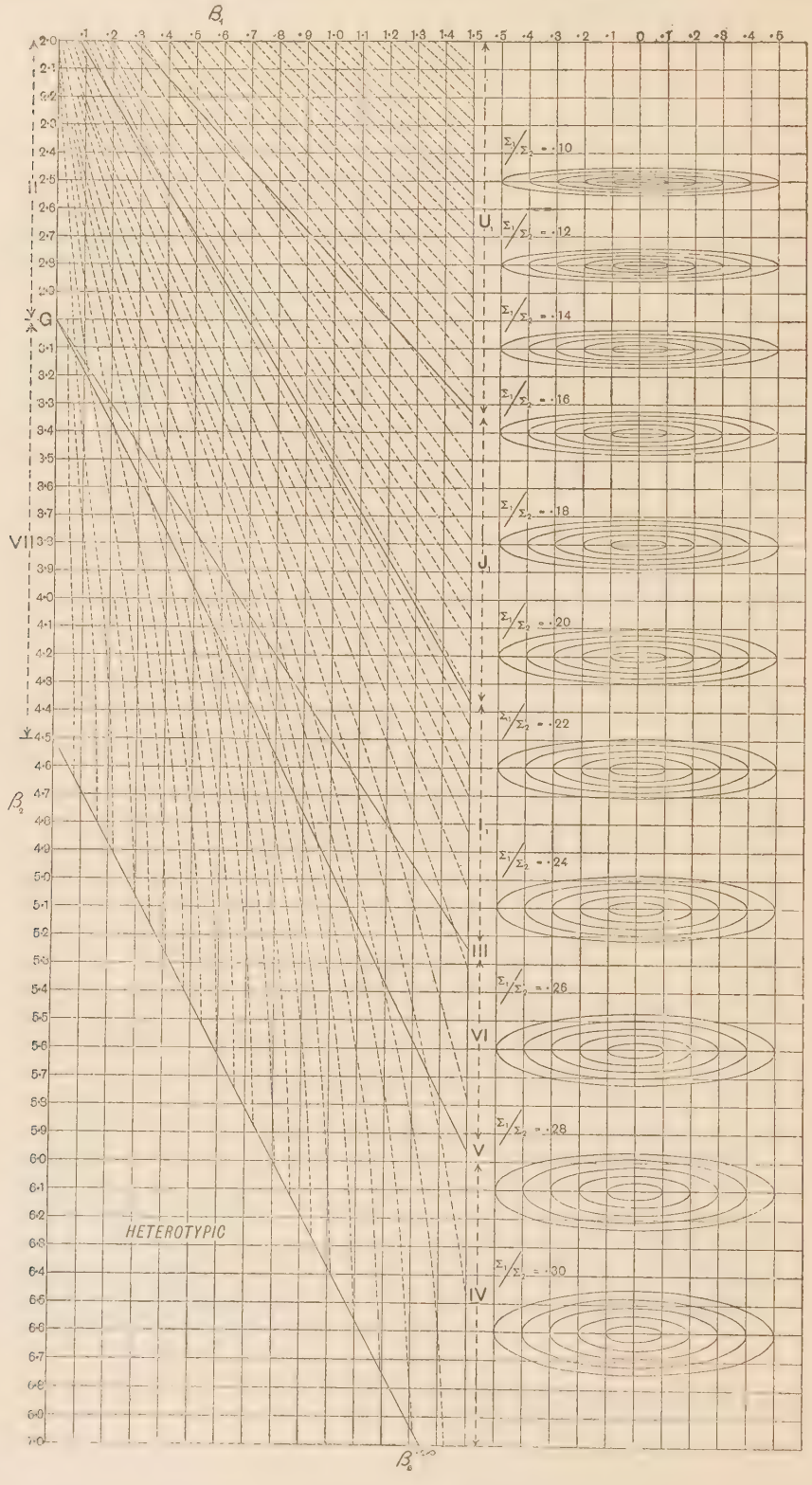


TABLE XLVIII. *Percentage Frequency of Successes in a Second Sample "m" after drawing "p" Successes in a First Sample "n".*

Successes		$p=0$	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$
$n=6$, $m=5$	0	58·3333	31·8182	15·9091	7·0707		
	1	26·5151	31·8182	26·5151	17·6768		
	2	10·6060	21·2121	26·5151	25·2525		
	3	3·5354	10·6060	18·9394	25·2525		
	4	·8838	3·7879	9·4697	17·6768		
	5	·1263	·7576	2·6515	7·0707		
$n=6$, $m=6$	0	53·8462	26·9231	12·2378	4·8951		
	1	26·9231	29·3706	22·0280	13·0536		
	2	12·2378	22·0280	24·4755	20·3963		
	3	4·8951	13·0536	20·3962	23·3100		
	4	1·6317	6·1189	13·1119	20·3963		
	5	·4079	2·0979	6·1189	13·0536		
	6	·0582	·4079	1·6317	4·8951		
$n=7$, $m=5$	0	61·5385	35·8974	19·5804	9·7902		
	1	25·6410	32·6340	29·3706	21·7560		
	2	9·3240	19·5804	26·1072	27·1950		
	3	2·7972	8·7024	16·3170	23·3100		
	4	·6216	2·7195	6·9930	13·5975		
	5	·0777	·4662	1·6317	4·3512		
$n=7$, $m=6$	0	57·1429	30·7692	15·3846	6·9930		
	1	26·3736	30·7692	25·1748	16·7832		
	2	10·9890	20·9790	25·1748	23·3100		
	3	3·9960	11·1888	18·6480	23·3100		
	4	1·1988	4·6620	10·4895	17·4825		
	5	·2664	1·3986	4·1958	9·3240		
	6	·0333	·2331	·9324	2·7972		
$n=7$, $m=7$	0	53·3333	26·6667	12·3077	5·1282		
	1	26·6667	28·7179	21·5385	13·0536		
	2	12·3077	21·5385	23·4965	19·5804		
	3	5·1282	13·0536	19·5804	21·7560		
	4	1·8648	6·5268	13·0536	19·0365		
	5	·5594	2·6107	6·8531	13·0536		
	6	·1243	·7615	2·6107	6·5268		
	7	·0155	·1243	·5594	1·8648		
$n=8$, $m=5$	0	64·2857	39·5604	23·0769	12·5874	6·2937	
	1	24·7253	32·9670	31·4685	25·1748	17·4825	
	2	8·2418	17·9820	25·1748	27·9720	26·2238	
	3	2·2478	7·1928	13·9860	20·9790	26·2238	
	4	·4495	1·9980	5·2448	10·4895	17·4825	
	5	·0499	·2997	1·0489	2·7972	6·2937	
$n=8$, $m=6$	0	60·0000	34·2857	18·4615	9·2308	4·1958	
	1	25·7143	31·6484	27·6923	20·1398	12·5874	
	2	9·8901	19·7802	25·1748	25·1748	20·9790	
	3	3·2967	9·5904	16·7832	22·3776	24·4755	
	4	·8991	3·5964	8·3916	14·6853	20·9790	
	5	·1798	·9590	2·9370	6·7133	12·5874	
	6	·0200	·1399	·5594	1·6783	4·1958	
$n=8$, $m=7$	0	56·2500	30·0000	15·0000	6·9231	2·8846	
	1	26·2500	30·0000	24·2308	16·1538	9·1783	
	2	11·2500	20·7692	24·2308	22·0280	16·5210	
	3	4·3269	11·5385	18·3566	22·0280	21·4161	
	4	1·4423	5·2448	11·0140	17·1329	21·4161	
	5	·3934	1·8881	5·1399	10·2797	16·5210	
	6	·0787	·4895	1·7132	4·4056	9·1783	
	7	·0087	·0699	·3147	1·0489	2·8846	

TABLE XLVIII—(*continued*). *Percentage Frequency of Successes in a Second Sample "m" after drawing "p" Successes in a First Sample "n".*

Successes		$p=0$	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$
$n=8$	0	52·9412	26·4706	12·3529	5·2941	2·0362	
$m=8$	1	26·4706	28·2353	21·1765	13·0317	6·7873	
	2	12·3529	21·1765	22·8054	19·0045	12·9576	
	3	5·2941	13·0317	19·0045	20·7322	18·1407	
	4	2·0362	6·7873	12·9576	18·1407	20·1563	
	5	·6787	2·9617	7·2563	12·9000	18·1407	
	6	·1851	1·0366	3·2250	7·2563	12·9576	
	7	·0370	·2633	1·0366	2·9617	6·7873	
	8	·0041	·0370	·1851	·6787	2·0362	
$n=9$	0	66·6667	42·8571	26·3736	15·3846	8·3916	
$m=5$	1	23·8095	32·9670	32·9670	27·9720	20·9790	
	2	7·3260	16·4835	23·9760	27·9720	27·9720	
	3	1·8315	5·9940	11·9880	18·6480	24·4755	
	4	·3330	1·4985	3·9960	8·1585	13·9860	
	5	·0333	·1998	·6993	1·8648	4·1958	
$n=9$	0	62·5000	37·5000	21·4286	11·5385	5·7692	
$m=6$	1	25·0000	32·1429	29·6703	23·0769	15·7343	
	2	8·9286	18·5439	24·7253	26·2238	23·6014	
	3	2·7472	8·2418	14·9850	20·9790	24·4755	
	4	·6868	2·8097	6·7433	12·2378	18·3566	
	5	·1249	·6743	2·0979	4·8951	9·4405	
	6	·0125	·0874	·3496	1·0489	2·6224	
$n=9$	0	58·8235	33·0882	17·6471	8·8235	4·0724	
$m=7$	1	25·7353	30·8824	26·4706	19·0045	11·8778	
	2	10·2941	19·8529	24·4344	23·7557	19·4364	
	3	3·6765	10·1810	16·9683	21·5961	22·6759	
	4	1·1312	4·2421	9·2554	15·1172	20·1563	
	5	·2828	1·3883	3·8873	8·0625	13·6055	
	6	·0514	·3239	1·1518	3·0234	6·4788	
	7	·0051	·0411	·1851	·6170	1·6968	
$n=9$	0	55·5555	29·4118	14·7059	6·8627	2·9412	
$m=8$	1	26·1438	29·4118	23·5294	15·6863	9·0498	
	2	11·4379	20·5882	23·5294	21·1161	15·8371	
	3	4·5752	11·7647	18·0995	21·1161	20·1563	
	4	1·6340	5·6561	11·3122	16·7969	20·1563	
	5	·5027	2·2624	5·7589	10·7500	16·1250	
	6	·1257	·7199	2·3036	5·3750	10·0782	
	7	·0229	·1645	·6582	1·9197	4·5249	
	8	·0023	·0206	·1028	·3771	1·1312	
$n=9$	0	52·6316	26·3158	12·3839	5·4180	2·1672	
$m=9$	1	26·3158	27·8638	20·8978	13·0031	6·9659	
	2	12·3839	20·8978	22·2910	18·5759	12·8602	
	3	5·4180	13·0031	18·5759	20·0047	17·5042	
	4	2·1672	6·9659	12·8602	17·5042	19·0955	
	5	·7740	3·2151	7·5018	12·7303	17·1859	
	6	·2381	1·2503	3·6372	7·6382	12·7303	
	7	·0595	·3897	1·4029	3·6372	7·5018	
	8	·0108	·0877	·3897	1·2503	3·2150	
	9	·0011	·0108	·0595	·2381	·7740	
$n=5$	0	54·5454	27·2727	12·1212	4·5454		
$m=5$	1	27·2727	30·3030	22·7273	12·9870		
	2	12·1212	22·7273	25·9740	21·6450		
	3	4·5454	12·9870	21·6450	25·9740		
	4	1·2987	5·4112	12·9870	22·7273		
	5	·2165	1·2987	4·5454	12·1212		

TABLE XLVIII—(*continued*).

Successes		$p=0$	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$
$n=10$	0	68·7500	45·8333	29·4643	18·1318	10·5769	5·7692		
$m=5$	1	22·9167	32·7381	33·9972	30·2198	24·0385	17·3077		
	2	6·5476	15·1099	22·6648	27·4725	28·8461	26·9230		
	3	1·5110	5·0366	10·3022	16·4835	22·4359	26·9230		
	4	·2518	1·1447	3·0907	6·4103	11·2179	17·3077		
	5	·0229	·1373	·4807	1·2820	2·8846	5·7692		
$n=10$	0	52·3809	26·1905	12·4060	5·5138	2·2704	·8514		
$m=10$	1	26·1905	27·5689	20·6767	12·9736	7·0949	3·4056		
	2	12·4060	20·6767	21·8930	18·2441	12·7709	7·6625		
	3	5·5138	12·9736	18·2441	19·4604	17·0278	12·5744		
	4	2·2704	7·0949	12·7709	17·0278	18·3377	16·5039		
	5	·8514	3·4056	7·6625	12·5744	16·5039	18·0043		
	6	·2838	1·4190	3·9295	7·8590	12·5030	16·5039		
	7	·0811	·4990	1·6841	4·0826	7·8590	12·5744		
	8	·0187	·1403	·5741	1·6841	3·9295	7·6625		
	9	·0031	·0284	·1403	·4990	1·4190	3·4056		
	10	·0003	·0031	·0187	·0811	·2838	·8514		
$n=15$	0	76·1905	57·1429	42·1053	30·4094	21·4654	14·7575	9·8383	6·3246
$m=5$	1	19·0476	30·0752	35·0877	35·7757	33·5397	29·5149	24·5958	19·4604
	2	4·0100	10·0251	16·5119	22·3598	26·8318	29·5149	30·2717	29·1906
	3	·6683	2·3588	5·1599	8·9439	13·4159	18·1631	22·7038	26·5369
	4	·0786	·3685	1·0320	2·2360	4·1280	6·8111	10·3199	14·5953
	5	·0049	·0295	·1032	·2752	·6192	1·2384	2·2704	3·8921
$n=15$	0	61·5384	36·9231	21·5385	12·1739	6·6403	3·4783	1·7391	·8238
$m=10$	1	24·6154	30·7692	28·0936	22·1344	15·8103	10·4348	6·4073	3·6613
	2	9·2308	18·0602	22·9857	23·7154	21·3439	17·2997	12·8146	8·7226
	3	3·2107	8·7565	14·5941	18·9723	20·9694	20·5034	18·0913	14·5376
	4	1·0216	3·6485	7·6619	12·2322	16·3095	18·9958	19·7873	18·6566
	5	·2919	1·3135	3·3874	6·5238	10·3613	14·2469	17·4128	19·1897
	6	·0730	·4032	1·2546	2·8781	5·3965	8·7064	12·4377	15·9914
	7	·0153	·1024	·3795	1·0279	2·2614	4·2644	7·1073	10·6609
	8	·0025	·0203	·0889	·2827	·7269	1·5991	3·1094	5·4516
	9	·0003	·0028	·0145	·0538	·1615	·4146	·9423	1·9383
	10	·0000	·0002	·0012	·0054	·0189	·0565	·1508	·3661
$n=15$	0	51·6129	25·8065	12·4583	5·7842	2·5707	1·0876	·4351	·1631
$m=15$	1	25·8065	26·6963	20·0222	12·8538	7·4156	3·9155	1·9033	·8512
	2	12·4583	20·0222	20·7638	17·3032	12·4583	7·9941	4·6342	2·4375
	3	5·7842	12·8538	17·3032	17·9953	15·7459	12·0490	8·2152	5·0297
	4	2·5707	7·4156	12·4583	15·7459	16·4305	14·7874	11·7361	8·2991
	5	1·0876	3·9155	7·9941	12·0490	14·7874	15·4916	14·2006	11·5313
	6	·4351	1·9033	4·6342	8·2152	11·7361	14·2006	14·9480	13·8803
	7	·1631	·8512	2·4375	5·0297	8·2991	11·5313	13·8803	14·6968
	8	·0567	·3482	1·1607	2·7663	5·2415	8·3282	11·4309	13·7783
	9	·0181	·1289	·4965	1·3589	2·9443	5·3344	8·3350	11·4309
	10	·0052	·0426	·1882	·5889	1·4548	3·0006	5·3344	8·3282
	11	·0013	·0122	·0618	·2204	·6200	1·4548	2·9443	5·2415
	12	·0003	·0029	·0169	·0689	·2204	·5889	1·3589	2·7664
	13	·0001	·0006	·0037	·0170	·0618	·1882	·4965	1·1607
	14	·0000	·0001	·0006	·0029	·0122	·0426	·1290	·3482
	15	·0000	·0000	·0000	·0003	·0013	·0052	·0181	·0567

TABLE XLVIII—(*continued*). *Percentage Frequency of Successes in a Second Sample "m" after drawing "p" Successes in a First Sample "n".*

Successes	$p=0$	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$
$n=20$, $m=5$ — 0	80·7692	64·6154	51·1538	40·0334	30·9349	23·5695
1	16·1538	26·9231	33·3612	36·3940	36·8273	35·3542
2	2·6923	7·0234	12·1313	17·3305	22·0964	26·0505
3	·3512	1·2770	2·8884	5·1992	8·1408	11·5780
4	·0319	·1520	·4333	·9577	1·8090	3·0647
5	·0015	·0091	·0319	·0851	·1915	·3831
$n=20$, $m=10$ — 0	67·7419	45·1613	29·5884	19·0211	11·9763	7·3700
1	22·5806	31·1457	31·7019	28·1795	23·0313	17·6880
2	7·0078	15·0167	21·1346	24·3861	24·8738	23·2155
3	2·0022	5·9325	10·8382	15·6071	19·3463	21·5333
4	·5191	1·9965	4·5521	7·9661	11·7760	15·4158
5	·1198	·5750	1·5932	3·3250	5·7809	8·8091
6	·0240	·1398	·4618	1·1335	2·2940	4·0375
7	·0040	·0278	·1079	·3084	·7210	1·4571
8	·0005	·0043	·0193	·0636	·1707	·3946
9	·0000	·0004	·0024	·0089	·0274	·0722
10	·0000	·0000	·0002	·0006	·0023	·0068
$n=20$, $m=15$ — 0	58·3333	33·3333	18·6275	10·1604	5·3977	2·7859
1	25·0000	29·4118	25·4011	19·0508	13·0590	8·3578
2	10·2941	18·7166	22·2259	21·5090	18·2826	14·1218
3	4·0553	10·1381	15·5343	18·6411	19·1232	17·4841
4	1·5207	4·9056	9·3206	13·4987	16·3913	17·4841
5	·5396	2·1584	4·9495	8·4849	12·0203	14·7942
6	·1799	·8683	2·3569	4·7138	7·7053	10·8491
7	·0558	·3190	1·0101	2·3310	4·3590	6·9744
8	·0159	·1063	·3885	1·0256	2·1795	3·9421
9	·0041	·0318	·1330	·3989	·9581	1·9511
10	·0010	·0084	·0399	·1353	·3658	·8362
11	·0002	·0019	·0102	·0391	·1188	·3041
12	·0000	·0004	·0022	·0093	·0317	·0907
13	·0000	·0001	·0004	·0017	·0065	·0209
14	·0000	·0000	·0000	·0002	·0009	·0033
15	·0000	·0000	·0000	·0000	·0001	·0003
$n=20$, $m=20$ — 0	51·2195	25·6098	12·4765	5·9099	2·7154	1·2068
1	25·6098	26·2664	19·6998	12·7783	7·5427	4·1377
2	12·4765	19·6998	20·2323	16·8602	12·2839	8·0929
3	5·9099	12·7783	16·8602	17·3419	15·1742	11·7715
4	2·7154	7·5427	12·2839	15·1742	15·6340	14·0706
5	1·2068	4·1377	8·0929	11·7715	14·0706	14·5245
6	·5172	2·1297	4·9048	8·2768	11·3473	13·3141
7	·2130	1·0326	2·7589	5·3399	8·3213	11·0186
8	·0839	·4719	1·4462	3·1817	5·5954	8·3131
9	·0315	·2030	·7070	1·7554	3·4638	5·7473
10	·0112	·0819	·3218	·8965	1·9757	3·6474
11	·0037	·0308	·1358	·4226	1·0362	2·1221
12	·0012	·0107	·0528	·1829	·4974	1·1274
13	·0003	·0034	·0188	·0720	·2168	·5429
14	·0001	·0010	·0060	·0255	·0848	·2345
15	·0000	·0003	·0017	·0080	·0293	·0893
16	—	·0000	·0004	·0022	·0087	·0293
17	—	—	·0001	·0005	·0022	·0080
18	—	—	·0000	·0001	·0004	·0017
19	—	—	—	·0000	·0001	·0003
20	—	—	—	—	·0000	·0000

TABLE XLVIII—(*continued*).

Successes		$p=0$	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$
$n=25$ $m=5$	0	83·8710	69·8925	57·8421	47·5131	38·7144	31·2693
	1	13·9785	24·1008	30·9868	35·1949	37·2254	37·5232
	2	1·9281	5·1645	9·1813	13·5365	17·8682	21·8885
	3	·2065	·7651	1·7656	3·2488	5·2115	7·6134
	4	·0153	·0736	·2119	·4738	·9064	1·5573
	5	·0006	·0035	·0123	·0329	·0741	·1483
$n=25$ $m=10$	0	72·2222	51·5873	36·4146	25·3798	17·4486	11·8200
	1	20·6349	30·3455	33·1041	31·7248	28·1430	23·6401
	2	5·4622	12·4141	18·6211	23·0261	25·3287	25·6780
	3	1·3242	4·1380	8·0091	12·2806	16·3035	19·5642
	4	·2897	1·1680	2·8032	5·1875	8·1518	11·4125
	5	·0561	·2803	·8119	1·7786	3·2607	5·2673
	6	·0093	·0564	·1933	·4940	1·0451	1·9313
	7	·0013	·0092	·0368	·1086	·2628	·5518
	8	·0001	·0011	·0053	·0179	·0493	·1170
	9	·0000	·0001	·0005	·0020	·0062	·0165
	10	·0000	·0000	·0000	·0001	·0004	·0012
$n=25$ $m=15$	0	63·4146	39·6341	24·3902	14·7625	8·7777	5·1203
	1	23·7805	30·4878	28·8832	23·9392	18·2869	13·1666
	2	8·5366	16·8485	21·8575	23·2742	21·9443	18·9753
	3	2·9204	7·8930	13·1550	17·2894	19·5777	19·9337
	4	·9472	3·2888	6·7654	10·6788	14·2383	16·8191
	5	·2894	1·2403	3·0643	5·6953	8·8100	11·9361
	6	·0827	·4256	1·2381	2·6697	4·7366	7·2943
	7	·0218	·1326	·4477	1·1072	2·2329	3·8807
	8	·0053	·0373	·1444	·4060	·9240	1·8017
	9	·0012	·0094	·0412	·1307	·3337	·7266
	10	·0002	·0020	·0102	·0364	·1038	·2515
	11	·0000	·0004	·0022	·0086	·0272	·0732
	12	·0000	·0001	·0004	·0016	·0058	·0173
	13	·0000	·0000	·0001	·0002	·0009	·0031
	14	—	·0000	·0000	·0000	·0001	·0004
	15	—	—	·0000	·0000	·0000	·0000
$n=25$ $m=20$	0	56·5217	31·4010	17·1278	9·1614	4·7988	2·4579
	1	25·1208	28·5463	23·8993	17·4502	11·7044	7·3738
	2	10·8476	18·9202	21·6231	20·2168	16·6788	12·5733
	3	4·5409	10·8116	15·8218	18·1951	17·9618	15·8820
	4	1·8380	5·6036	10·0864	13·8796	16·0711	16·4186
	5	·7172	2·6897	5·7932	9·3504	12·5094	14·5943
	6	·2690	1·2069	3·0491	5·6861	8·6871	11·4669
	7	·0965	·5082	1·4833	3·1589	5·4604	8·0943
	8	·0330	·2009	·6695	1·6133	3·1317	5·1816
	9	·0107	·0744	·2806	·7592	1·6449	3·0226
	10	·0033	·0257	·1089	·3290	·7916	1·6088
	11	·0009	·0082	·0390	·1308	·3482	·7800
	12	·0002	·0024	·0128	·0475	·1393	·3429
	13	·0001	·0006	·0038	·0156	·0502	·1357
	14	·0000	·0002	·0010	·0046	·0162	·0477
	15	—	·0000	·0002	·0012	·0045	·0147
	16	—	—	·0001	·0002	·0011	·0039
	17	—	—	—	·0000	·0002	·0008
	18	—	—	—	—	·0000	·0001
	19	—	—	—	—	—	·0000
	20	—	—	—	—	—	—

TABLE XLVIII—(*continued*). *Percentage Frequency of Successes in a Second Sample "m" after drawing "p" Successes in a First Sample "n".*

Successes		$p=0$	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$
$n=25$	0	50·9804	25·4902	12·4850	5·9824	2·8003	1·2784
$m=25$	1	25·4902	26·0104	19·5078	12·7285	7·6094	4·2613
	2	12·4850	19·5078	19·9229	16·6024	12·1751	8·1352
	3	5·9824	12·7285	16·6024	16·9713	14·8499	11·6037
	4	2·8003	7·6094	12·1751	14·8499	15·1953	13·6757
	5	1·2784	4·2613	8·1352	11·6037	13·6757	14·0093
	6	·5682	2·2598	5·0451	8·2883	11·1185	12·8418
	7	·2453	1·1411	2·9344	5·4870	8·2990	10·7251
	8	·1027	·5502	1·6103	3·3951	5·7456	8·2555
	9	·0416	·2535	·8365	1·9732	3·7128	5·9003
	10	·0162	·1115	·4118	1·0801	2·2477	3·9335
	11	·0061	·0468	·1921	·5573	1·2771	2·4521
	12	·0022	·0187	·0848	·2709	·6811	1·4304
	13	·0007	·0070	·0353	·1238	·3406	·7802
	14	·0002	·0025	·0138	·0531	·1592	·3971
	15	—	·0008	·0051	·0212	·0693	·1879
	16	—	·0003	·0017	·0079	·0280	·0822
	17	—	·0001	·0005	·0027	·0104	·0330
	18	—	—	·0002	·0008	·0035	·0120
	19	—	—	·0000	·0002	·0010	·0039
	20	—	—	—	·0001	·0003	·0011
	21	—	—	—	—	·0001	·0003
	22	—	—	—	—	—	·0001
	23	—	—	—	—	—	—
	24	—	—	—	—	—	—
	25	—	—	—	—	—	—
$n=50$	0	91·0714	82·7922	75·1263	68·0389	61·4967	55·4676
$m=5$	1	8·2792	15·3319	21·2621	26·1688	30·1454	33·2805
	2	·6133	1·7357	3·2711	5·1311	7·2349	9·5087
	3	·0347	·1335	·3207	·6157	1·0335	1·5848
	4	·0013	·0065	·0192	·0440	·0861	·1517
	5	·0000	·0001	·0005	·0014	·0033	·0066
$n=50$	0	83·6065	69·6721	57·8633	47·8869	39·4857	32·4346
$m=10$	1	13·9344	23·6177	29·9293	33·6048	35·2551	35·3833
	2	2·1256	5·4972	9·4513	13·5019	17·3070	20·6402
	3	·2932	1·0287	2·2503	3·9278	5·9827	8·3080
	4	·0360	·1607	·4296	·8910	1·5803	2·5164
	5	·0039	·0210	·0668	·1614	·3282	·5921
	6	·0003	·0023	·0084	·0233	·0536	·1085
	7	·0000	·0002	·0008	·0026	·0067	·0152
	8	·0000	·0000	·0001	·0002	·0006	·0015
	9	·0000	·0000	·0000	·0000	·0000	·0001
	10	·0000	·0000	·0000	·0000	·0000	·0000
$n=50$	0	77·2727	59·4406	45·5092	34·6737	26·2849	19·8214
$m=15$	1	17·8322	27·8628	32·5066	33·5552	32·3175	29·7321
	2	3·9008	9·2876	14·6804	19·2530	22·6222	24·6927
	3	·8049	2·5965	5·2143	8·3429	11·6306	14·7589
	4	·1558	·6385	1·5643	2·9695	4·8127	6·9910
	5	·0281	·1405	·4083	·9011	1·6718	2·7465
	6	·0047	·0278	·0939	·2371	·4976	·9155
	7	·0007	·0049	·0191	·0544	·1279	·2616
	8	·0001	·0008	·0034	·0109	·0284	·0642
	9	—	·0001	·0005	·0019	·0054	·0132
	10	—	—	·0001	·0003	·0009	·0024
	11	—	—	—	—	·0001	·0003
	12	—	—	—	—	—	·0000
13, 14, 15		—	—	—	—	—	—

TABLE XLVIII—(*continued*).

	Successes	$p=0$	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$
$n=50$	0	71·8310	51·3078	36·4360	25·7195	18·0421	12·5748
$m=20$	1	20·5231	29·7437	32·1494	30·7099	27·3365	23·2150
	2	5·6513	12·4661	18·2340	22·1018	23·9720	24·1218
	3	1·4959	4·4655	8·2882	12·2410	15·7316	18·3785
	4	·3796	1·4377	3·2515	5·6902	8·4901	11·3384
	5	·0920	·4247	1·1380	2·3122	3·9438	5·9480
	6	·0212	·1161	·3613	·8391	1·6163	2·7262
	7	·0046	·0295	·1049	·2751	·5926	1·1089
	8	·0010	·0070	·0279	·0820	·1959	·4039
	9	·0002	·0015	·0068	·0222	·0585	·1323
	10	·0000	·0003	·0015	·0055	·0158	·0390
	11	—	·0001	·0003	·0012	·0039	·0103
	12	—	—	·0001	·0002	·0008	·0024
	13	—	—	—	·0000	·0002	·0005
	14	—	—	—	—	·0000	·0001
	15—20	—	—	—	—	—	·0000
$n=50$	0	67·1053	44·7368	29·6230	19·4782	12·7149	8·2378
$m=25$	1	22·3684	30·2276	30·4346	27·0530	22·3854	17·6525
	2	7·2546	14·9068	20·2898	22·8617	23·0250	21·4900
	3	2·2857	6·3492	10·9546	15·0234	17·9083	19·3831
	4	·6984	2·4592	5·1643	8·3826	11·5877	14·3204
	5	·2066	·8853	2·2004	4·1420	6·5376	9·1130
	6	·0590	·2994	·8629	1·8546	3·3018	5·1406
	7	·0163	·0956	·3146	·7627	1·5167	2·6162
	8	·0043	·0289	·1073	·2904	·6398	1·2147
	9	·0011	·0083	·0343	·1029	·2494	·5181
	10	·0003	·0022	·0103	·0340	·0901	·2038
	11	·0001	·0006	·0029	·0105	·0302	·0741
	12	·0000	·0001	·0008	·0030	·0094	·0249
	13	·0000	·0000	·0002	·0008	·0027	·0077
	14	—	·0000	·0000	·0002	·0007	·0022
	15	—	—	·0000	·0000	·0002	·0006
	16	—	—	—	·0000	·0000	·0001
	17	—	—	—	—	·0000	·0000
	18—25	—	—	—	—	—	·0000
$n=50$	0	50·4950	25·2475	12·4963	6·1206	2·9657	1·4210
$m=50$	1	25·2475	25·5026	19·1269	12·6198	7·7231	4·4875
	2	12·4963	19·1269	19·3241	16·1034	11·9504	8·1873
	3	6·1206	12·6198	16·1034	16·2729	14·2388	11·2686
	4	2·9657	7·7231	11·9504	14·2388	14·3919	12·9527
	5	1·4210	4·4875	8·1873	11·2686	12·9527	13·0951
	6	·6731	2·5063	5·2821	8·2677	10·6753	12·0038
	7	·3151	1·3552	3·2480	5·7108	8·2014	10·1734
	8	·1457	·7126	1·9185	3·7517	5·9437	8·0780
	9	·0665	·3654	1·0942	2·3606	4·0975	6·0662
	10	·0300	·1831	·6049	1·4298	2·7034	4·3381
	11	·0133	·0898	·3250	·8367	1·7147	2·9694
	12	·0058	·0431	·1699	·4743	1·0490	1·9531
	13	·0025	·0203	·0866	·2610	·6205	1·2381
	14	·0011	·0093	·0431	·1396	·3557	·7582
	15	·0004	·0042	·0209	·0726	·1978	·4493
	16	·0002	·0019	·0099	·0368	·1068	·2580
	17	·0001	·0008	·0046	·0182	·0561	·1437
	18	—	·0003	·0021	·0088	·0286	·0777
	19	—	·0001	·0009	·0041	·0142	·0408
	20	—	—	·0004	·0019	·0069	·0208
	21	—	—	·0002	·0008	·0032	·0103
	22	—	—	·0001	·0004	·0015	·0050
	23	—	—	—	·0002	·0007	·0023
	24	—	—	—	·0001	·0003	·0010
	25	—	—	—	·0000	·0001	·0005
	26	—	—	—	—	·0000	·0002
	27	—	—	—	—	·0000	·0001
	28—50	—	—	—	—	—	·0000

TABLE XLVIII—(*continued*). *Percentage Frequency of Successes in a Second Sample "m" after drawing "p" Successes in a First Sample "n".*

Successes		$p=0$	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$	$p=8$	$p=9$
$n=100$, $m=10$	0	90·9910	82·7191	75·1302	68·1737	61·8023	55·9719	50·6412	45·7719	41·3280	37·2763
	1	8·2719	15·1778	20·8695	25·4855	29·1520	31·9839	34·0855	35·5510	36·4659	36·9072
	2	·6830	1·8972	3·5107	5·4097	7·4962	9·6874	11·9134	14·1158	16·2472	18·2690
	3	·0506	·1891	·4416	·8243	1·3455	2·0065	2·8031	3·7269	4·7658	5·9052
	4	·0033	·0156	·0442	·0971	·1829	·3098	·4857	·7174	1·0109	1·3708
	5	·0002	·0011	·0036	·0090	·0194	·0368	·0641	·1044	·1609	·2374
	6	·0000	·0001	·0002	·0007	·0016	·0034	·0065	·0115	·0194	·0309
	7	—	·0000	·0000	·0000	·0001	·0002	·0005	·0010	·0017	·0030
	8	—	—	—	—	·0000	·0000	·0000	·0001	·0001	·0002
	9	—	—	—	—	—	—	—	·0000	·0000	·0000
	10	—	—	—	—	—	—	—	—	—	—

	$p=10$	$p=15$	$p=20$	$p=25$	$p=30$	$p=35$	$p=40$	$p=45$	$p=50$
0	33·5855	19·6056	11·0992	6·0712	3·1945	1·6083	·7697	·3473	·1463
1	36·9441	33·0200	25·8982	18·5708	12·3788	7·7198	4·5082	2·4580	1·2434
2	20·1513	26·8727	28·8081	26·8613	22·5639	17·3696	12·3485	8·1229	4·9313
3	7·1284	13·8698	20·0784	24·1644	25·4567	24·1112	20·8229	16·5036	12·0167
4	1·8005	5·0127	9·6930	14·9554	19·6711	22·8554	23·9308	22·8255	19·9224
5	·3376	1·3220	3·3813	6·6468	10·8709	15·4516	19·5797	22·4513	23·4799
6	·0474	·2571	·8619	2·1464	4·3483	7·5418	11·5470	15·9030	19·9224
7	·0049	·0363	·1583	·4968	1·2424	2·6232	4·8456	8·0093	12·0167
8	·0003	·0036	·0200	·0788	·2425	·6221	1·3845	2·7446	4·9313
9	·0000	·0002	·0015	·0077	·0292	·0908	·2432	·5778	1·2434
10	—	·0000	·0001	·0004	·0017	·0062	·0199	·0567	·1463

Successes		$p=0$	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$	$p=8$	$p=9$	$p=10$
$n=100$, $m=5$	0	95·2830	90·7457	86·3830	82·1896	78·1607	74·2914	70·5768	67·0123	63·5933	60·3153	57·1739
	1	4·5373	8·7256	12·5800	16·1156	19·3467	22·2874	24·9514	27·3520	29·5020	31·4142	33·1007
	2	·1745	·5083	·9867	1·5956	2·3216	3·1517	4·0737	5·0756	6·1463	7·2749	8·4512
	3	·0051	·0199	·0488	·0957	·1642	·2573	·3780	·5287	·7117	·9287	1·1813
	4	·0001	·0005	·0015	·0034	·0067	·0119	·0197	·0306	·0454	·0649	·0899
	5	·0000	·0000	·0000	·0001	·0001	·0003	·0004	·0008	·0013	·0020	·0030
$n=100$, $m=15$	0	87·0690	75·7121	65·7500	57·0221	49·3852	42·7116	36·8873	31·8110	27·3928	23·5527	20·2198
	1	11·3568	19·9243	26·1836	30·5476	33·3684	34·9458	35·5336	35·3456	34·5611	33·3293	31·7739
	2	1·3947	3·7027	6·5459	9·6321	12·7407	15·7096	18·4248	20·8110	22·8233	24·4415	25·6636
	3	·1604	·5730	1·2777	2·2767	3·5456	5·0426	6·7156	8·5076	10·3611	12·2207	14·0361
	4	·0172	·0774	·2091	·4386	·7879	1·2724	1·9006	2·6738	3·5865	4·6273	5·7796
	5	·0017	·0093	·0295	·0715	·1458	·2641	·4381	·6787	·9959	1·3973	1·8884
	6	·0002	·0010	·0037	·0100	·0229	·0461	·0842	·1428	·2278	·3459	·5036
	7	·0000	·0001	·0004	·0012	·0031	·0068	·0137	·0252	·0435	·0711	·1112
	8	—	·0000	·0000	·0001	·0004	·0009	·0019	·0037	·0070	·0122	·0204
	9	—	—	—	·0000	·0000	·0001	·0002	·0005	·0009	·0017	·0031
	10	—	—	—	—	—	·0000	·0000	·0001	·0001	·0002	·0004
	11—15	—	—	—	—	—	—	—	·0000	·0000	·0000	·0000
$n=100$, $m=20$	0	83·4711	69·5592	57·8686	48·0604	39·8449	32·9751	27·2403	22·4613	18·4859	15·1848	12·4488
	1	13·9119	23·3813	29·4247	32·8618	34·3491	34·4088	33·4530	31·8036	29·7094	27·3600	24·8976
	2	2·2212	5·6472	9·5567	13·4563	17·0252	20·0718	22·4994	24·2787	25·4270	25·9920	26·0397
	3	·3388	1·1584	2·4716	4·2124	6·2724	8·5261	10·8479	13·1236	15·2562	17·1690	18·8065
	4	·0492	·2122	·5480	1·0993	1·8873	2·9118	4·1535	5·5775	7·1382	8·7832	10·4578
	5	·0068	·0354	·1077	·2490	·4853	·8394	1·3291	1·9649	2·7495	3·6775	4·7356
	6	·0009	·0054	·0191	·0500	·1093	·2099	·3658	·5913	·8994	1·3010	1·8040
	7	·0001	·0008	·0031	·0090	·0219	·0462	·0881	·1547	·2545	·3965	·5898
	8	·0000	·0001	·0004	·0015	·0039	·0090	·0187	·0356	·0630	·1053	·1675
	9	—	·0000	·0001	·0002	·0006	·0016	·0035	·0072	·0137	·0245	·0416
	10	—	—	·0000	·0000	·0001	·0002	·0006	·0013	·0026	·0050	·0091
	11	—	—	—	—	·0000	·0000	·0001	·0002	·0005	·0009	·0017
	12	—	—	—	—	—	—	·0000	·0000	·0001	·0001	·0003
	13—20	—	—	—	—	—	—	—	—	·0000	·0000	·0000

TABLE XLVIII—(*continued*).

Successes		$p=0$	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$	$p=8$	$p=9$	$p=10$
$n=100$	0	80·1587	64·1270	51·1981	40·7920	32·4330	25·7320	20·3711	16·0915	12·6823	9·9724	7·8232
$m=25$	1	16·0317	25·8576	31·2184	33·4361	33·5052	32·1649	29·9575	27·2737	24·3890	21·4922	18·7076
	2	3·1029	7·5681	12·2826	16·5799	20·1031	22·7047	24·3723	25·1757	25·2300	24·6693	23·6306
	3	·5802	1·9024	3·8912	6·3556	9·0661	11·8013	14·3734	16·6391	18·5020	19·9086	20·8423
	4	·1046	·4323	1·0701	2·0562	3·3806	4·9929	6·8150	8·7536	10·7117	12·5970	14·3291
	5	·0182	·0908	·2644	·5855	1·0922	1·8077	2·7378	3·8700	5·1757	6·6134	8·1327
	6	·0030	·0178	·0597	·1501	·3138	·5764	·9606	1·4841	2·1565	2·9790	3·9431
	7	·0005	·0033	·0125	·0351	·0815	·1647	·3000	·5035	·7910	1·1761	1·6693
	8	·0001	·0005	·0024	·0075	·0193	·0426	·0844	·1531	·2589	·4127	·6260
	9	·0000	·0001	·0004	·0015	·0042	·0100	·0215	·0421	·0763	·1299	·2099
	10	—	·0000	·0001	·0003	·0008	·0022	·0050	·0105	·0203	·0369	·0634
	11	—	—	·0000	·0001	·0001	·0004	·0011	·0024	·0049	·0095	·0173
	12	—	—	—	·0000	·0000	·0001	·0002	·0005	·0011	·0022	·0043
	13	—	—	—	—	—	·0000	·0000	·0001	·0002	·0005	·0009
	14	—	—	—	—	—	—	—	·0000	·0000	·0001	·0002
	15—25	—	—	—	—	—	—	—	—	—	·0000	·0000
$n=100$	0	66·8874	44·5916	29·6280	19·6185	12·9456	8·5121	5·5769	3·6405	2·3676	1·5339	·9900
$m=50$	1	22·2958	29·9273	30·0284	26·6919	22·1671	17·6113	13·5550	10·1832	7·5029	5·4395	3·8892
	2	7·3322	14·8625	20·0189	22·3956	22·4728	20·9746	18·5789	15·8126	13·0370	10·4710	8·2261
	3	2·3780	6·4708	10·9693	14·8274	17·4789	18·7745	18·8406	17·9434	16·3894	14·4635	12·3988
	4	·7603	2·6038	5·3333	8·4691	11·4896	13·9817	15·7005	16·5656	16·6252	16·0095	14·8876
	5	·2396	·9912	2·3852	4·3589	6·6996	9·1228	11·3492	13·1572	14·4085	15·0512	15·1066
	6	·0743	·3614	1·0008	2·0720	3·5636	5·3759	7·3484	9·2958	11·0430	12·4505	13·4281
	7	·0227	·1271	·3987	·9237	1·7600	2·9173	4·3512	5·9710	7·6559	9·2753	10·7081
	8	·0068	·0433	·1520	·3901	·8167	1·4771	2·3900	3·5398	4·8771	6·3248	7·7895
	9	·0020	·0143	·0557	·1572	·3590	·7044	1·2301	1·9578	2·8874	3·9946	5·2324
	10	·0006	·0046	·0197	·0607	·1504	·3185	·5978	1·0184	1·6022	2·3574	3·2752
	11	·0002	·0014	·0068	·0226	·0603	·1373	·2758	·5012	·8386	1·3088	1·9239
	12	—	·0004	·0022	·0081	·0232	·0566	·1213	·2345	·4161	·6871	1·0664
	13	—	·0001	·0007	·0028	·0086	·0224	·0510	·1046	·1965	·3425	·5601
	14	—	—	·0002	·0009	·0031	·0085	·0206	·0447	·0886	·1627	·2797
	15	—	—	·0001	·0003	·0011	·0031	·0080	·0183	·0382	·0738	·1332
	16	—	—	—	·0001	·0004	·0011	·0030	·0072	·0158	·0320	·0606
	17	—	—	—	—	·0001	·0004	·0011	·0027	·0063	·0133	·0264
	18	—	—	—	—	—	·0001	·0004	·0010	·0024	·0053	·0110
	19	—	—	—	—	—	—	·0001	·0003	·0009	·0020	·0044
	20	—	—	—	—	—	—	—	·0001	·0003	·0008	·0017
	21	—	—	—	—	—	—	—	—	·0001	·0003	·0006
	22	—	—	—	—	—	—	—	—	—	·0001	·0002
	23	—	—	—	—	—	—	—	—	—	—	·0001
	24—50	—	—	—	—	—	—	—	—	—	—	—

TABLE XLIX. *Logarithms of Factorials.*

1—250 $\log \lfloor n$ *from* $n=1$ *to* $n=1000$.

n	$\log \lfloor n$	n	$\log \lfloor n$	n	$\log \lfloor n$	n	$\log \lfloor n$	n	$\log \lfloor n$
1	·000 0000	51	66·190 6450	101	159·974 3250	151	264·935 8704	201	377·200 0847
2	·301 0300	52	67·906 6484	102	161·982 9252	152	267·117 7139	202	379·505 4361
3	·778 1513	53	69·630 9243	103	163·995 7624	153	269·302 4054	203	381·812 9321
4	1·380 2112	54	71·363 3180	104	166·012 7958	154	271·489 9261	204	384·122 5623
5	2·079 1812	55	73·103 6807	105	168·033 9851	155	273·680 2578	205	386·434 3161
6	2·857 3325	56	74·851 8687	106	170·059 2909	156	275·873 3824	206	388·748 1834
7	3·702 4305	57	76·607 7436	107	172·088 6747	157	278·069 2820	207	391·064 1537
8	4·605 5205	58	78·371 1716	108	174·122 0985	158	280·267 9391	208	393·382 2170
9	5·559 7630	59	80·142 0236	109	176·159 5250	159	282·469 3363	209	395·702 3633
10	6·559 7630	60	81·920 1748	110	178·200 9176	160	284·673 4562	210	398·024 5826
11	7·601 1557	61	83·705 5047	111	180·246 2406	161	286·880 2821	211	400·348 8651
12	8·680 3370	62	85·497 8964	112	182·295 4586	162	289·089 7971	212	402·675 2009
13	9·794 2803	63	87·297 2369	113	184·348 5371	163	291·301 9847	213	405·003 5805
14	10·940 4084	64	89·103 4169	114	186·405 4419	164	293·516 8286	214	407·333 9943
15	12·116 4996	65	90·916 3303	115	188·466 1398	165	295·734 3125	215	409·666 4328
16	13·320 6196	66	92·735 8742	116	190·530 5978	166	297·954 4206	216	412·000 8865
17	14·551 0685	67	94·561 9490	117	192·598 7836	167	300·177 1371	217	414·337 3463
18	15·806 3410	68	96·394 4579	118	194·670 6656	168	302·402 4464	218	416·675 8027
19	17·085 0946	69	98·233 3070	119	196·746 2126	169	304·630 3331	219	419·016 2469
20	18·386 1246	70	100·078 4050	120	198·825 3938	170	306·860 7820	220	421·358 6695
21	19·708 3439	71	101·929 6634	121	200·908 1792	171	309·093 7781	221	423·703 0618
22	21·050 7666	72	103·786 9959	122	202·994 5390	172	311·329 3066	222	426·049 4148
23	22·412 4944	73	105·650 3187	123	205·084 4442	173	313·567 3527	223	428·397 7197
24	23·792 7057	74	107·519 5505	124	207·177 8658	174	315·807 9019	224	430·747 9677
25	25·190 6457	75	109·394 6117	125	209·274 7759	175	318·050 9400	225	433·100 1502
26	26·605 6190	76	111·275 4253	126	211·375 1464	176	320·296 4526	226	435·454 2586
27	28·036 9828	77	113·161 9160	127	213·478 9501	177	322·544 4259	227	437·810 2845
28	29·484 1408	78	115·054 0106	128	215·586 1601	178	324·794 8459	228	440·168 2193
29	30·946 5388	79	116·951 6377	129	217·696 7498	179	327·047 6989	229	442·528 0548
30	32·423 6601	80	118·854 7277	130	219·810 6932	180	329·302 9714	230	444·889 7827
31	33·915 0218	81	120·763 2127	131	221·927 9645	181	331·560 6500	231	447·253 3946
32	35·420 1717	82	122·677 0266	132	224·048 5384	182	333·820 7214	232	449·618 8826
33	36·938 6857	83	124·596 1047	133	226·172 3900	183	336·083 1725	233	451·986 2385
34	38·470 1646	84	126·520 3840	134	228·299 4948	184	338·347 9903	234	454·355 4544
35	40·014 2326	85	128·449 8029	135	230·429 8286	185	340·615 1620	235	456·726 5223
36	41·570 5351	86	130·384 3013	136	232·563 3675	186	342·884 6750	236	459·099 4343
37	43·138 7369	87	132·323 8206	137	234·700 0881	187	345·156 5166	237	461·474 1826
38	44·718 5205	88	134·268 3033	138	236·839 9672	188	347·430 6744	238	463·850 7596
39	46·309 5851	89	136·217 6933	139	238·982 9820	189	349·707 1362	239	466·229 1575
40	47·911 6451	90	138·171 9358	140	241·129 1100	190	351·985 8898	240	468·609 3687
41	49·524 4289	91	140·130 9772	141	243·278 3291	191	354·266 9232	241	470·991 3857
42	51·147 6782	92	142·094 7650	142	245·430 6174	192	356·550 2244	242	473·375 2011
43	52·781 1467	93	144·063 2480	143	247·585 9535	193	358·835 7817	243	475·760 8074
44	54·424 5993	94	146·036 3758	144	249·744 3160	194	361·123 5835	244	478·148 1972
45	56·077 8119	95	148·014 0994	145	251·905 6840	195	363·413 6181	245	480·537 3633
46	57·740 5697	96	149·996 3707	146	254·070 0368	196	365·705 8742	246	482·928 2984
47	59·412 6676	97	151·983 1424	147	256·237 3542	197	368·000 3404	247	485·320 9954
48	61·093 9088	98	153·974 3685	148	258·407 6159	198	370·297 0056	248	487·715 4470
49	62·784 1049	99	155·970 0037	149	260·580 8022	199	372·595 8586	249	490·111 6464
50	64·483 0749	100	157·970 0037	150	262·756 8934	200	374·896 8886	250	492·509 5864

TABLE XLIX—(*continued*). 251—500

n	$\log \lfloor n$
251	494·909 2601
252	497·310 6607
253	499·713 7812
254	502·118 6149
255	504·525 1551
256	506·933 3950
257	509·343 3282
258	511·754 9479
259	514·168 2476
260	516·583 2210
261	518·999 8615
262	521·418 1628
263	523·838 1185
264	526·259 7225
265	528·682 9683
266	531·107 8500
267	533·534 3612
268	535·962 4960
269	538·392 2483
270	540·823 6121
271	543·256 5814
272	545·691 1503
273	548·127 3129
274	550·565 0635
275	553·004 3962
276	555·445 3052
277	557·887 7850
278	560·331 8298
279	562·777 4340
280	565·224 5920
281	567·673 2984
282	570·123 5475
283	572·575 3339
284	575·028 6523
285	577·483 4971
286	579·939 8631
287	582·397 7450
288	584·857 1375
289	587·318 0354
290	589·780 4334
291	592·244 3264
292	594·709 7092
293	597·176 5768
294	599·644 9242
295	602·114 7462
296	604·586 0379
297	607·058 7943
298	609·533 0106
299	612·008 6818
300	614·485 8030

n	$\log \lfloor n$
301	616·964 3695
302	619·444 3765
303	621·925 8191
304	624·408 6927
305	626·892 9925
306	629·378 7140
307	631·865 8523
308	634·354 4031
309	636·844 3615
310	639·335 7232
311	641·828 4836
312	644·322 6382
313	646·818 1825
314	649·315 1122
315	651·813 4227
316	654·313 1098
317	656·814 1691
318	659·316 5962
319	661·820 3869
320	664·325 5369
321	666·832 0419
322	669·339 8978
323	671·849 1003
324	674·359 6453
325	676·871 5287
326	679·384 7463
327	681·899 2940
328	684·415 1679
329	686·932 3638
330	689·450 8777
331	691·970 7057
332	694·491 8438
333	697·014 2880
334	699·538 0345
335	702·063 0793
336	704·589 4186
337	707·117 0485
338	709·645 9652
339	712·176 1649
340	714·707 6438
341	717·240 3982
342	719·774 4243
343	722·309 7184
344	724·846 2768
345	727·384 0959
346	729·923 1720
347	732·463 5015
348	735·005 0807
349	737·547 9062
350	740·091 9742

n	$\log \lfloor n$
351	742·637 2813
352	745·183 8240
353	747·731 5987
354	750·280 6020
355	752·830 8303
356	755·382 2803
357	757·934 9485
358	760·488 8316
359	763·043 9260
360	765·600 2285
361	768·157 7357
362	770·716 4443
363	773·276 3509
364	775·837 4523
365	778·399 7452
366	780·963 2262
367	783·527 8923
368	786·093 7401
369	788·660 7665
370	791·228 9682
371	793·798 3421
372	796·368 8851
373	798·940 5939
374	801·513 4655
375	804·087 4968
376	806·662 6846
377	809·239 0260
378	811·816 5178
379	814·395 1570
380	816·974 9406
381	819·555 8655
382	822·137 9289
383	824·721 1277
384	827·305 4589
385	829·890 9196
386	832·477 5069
387	835·065 2179
388	837·654 0496
389	840·243 9992
390	842·835 0638
391	845·427 2406
392	848·020 5267
393	850·614 9192
394	853·210 4154
395	855·807 0125
396	858·404 7077
397	861·003 4982
398	863·603 3813
399	866·204 3542
400	868·806 4142

n	$\log \lfloor n$
401	871·409 5586
402	874·013 7846
403	876·619 0896
404	879·225 4710
405	881·832 9260
406	884·441 4521
407	887·051 0465
408	889·661 7066
409	892·273 4300
410	894·886 2138
411	897·500 0556
412	900·114 9528
413	902·730 9029
414	905·347 9032
415	907·965 9513
416	910·585 0447
417	913·205 1807
418	915·826 3570
419	918·448 5710
420	921·071 8203
421	923·696 1024
422	926·321 4149
423	928·947 7552
424	931·575 1211
425	934·203 5100
426	936·832 9196
427	939·463 3475
428	942·094 7913
429	944·727 2486
430	947·360 7170
431	949·995 1943
432	952·630 6780
433	955·267 1659
434	957·904 6557
435	960·543 1449
436	963·182 6314
437	965·823 1128
438	968·464 5869
439	971·107 0515
440	973·750 5041
441	976·394 9427
442	979·040 3650
443	981·686 7687
444	984·334 1517
445	986·982 5117
446	989·631 8466
447	992·282 1541
448	994·933 4321
449	997·585 6784
450	1000·238 8910

n	$\log \lfloor n$
451	1002·893 0675
452	1005·548 2059
453	1008·204 3041
454	1010·861 3600
455	1013·519 3714
456	1016·178 3362
457	1018·838 2524
458	1021·499 1179
459	1024·160 9306
460	1026·823 6884
461	1029·487 3893
462	1032·152 0313
463	1034·817 6123
464	1037·484 1303
465	1040·151 5832
466	1042·819 9692
467	1045·489 2860
468	1048·159 5319
469	1050·830 7047
470	1053·502 8026
471	1056·175 8235
472	1058·849 7655
473	1061·524 6266
474	1064·200 4050
475	1066·877 0986
476	1069·554 7056
477	1072·233 2239
478	1074·912 6518
479	1077·592 9873
480	1080·274 2286
481	1082·956 3737
482	1085·639 4207
483	1088·323 3678
484	1091·008 2132
485	1093·693 9549
486	1096·380 5912
487	1099·068 1202
488	1101·756 5400
489	1104·445 8488
490	1107·136 0449
491	1109·827 1264
492	1112·519 0915
493	1115·211 9384
494	1117·905 6654
495	1120·600 2706
496	1123·295 7523
497	1125·992 1086
498	1128·689 3380
499	1131·387 4385
500	1134·086 4085

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501—750 *Table of* $\log \lfloor n$ *from* $n=1$ *to* $n=1000$.

n	$\log \lfloor n$
501	1136·786 2463
502	1139·486 9500
503	1142·188 5180
504	1144·890 9485
505	1147·594 2399
506	1150·298 3904
507	1153·003 3984
508	1155·709 2621
509	1158·415 9798
510	1161·123 5500
511	1163·831 9709
512	1166·541 2409
513	1169·251 3583
514	1171·962 3214
515	1174·674 1286
516	1177·386 7783
517	1180·100 2688
518	1182·814 5986
519	1185·529 7660
520	1188·245 7693
521	1190·962 6070
522	1193·680 2775
523	1196·398 7792
524	1199·118 1105
525	1201·838 2698
526	1204·559 2556
527	1207·281 0662
528	1210·003 7001
529	1212·727 1558
530	1215·451 4316
531	1218·176 5262
532	1220·902 4378
533	1223·629 1650
534	1226·356 7063
535	1229·085 0600
536	1231·814 2248
537	1234·544 1991
538	1237·274 9814
539	1240·006 5702
540	1242·738 9639
541	1245·472 1612
542	1248·206 1605
543	1250·940 9603
544	1253·676 5592
545	1256·412 9557
546	1259·150 1483
547	1261·888 1357
548	1264·626 9162
549	1267·366 4886
550	1270·106 8513

n	$\log \lfloor n$
551	1272·848 0029
552	1275·589 9419
553	1278·332 6671
554	1281·076 1768
555	1283·820 4698
556	1286·565 5446
557	1289·311 3998
558	1292·058 0340
559	1294·805 4458
560	1297·553 6338
561	1300·302 5967
562	1303·052 3330
563	1305·802 8414
564	1308·554 1205
565	1311·306 1690
566	1314·058 9854
567	1316·812 5684
568	1319·566 9168
569	1322·322 0290
570	1325·077 9039
571	1327·834 5400
572	1330·591 9360
573	1333·350 0907
574	1336·109 0026
575	1338·868 6704
576	1341·629 0929
577	1344·390 2687
578	1347·152 1965
579	1349·914 8751
580	1352·678 3031
581	1355·442 4792
582	1358·207 4022
583	1360·973 0708
584	1363·739 4836
585	1366·506 6395
586	1369·274 5371
587	1372·043 1752
588	1374·812 5525
589	1377·582 6678
590	1380·353 5198
591	1383·125 1073
592	1385·897 4290
593	1388·670 4837
594	1391·444 2702
595	1394·218 7871
596	1396·994 0334
597	1399·770 0077
598	1402·546 7089
599	1405·324 1357
600	1408·102 2870

n	$\log \lfloor n$
601	1410·881 1614
602	1413·660 7579
603	1416·441 0752
604	1419·222 1122
605	1422·003 8676
606	1424·786 3402
607	1427·569 5289
608	1430·353 4324
609	1433·138 0497
610	1435·923 3796
611	1438·709 4208
612	1441·496 1722
613	1444·283 6327
614	1447·071 8011
615	1449·860 6762
616	1452·650 2569
617	1455·440 5420
618	1458·231 5305
619	1461·023 2212
620	1463·815 6129
621	1466·608 7045
622	1469·402 4948
623	1472·196 9829
624	1474·992 1675
625	1477·788 0475
626	1480·584 6218
627	1483·381 8894
628	1486·179 8490
629	1488·978 4997
630	1491·777 8402
631	1494·577 8696
632	1497·378 5866
633	1500·179 9904
634	1502·982 0796
635	1505·784 8533
636	1508·588 3105
637	1511·392 4499
638	1514·197 2706
639	1517·002 7714
640	1519·808 9514
641	1522·615 8094
642	1525·423 3445
643	1528·231 5554
644	1531·040 4413
645	1533·850 0010
646	1536·660 2335
647	1539·471 1378
648	1542·282 7128
649	1545·094 9575
650	1547·907 8709

n	$\log \lfloor n$
651	1550·721 4519
652	1553·535 6995
653	1556·350 6126
654	1559·166 1904
655	1561·982 4317
656	1564·799 3355
657	1567·616 9009
658	1570·435 1268
659	1573·254 0122
660	1576·073 5561
661	1578·893 7576
662	1581·714 6156
663	1584·536 1291
664	1587·358 2972
665	1590·181 1188
666	1593·004 5931
667	1595·828 7189
668	1598·653 4954
669	1601·478 9215
670	1604·304 9963
671	1607·131 7188
672	1609·959 0881
673	1612·787 1031
674	1615·615 7630
675	1618·445 0668
676	1621·275 0135
677	1624·105 6022
678	1626·936 8319
679	1629·768 7016
680	1632·601 2106
681	1635·434 3577
682	1638·268 1420
683	1641·102 5627
684	1643·937 6189
685	1646·773 3094
686	1649·609 6335
687	1652·446 5903
688	1655·284 1787
689	1658·122 3979
690	1660·961 2470
691	1663·800 7251
692	1666·640 8312
693	1669·481 5644
694	1672·322 9239
695	1675·164 9087
696	1678·007 5179
697	1680·850 7507
698	1683·694 6061
699	1686·539 0833
700	1689·384 1813

n	$\log \lfloor n$
701	1692·229 8994
702	1695·076 2365
703	1697·923 1918
704	1700·770 7644
705	1703·618 9536
706	1706·467 7583
707	1709·317 1777
708	1712·167 2109
709	1715·017 8572
710	1717·869 1155
711	1720·720 9851
712	1723·573 4651
713	1726·426 5546
714	1729·280 2529
715	1732·134 5589
716	1734·989 4719
717	1737·844 9911
718	1740·701 1155
719	1743·557 8444
720	1746·415 1769
721	1749·273 1122
722	1752·131 6494
723	1754·990 7877
724	1757·850 5262
725	1760·710 8642
726	1763·571 8009
727	1766·433 3353
728	1769·295 4667
729	1772·158 1942
730	1775·021 5170
731	1777·885 4344
732	1780·749 9455
733	1783·615 0495
734	1786·480 7455
735	1789·347 0329
736	1792·213 9107
737	1795·081 3782
738	1797·949 4345
739	1800·818 0790
740	1803·687 3107
741	1806·557 1289
742	1809·427 5328
743	1812·298 5216
744	1815·170 0946
745	1818·042 2508
746	1820·914 9897
747	1823·788 3103
748	1826·662 2119
749	1829·536 6937
750	1832·411 7549

TABLE XLIX—(*continued*). 751—1000

n	$\log \lfloor n$	n	$\log \lfloor n$	n	$\log \lfloor n$	n	$\log \lfloor n$	n	$\log \lfloor n$
751	1835·287 3949	801	1979·790 7168	851	2125·649 5488	901	2272·784 2010	951	2421·123 8376
752	1838·163 6127	802	1982·694 8911	852	2128·579 9884	902	2275·739 4075	952	2424·102 4745
753	1841·040 4077	803	1985·599 6067	853	2131·510 9374	903	2278·695 0953	953	2427·081 5674
754	1843·917 7790	804	1988·504 8627	854	2134·442 3953	904	2281·651 2637	954	2430·061 1158
755	1846·795 7260	805	1991·410 6586	855	2137·374 3614	905	2284·607 9123	955	2433·041 1192
756	1849·674 2478	806	1994·316 9936	856	2140·306 8352	906	2287·565 0405	956	2436·021 5771
757	1852·553 3437	807	1997·223 8672	857	2143·239 8160	907	2290·522 6478	957	2439·002 4890
758	1855·433 0129	808	2000·131 2785	858	2146·173 3033	908	2293·480 7336	958	2441·983 8545
759	1858·313 2546	809	2003·039 2271	859	2149·107 2964	909	2296·439 2975	959	2444·965 6731
760	1861·194 0682	810	2005·947 7121	860	2152·041 7949	910	2299·398 3389	960	2447·947 9443
761	1864·075 4529	811	2008·856 7329	861	2154·976 7980	911	2302·357 8573	961	2450·930 6677
762	1866·957 4079	812	2011·766 2890	862	2157·912 3053	912	2305·317 8521	962	2453·913 8428
763	1869·839 9324	813	2014·676 3795	863	2160·848 3161	913	2308·278 3229	963	2456·897 4691
764	1872·723 0258	814	2017·587 0039	864	2163·784 8298	914	2311·239 2691	964	2459·881 5461
765	1875·606 6872	815	2020·498 1615	865	2166·721 8459	915	2314·200 6902	965	2462·866 0734
766	1878·490 9160	816	2023·409 8517	866	2169·659 3638	916	2317·162 5856	966	2465·851 0506
767	1881·375 7113	817	2026·322 0737	867	2172·597 3829	917	2320·124 9550	967	2468·836 4770
768	1884·261 0726	818	2029·234 8270	868	2175·535 9027	918	2323·087 7977	968	2471·822 3524
769	1887·146 9989	819	2032·148 1109	869	2178·474 9224	919	2326·051 1132	969	2474·808 6762
770	1890·033 4896	820	2035·061 9248	870	2181·414 4417	920	2329·014 9010	970	2477·795 4479
771	1892·920 5440	821	2037·976 2679	871	2184·354 4598	921	2331·979 1606	971	2480·782 6671
772	1895·808 1613	822	2040·891 1398	872	2187·294 9763	922	2334·943 8915	972	2483·770 3334
773	1898·696 3408	823	2043·806 5396	873	2190·235 9906	923	2337·909 0932	973	2486·758 4462
774	1901·585 0817	824	2046·722 4668	874	2193·177 5020	924	2340·874 7652	974	2489·747 0052
775	1904·474 3835	825	2049·638 9208	875	2196·119 5101	925	2343·840 9069	975	2492·736 0098
776	1907·364 2452	826	2052·555 9008	876	2199·062 0142	926	2346·807 5179	976	2495·725 4596
777	1910·254 6662	827	2055·473 4063	877	2202·005 0138	927	2349·774 5977	977	2498·715 3542
778	1913·145 6458	828	2058·391 4367	878	2204·948 5083	928	2352·742 1456	978	2501·705 6930
779	1916·037 1832	829	2061·309 9912	879	2207·892 4971	929	2355·710 1614	979	2504·696 4757
780	1918·929 2778	830	2064·229 0693	880	2210·836 9798	930	2358·678 6443	980	2507·687 7018
781	1921·821 9289	831	2067·148 6703	881	2213·781 9557	931	2361·647 5940	981	2510·679 3708
782	1924·715 1356	832	2070·068 7936	882	2216·727 4243	932	2364·617 0099	982	2513·671 4823
783	1927·608 8974	833	2072·989 4386	883	2219·673 3850	933	2367·586 8915	983	2516·664 0358
784	1930·503 2135	834	2075·910 6047	884	2222·619 8373	934	2370·557 2384	984	2519·657 0309
785	1933·398 0831	835	2078·832 2912	885	2225·566 7805	935	2373·528 0500	985	2522·650 4672
786	1936·293 5057	836	2081·754 4974	886	2228·514 2143	936	2376·499 3259	986	2525·644 3441
787	1939·189 4804	837	2084·677 2229	887	2231·462 1379	937	2379·471 0655	987	2528·638 6612
788	1942·086 0066	838	2087·600 4669	888	2234·410 5509	938	2382·443 2683	988	2531·633 4182
789	1944·983 0836	839	2090·524 2289	889	2237·359 4526	939	2385·415 9339	989	2534·628 6145
790	1947·880 7107	840	2093·448 5082	890	2240·308 8426	940	2388·389 0618	990	2537·624 2497
791	1950·778 8872	841	2096·373 3042	891	2243·258 7203	941	2391·362 6514	991	2540·620 3233
792	1953·677 6124	842	2099·298 6162	892	2246·209 0852	942	2394·336 7023	992	2543·616 8350
793	1956·576 8856	843	2102·224 4438	893	2249·159 9366	943	2397·311 2140	993	2546·613 7842
794	1959·476 7061	844	2105·150 7863	894	2252·111 2742	944	2400·286 1860	994	2549·611 1706
795	1962·377 0732	845	2108·077 6430	895	2255·063 0972	945	2403·261 6178	995	2552·608 9937
796	1965·277 9863	846	2111·005 0133	896	2258·015 4052	946	2406·237 5089	996	2555·607 2530
797	1968·179 4446	847	2113·932 8967	897	2260·968 1976	947	2409·213 8589	997	2558·605 9482
798	1971·081 4475	848	2116·861 2926	898	2263·921 4740	948	2412·190 6672	998	2561·605 0787
799	1973·983 9943	849	2119·790 2003	899	2266·875 2337	949	2415·167 9334	999	2564·604 6442
800	1976·887 0842	850	2122·719 6192	900	2269·829 4762	950	2418·145 6570	1000	2567·604 6442

TABLE L. *Table of Fourth-Moments of Subgroup-Frequencies.*

Ordinate 2—11. Frequency 1—50.

n	$x=2$	$x=3$	$x=4$	$x=5$	$x=6$	$x=7$	$x=8$	$x=9$	$x=11$	n
1	16	81	256	625	1296	2401	4096	6561	14641	*1*
2	32	162	512	1250	2592	4802	8192	13122	29282	*2*
3	48	243	768	1875	3888	7203	12288	19683	43923	*3*
4	64	324	1024	2500	5184	9604	16384	26244	58564	*4*
5	80	405	1280	3125	6480	12005	20480	32805	73205	*5*
6	96	486	1536	3750	7776	14406	24576	39366	87846	*6*
7	112	567	1792	4375	9072	16807	28672	45927	102487	*7*
8	128	648	2048	5000	10368	19208	32768	52488	117128	*8*
9	144	729	2304	5625	11664	21609	36864	59049	131769	*9*
10	160	810	2560	6250	12960	24010	40960	65610	146410	*10*
11	176	891	2816	6875	14256	26411	45056	72171	161051	*11*
12	192	972	3072	7500	15552	28812	49152	78732	175692	*12*
13	208	1053	3328	8125	16848	31213	53248	85293	190333	*13*
14	224	1134	3584	8750	18144	33614	57344	91854	204974	*14*
15	240	1215	3840	9375	19440	36015	61440	98415	219615	*15*
16	256	1296	4096	10000	20736	38416	65536	104976	234256	*16*
17	272	1377	4352	10625	22032	40817	69632	111537	248897	*17*
18	288	1458	4608	11250	23328	43218	73728	118098	263538	*18*
19	304	1539	4864	11875	24624	45619	77824	124659	278179	*19*
20	320	1620	5120	12500	25920	48020	81920	131220	292820	*20*
21	336	1701	5376	13125	27216	50421	86016	137781	307461	*21*
22	352	1782	5632	13750	28512	52822	90112	144342	322102	*22*
23	368	1863	5888	14375	29808	55223	94208	150903	336743	*23*
24	384	1944	6144	15000	31104	57624	98304	157464	351384	*24*
25	400	2025	6400	15625	32400	60025	102400	164025	366025	*25*
26	416	2106	6656	16250	33696	62426	106496	170586	380666	*26*
27	432	2187	6912	16875	34992	64827	110592	177147	395307	*27*
28	448	2268	7168	17500	36288	67228	114688	183708	409948	*28*
29	464	2349	7424	18125	37584	69629	118784	190269	424589	*29*
30	480	2430	7680	18750	38880	72030	122880	196830	439230	*30*
31	496	2511	7936	19375	40176	74431	126976	203391	453871	*31*
32	512	2592	8192	20000	41472	76832	131072	209952	468512	*32*
33	528	2673	8448	20625	42768	79233	135168	216513	483153	*33*
34	544	2754	8704	21250	44064	81634	139264	223074	497794	*34*
35	560	2835	8960	21875	45360	84035	143360	229635	512435	*35*
36	576	2916	9216	22500	46656	86436	147456	236196	527076	*36*
37	592	2997	9472	23125	47952	88837	151552	242757	541717	*37*
38	608	3078	9728	23750	49248	91238	155648	249318	556358	*38*
39	624	3159	9984	24375	50544	93639	159744	255879	570999	*39*
40	640	3240	10240	25000	51840	96040	163840	262440	585640	*40*
41	656	3321	10496	25625	53136	98441	167936	269001	600281	*41*
42	672	3402	10752	26250	54432	100842	172032	275562	614922	*42*
43	688	3483	11008	26875	55728	103243	176128	282123	629563	*43*
44	704	3564	11264	27500	57024	105644	180224	288684	644204	*44*
45	720	3645	11520	28125	58320	108045	184320	295245	658845	*45*
46	736	3726	11776	28750	59616	110446	188416	301806	673486	*46*
47	752	3807	12032	29375	60912	112847	192512	308367	688127	*47*
48	768	3888	12288	30000	62208	115248	196608	314928	702768	*48*
49	784	3969	12544	30625	63504	117649	200704	321489	717409	*49*
50	800	4050	12800	31250	64800	120050	204800	328050	732050	*50*

TABLE L—(*continued*).

Ordinate 12—19. Frequency 1—50.

n	$x=12$	$x=13$	$x=14$	$x=15$	$x=16$	$x=17$	$x=18$	$x=19$	n
1	20736	28561	38416	50625	65536	83521	104976	130321	*1*
2	41472	57122	76832	101250	131072	167042	209952	260642	*2*
3	62208	85683	115248	151875	196608	250563	314928	390963	*3*
4	82944	114244	153664	202500	262144	334084	419904	521284	*4*
5	103680	142805	192080	253125	327680	417605	524880	651605	*5*
6	124416	171366	230496	303750	393216	501126	629856	781926	*6*
7	145152	199927	268912	354375	458752	584647	734832	912247	*7*
8	165888	228488	307328	405000	524288	668168	839808	1042568	*8*
9	186624	257049	345744	455625	589824	751689	944784	1172889	*9*
10	207360	285610	384160	506250	655360	835210	1049760	1303210	*10*
11	228096	314171	422576	556875	720896	918731	1154736	1433531	*11*
12	248832	342732	460992	607500	786432	1002252	1259712	1563852	*12*
13	269568	371293	499408	658125	851968	1085773	1364688	1694173	*13*
14	290304	399854	537824	708750	917504	1169294	1469664	1824494	*14*
15	311040	428415	576240	759375	983040	1252815	1574640	1954815	*15*
16	331776	456976	614656	810000	1048576	1336336	1679616	2085136	*16*
17	352512	485537	653072	860625	1114112	1419857	1784592	2215457	*17*
18	373248	514098	691488	911250	1179648	1503378	1889568	2345778	*18*
19	393984	542659	729904	961875	1245184	1586899	1994544	2476099	*19*
20	414720	571220	768320	1012500	1310720	1670420	2099520	2606420	*20*
21	435456	599781	806736	1063125	1376256	1753941	2204496	2736741	*21*
22	456192	628342	845152	1113750	1441792	1837462	2309472	2867062	*22*
23	476928	656903	883568	1164375	1507328	1920983	2414448	2997383	*23*
24	497664	685464	921984	1215000	1572864	2004504	2519424	3127704	*24*
25	518400	714025	960400	1265625	1638400	2088025	2624400	3258025	*25*
26	539136	742586	998816	1316250	1703936	2171546	2729376	3388346	*26*
27	559872	771147	1037232	1366875	1769472	2255067	2834352	3518667	*27*
28	580608	799708	1075648	1417500	1835008	2338588	2939328	3648988	*28*
29	601344	828269	1114064	1468125	1900544	2422109	3044304	3779309	*29*
30	622080	856830	1152480	1518750	1966080	2505630	3149280	3909630	*30*
31	642816	885391	1190896	1569375	2031616	2589151	3254256	4039951	*31*
32	663552	913952	1229312	1620000	2097152	2672672	3359232	4170272	*32*
33	684288	942513	1267728	1670625	2162688	2756193	3464208	4300593	*33*
34	705024	971074	1306144	1721250	2228224	2839714	3569184	4430914	*34*
35	725760	999635	1344560	1771875	2293760	2923235	3674160	4561235	*35*
36	746496	1028196	1382976	1822500	2359296	3006756	3779136	4691556	*36*
37	767232	1056757	1421392	1873125	2424832	3090277	3884112	4821877	*37*
38	787968	1085318	1459808	1923750	2490368	3173798	3989088	4952198	*38*
39	808704	1113879	1498224	1974375	2555904	3257319	4094064	5082519	*39*
40	829440	1142440	1536640	2025000	2621440	3340840	4199040	5212840	*40*
41	850176	1171001	1575056	2075625	2686976	3424361	4304016	5343161	*41*
42	870912	1199562	1613472	2126250	2752512	3507882	4408992	5473482	*42*
43	891648	1228123	1651888	2176875	2818048	3591403	4513968	5603803	*43*
44	912384	1256684	1690304	2227500	2883584	3674924	4618944	5734124	*44*
45	933120	1285245	1728720	2278125	2949120	3758445	4723920	5864445	*45*
46	953856	1313806	1767136	2328750	3014656	3841966	4828896	5994766	*46*
47	974592	1342367	1805552	2379375	3080192	3925487	4933872	6125087	*47*
48	995328	1370928	1843968	2430000	3145728	4009008	5038848	6255408	*48*
49	1016064	1399489	1882384	2480625	3211264	4092529	5143824	6385729	*49*
50	1036800	1428050	1920800	2531250	3276800	4176050	5248800	6516050	*50*

TABLE L—(*continued*).

Ordinate 2—11. Frequency 51—100.

n	$x=2$	$x=3$	$x=4$	$x=5$	$x=6$	$x=7$	$x=8$	$x=9$	$x=11$	n
51	816	4131	13056	31875	66096	122451	208896	334611	746691	*51*
52	832	4212	13312	32500	67392	124852	212992	341172	761332	*52*
53	848	4293	13568	33125	68688	127253	217088	347733	775973	*53*
54	864	4374	13824	33750	69984	129654	221184	354294	790614	*54*
55	880	4455	14080	34375	71280	132055	225280	360855	805255	*55*
56	896	4536	14336	35000	72576	134456	229376	367416	819896	*56*
57	912	4617	14592	35625	73872	136857	233472	373977	834537	*57*
58	928	4698	14848	36250	75168	139258	237568	380538	849178	*58*
59	944	4779	15104	36875	76464	141659	241664	387099	863819	*59*
60	960	4860	15360	37500	77760	144060	245760	393660	878460	*60*
61	976	4941	15616	38125	79056	146461	249856	400221	893101	*61*
62	992	5022	15872	38750	80352	148862	253952	406782	907742	*62*
63	1008	5103	16128	39375	81648	151263	258048	413343	922383	*63*
64	1024	5184	16384	40000	82944	153664	262144	419904	937024	*64*
65	1040	5265	16640	40625	84240	156065	266240	426465	951665	*65*
66	1056	5346	16896	41250	85536	158466	270336	433026	966306	*66*
67	1072	5427	17152	41875	86832	160867	274432	439587	980947	*67*
68	1088	5508	17408	42500	88128	163268	278528	446148	995588	*68*
69	1104	5589	17664	43125	89424	165669	282624	452709	1010229	*69*
70	1120	5670	17920	43750	90720	168070	286720	459270	1024870	*70*
71	1136	5751	18176	44375	92016	170471	290816	465831	1039511	*71*
72	1152	5832	18432	45000	93312	172872	294912	472392	1054152	*72*
73	1168	5913	18688	45625	94608	175273	299008	478953	1068793	*73*
74	1184	5994	18944	46250	95904	177674	303104	485514	1083434	*74*
75	1200	6075	19200	46875	97200	180075	307200	492075	1098075	*75*
76	1216	6156	19456	47500	98496	182476	311296	498636	1112716	*76*
77	1232	6237	19712	48125	99792	184877	315392	505197	1127357	*77*
78	1248	6318	19968	48750	101088	187278	319488	511758	1141998	*78*
79	1264	6399	20224	49375	102384	189679	323584	518319	1156639	*79*
80	1280	6480	20480	50000	103680	192080	327680	524880	1171280	*80*
81	1296	6561	20736	50625	104976	194481	331776	531441	1185921	*81*
82	1312	6642	20992	51250	106272	196882	335872	538002	1200562	*82*
83	1328	6723	21248	51875	107568	199283	339968	544563	1215203	*83*
84	1344	6804	21504	52500	108864	201684	344064	551124	1229844	*84*
85	1360	6885	21760	53125	110160	204085	348160	557685	1244485	*85*
86	1376	6966	22016	53750	111456	206486	352256	564246	1259126	*86*
87	1392	7047	22272	54375	112752	208887	356352	570807	1273767	*87*
88	1408	7128	22528	55000	114048	211288	360448	577368	1288408	*88*
89	1424	7209	22784	55625	115344	213689	364544	583929	1303049	*89*
90	1440	7290	23040	56250	116640	216090	368640	590490	1317690	*90*
91	1456	7371	23296	56875	117936	218491	372736	597051	1332331	*91*
92	1472	7452	23552	57500	119232	220892	376832	603612	1346972	*92*
93	1488	7533	23808	58125	120528	223293	380928	610173	1361613	*93*
94	1504	7614	24064	58750	121824	225694	385024	616734	1376254	*94*
95	1520	7695	24320	59375	123120	228095	389120	623295	1390895	*95*
96	1536	7776	24576	60000	124416	230496	393216	629856	1405536	*96*
97	1552	7857	24832	60625	125712	232897	397312	636417	1420177	*97*
98	1568	7938	25088	61250	127008	235298	401408	642978	1434818	*98*
99	1584	8019	25344	61875	128304	237699	405504	649539	1449459	*99*
100	1600	8100	25600	62500	129600	240100	409600	656100	1464100	*100*

TABLE L—(*continued*).

Ordinate 12—19. Frequency 51—100.

n	$x=12$	$x=13$	$x=14$	$x=15$	$x=16$	$x=17$	$x=18$	$x=19$	n
51	1057536	1456611	1959216	2581875	3342336	4259571	5353776	6646371	*51*
52	1078272	1485172	1997632	2632500	3407872	4343092	5458752	6776692	*52*
53	1099008	1513733	2036048	2683125	3473408	4426613	5563728	6907013	*53*
54	1119744	1542294	2074464	2733750	3538944	4510134	5668704	7037334	*54*
55	1140480	1570855	2112880	2784375	3604480	4593655	5773680	7167655	*55*
56	1161216	1599416	2151296	2835000	3670016	4677176	5878656	7297976	*56*
57	1181952	1627977	2189712	2885625	3735552	4760697	5983632	7428297	*57*
58	1202688	1656538	2228128	2936250	3801088	4844218	6088608	7558618	*58*
59	1223424	1685099	2266544	2986875	3866624	4927739	6193584	7688939	*59*
60	1244160	1713660	2304960	3037500	3932160	5011260	6298560	7819260	*60*
61	1264896	1742221	2343376	3088125	3997696	5094781	6403536	7949581	*61*
62	1285632	1770782	2381792	3138750	4063232	5178302	6508512	8079902	*62*
63	1306368	1799343	2420208	3189375	4128768	5261823	6613488	8210223	*63*
64	1327104	1827904	2458624	3240000	4194304	5345344	6718464	8340544	*64*
65	1347840	1856465	2497040	3290625	4259840	5428865	6823440	8470865	*65*
66	1368576	1885026	2535456	3341250	4325376	5512386	6928416	8601186	*66*
67	1389312	1913587	2573872	3391875	4390912	5595907	7033392	8731507	*67*
68	1410048	1942148	2612288	3442500	4456448	5679428	7138368	8861828	*68*
69	1430784	1970709	2650704	3493125	4521984	5762949	7243344	8992149	*69*
70	1451520	1999270	2689120	3543750	4587520	5846470	7348320	9122470	*70*
71	1472256	2027831	2727536	3594375	4653056	5929991	7453296	9252791	*71*
72	1492992	2056392	2765952	3645000	4718592	6013512	7558272	9383112	*72*
73	1513728	2084953	2804368	3695625	4784128	6097033	7663248	9513433	*73*
74	1534464	2113514	2842784	3746250	4849664	6180554	7768224	9643754	*74*
75	1555200	2142075	2881200	3796875	4915200	6264075	7873200	9774075	*75*
76	1575936	2170636	2919616	3847500	4980736	6347596	7978176	9904396	*76*
77	1596672	2199197	2958032	3898125	5046272	6431117	8083152	10034717	*77*
78	1617408	2227758	2996448	3948750	5111808	6514638	8188128	10165038	*78*
79	1638144	2256319	3034864	3999375	5177344	6598159	8293104	10295359	*79*
80	1658880	2284880	3073280	4050000	5242880	6681680	8398080	10425680	*80*
81	1679616	2313441	3111696	4100625	5308416	6765201	8503056	10556001	*81*
82	1700352	2342002	3150112	4151250	5373952	6848722	8608032	10686322	*82*
83	1721088	2370563	3188528	4201875	5439488	6932243	8713008	10816643	*83*
84	1741824	2399124	3226944	4252500	5505024	7015764	8817984	10946964	*84*
85	1762560	2427685	3265360	4303125	5570560	7099285	8922960	11077285	*85*
86	1783296	2456246	3303776	4353750	5636096	7182806	9027936	11207606	*86*
87	1804032	2484807	3342192	4404375	5701632	7266327	9132912	11337927	*87*
88	1824768	2513368	3380608	4455000	5767168	7349848	9237888	11468248	*88*
89	1845504	2541929	3419024	4505625	5832704	7433369	9342864	11598569	*89*
90	1866240	2570490	3457440	4556250	5898240	7516890	9447840	11728890	*90*
91	1886976	2599051	3495856	4606875	5963776	7600411	9552816	11859211	*91*
92	1907712	2627612	3534272	4657500	6029312	7683932	9657792	11989532	*92*
93	1928448	2656173	3572688	4708125	6094848	7767453	9762768	12119853	*93*
94	1949184	2684734	3611104	4758750	6160384	7850974	9867744	12250174	*94*
95	1969920	2713295	3649520	4809375	6225920	7934495	9972720	12380495	*95*
96	1990656	2741856	3687936	4860000	6291456	8018016	10077696	12510816	*96*
97	2011392	2770417	3726352	4910625	6356992	8101537	10182672	12641137	*97*
98	2032128	2798978	3764768	4961250	6422528	8185058	10287648	12771458	*98*
99	2052864	2827539	3803184	5011875	6488064	8268579	10392624	12901779	*99*
100	2073600	2856100	3841600	5062500	6553600	8352100	10497600	13032100	*100*

TABLE L—(*continued*).

Ordinate 2—7. Frequency 101—150.

n	$x=2$	$x=3$	$x=4$	$x=5$	$x=6$	$x=7$	n
101	1616	8181	25856	63125	130896	242501	*101*
102	1632	8262	26112	63750	132192	244902	*102*
103	1648	8343	26368	64375	133488	247303	*103*
104	1664	8424	26624	65000	134784	249704	*104*
105	1680	8505	26880	65625	136080	252105	*105*
106	1696	8586	27136	66250	137376	254506	*106*
107	1712	8667	27392	66875	138672	256907	*107*
108	1728	8748	27648	67500	139968	259308	*108*
109	1744	8829	27904	68125	141264	261709	*109*
110	1760	8910	28160	68750	142560	264110	*110*
111	1776	8991	28416	69375	143856	266511	*111*
112	1792	9072	28672	70000	145152	268912	*112*
113	1808	9153	28928	70625	146448	271313	*113*
114	1824	9234	29184	71250	147744	273714	*114*
115	1840	9315	29440	71875	149040	276115	*115*
116	1856	9396	29696	72500	150336	278516	*116*
117	1872	9477	29952	73125	151632	280917	*117*
118	1888	9558	30208	73750	152928	283318	*118*
119	1904	9639	30464	74375	154224	285719	*119*
120	1920	9720	30720	75000	155520	288120	*120*
121	1936	9801	30976	75625	156816	290521	*121*
122	1952	9882	31232	76250	158112	292922	*122*
123	1968	9963	31488	76875	159408	295323	*123*
124	1984	10044	31744	77500	160704	297724	*124*
125	2000	10125	32000	78125	162000	300125	*125*
126	2016	10206	32256	78750	163296	302526	*126*
127	2032	10287	32512	79375	164592	304927	*127*
128	2048	10368	32768	80000	165888	307328	*128*
129	2064	10449	33024	80625	167184	309729	*129*
130	2080	10530	33280	81250	168480	312130	*130*
131	2096	10611	33536	81875	169776	314531	*131*
132	2112	10692	33792	82500	171072	316932	*132*
133	2128	10773	34048	83125	172368	319333	*133*
134	2144	10854	34304	83750	173664	321734	*134*
135	2160	10935	34560	84375	174960	324135	*135*
136	2176	11016	34816	85000	176256	326536	*136*
137	2192	11097	35072	85625	177552	328937	*137*
138	2208	11178	35328	86250	178848	331338	*138*
139	2224	11259	35584	86875	180144	333739	*139*
140	2240	11340	35840	87500	181440	336140	*140*
141	2256	11421	36096	88125	182736	338541	*141*
142	2272	11502	36352	88750	184032	340942	*142*
143	2288	11583	36608	89375	185328	343343	*143*
144	2304	11664	36864	90000	186624	345744	*144*
145	2320	11745	37120	90625	187920	348145	*145*
146	2336	11826	37376	91250	189216	350546	*146*
147	2352	11907	37632	91875	190512	352947	*147*
148	2368	11988	37888	92500	191808	355348	*148*
149	2384	12069	38144	93125	193104	357749	*149*
150	2400	12150	38400	93750	194400	360150	*150*

TABLE L—(*continued*).

Ordinate 8—14. Frequency 101—150.

n	$x=8$	$x=9$	$x=11$	$x=12$	$x=13$	$x=14$	n
101	413696	662661	1478741	2094336	2884661	3880016	*101*
102	417792	669222	1493382	2115072	2913222	3918432	*102*
103	421888	675783	1508023	2135808	2941783	3956848	*103*
104	425984	682344	1522664	2156544	2970344	3995264	*104*
105	430080	688905	1537305	2177280	2998905	4033680	*105*
106	434176	695466	1551946	2198016	3027466	4072096	*106*
107	438272	702027	1566587	2218752	3056027	4110512	*107*
108	442368	708588	1581228	2239488	3084588	4148928	*108*
109	446464	715149	1595869	2260224	3113149	4187344	*109*
110	450560	721710	1610510	2280960	3141710	4225760	*110*
111	454656	728271	1625151	2301696	3170271	4264176	*111*
112	458752	734832	1639792	2322432	3198832	4302592	*112*
113	462848	741393	1654433	2343168	3227393	4341008	*113*
114	466944	747954	1669074	2363904	3255954	4379424	*114*
115	471040	754515	1683715	2384640	3284515	4417840	*115*
116	475136	761076	1698356	2405376	3313076	4456256	*116*
117	479232	767637	1712997	2426112	3341637	4494672	*117*
118	483328	774198	1727638	2446848	3370198	4533088	*118*
119	487424	780759	1742279	2467584	3398759	4571504	*119*
120	491520	787320	1756920	2488320	3427320	4609920	*120*
121	495616	793881	1771561	2509056	3455881	4648336	*121*
122	499712	800442	1786202	2529792	3484442	4686752	*122*
123	503808	807003	1800843	2550528	3513003	4725168	*123*
124	507904	813564	1815484	2571264	3541564	4763584	*124*
125	512000	820125	1830125	2592000	3570125	4802000	*125*
126	516096	826686	1844766	2612736	3598686	4840416	*126*
127	520192	833247	1859407	2633472	3627247	4878832	*127*
128	524288	839808	1874048	2654208	3655808	4917248	*128*
129	528384	846369	1888689	2674944	3684369	4955664	*129*
130	532480	852930	1903330	2695680	3712930	4994080	*130*
131	536576	859491	1917971	2716416	3741491	5032496	*131*
132	540672	866052	1932612	2737152	3770052	5070912	*132*
133	544768	872613	1947253	2757888	3798613	5109328	*133*
134	548864	879174	1961894	2778624	3827174	5147744	*134*
135	552960	885735	1976535	2799360	3855735	5186160	*135*
136	557056	892296	1991176	2820096	3884296	5224576	*136*
137	561152	898857	2005817	2840832	3912857	5262992	*137*
138	565248	905418	2020458	2861568	3941418	5301408	*138*
139	569344	911979	2035099	2882304	3969979	5339824	*139*
140	573440	918540	2049740	2903040	3998540	5378240	*140*
141	577536	925101	2064381	2923776	4027101	5416656	*141*
142	581632	931662	2079022	2944512	4055662	5455072	*142*
143	585728	938223	2093663	2965248	4084223	5493488	*143*
144	589824	944784	2108304	2985984	4112784	5531904	*144*
145	593920	951345	2122945	3006720	4141345	5570320	*145*
146	598016	957906	2137586	3027456	4169906	5608736	*146*
147	602112	964467	2152227	3048192	4198467	5647152	*147*
148	606208	971028	2166868	3068928	4227028	5685568	*148*
149	610304	977589	2181509	3089664	4255589	5723984	*149*
150	614400	984150	2196150	3110400	4284150	5762400	*150*

TABLE L—(*continued*).

Ordinate 2—12. Frequency 151—200.

n	$x=2$	$x=3$	$x=4$	$x=5$	$x=6$	$x=7$	$x=8$	$x=9$	$x=11$	$x=12$	n
151	2416	12231	38656	94375	195696	362551	618496	990711	2210791	3131136	*151*
152	2432	12312	38912	95000	196992	364952	622592	997272	2225432	3151872	*152*
153	2448	12393	39168	95625	198288	367353	626688	1003833	2240073	3172608	*153*
154	2464	12474	39424	96250	199584	369754	630784	1010394	2254714	3193344	*154*
155	2480	12555	39680	96875	200880	372155	634880	1016955	2269355	3214080	*155*
156	2496	12636	39936	97500	202176	374556	638976	1023516	2283996	3234816	*156*
157	2512	12717	40192	98125	203472	376957	643072	1030077	2298637	3255552	*157*
158	2528	12798	40448	98750	204768	379358	647168	1036638	2313278	3276288	*158*
159	2544	12879	40704	99375	206064	381759	651264	1043199	2327919	3297024	*159*
160	2560	12960	40960	100000	207360	384160	655360	1049760	2342560	3317760	*160*
161	2576	13041	41216	100625	208656	386561	659456	1056321	2357201	3338496	*161*
162	2592	13122	41472	101250	209952	388962	663552	1062882	2371842	3359232	*162*
163	2608	13203	41728	101875	211248	391363	667648	1069443	2386483	3379968	*163*
164	2624	13284	41984	102500	212544	393764	671744	1076004	2401124	3400704	*164*
165	2640	13365	42240	103125	213840	396165	675840	1082565	2415765	3421440	*165*
166	2656	13446	42496	103750	215136	398566	679936	1089126	2430406	3442176	*166*
167	2672	13527	42752	104375	216432	400967	684032	1095687	2445047	3462912	*167*
168	2688	13608	43008	105000	217728	403368	688128	1102248	2459688	3483648	*168*
169	2704	13689	43264	105625	219024	405769	692224	1108809	2474329	3504384	*169*
170	2720	13770	43520	106250	220320	408170	696320	1115370	2488970	3525120	*170*
171	2736	13851	43776	106875	221616	410571	700416	1121931	2503611	3545856	*171*
172	2752	13932	44032	107500	222912	412972	704512	1128492	2518252	3566592	*172*
173	2768	14013	44288	108125	224208	415373	708608	1135053	2532893	3587328	*173*
174	2784	14094	44544	108750	225504	417774	712704	1141614	2547534	3608064	*174*
175	2800	14175	44800	109375	226800	420175	716800	1148175	2562175	3628800	*175*
176	2816	14256	45056	110000	228096	422576	720896	1154736	2576816	3649536	*176*
177	2832	14337	45312	110625	229392	424977	724992	1161297	2591457	3670272	*177*
178	2848	14418	45568	111250	230688	427378	729088	1167858	2606098	3691008	*178*
179	2864	14499	45824	111875	231984	429779	733184	1174419	2620739	3711744	*179*
180	2880	14580	46080	112500	233280	432180	737280	1180980	2635380	3732480	*180*
181	2896	14661	46336	113125	234576	434581	741376	1187541	2650021	3753216	*181*
182	2912	14742	46592	113750	235872	436982	745472	1194102	2664662	3773952	*182*
183	2928	14823	46848	114375	237168	439383	749568	1200663	2679303	3794688	*183*
184	2944	14904	47104	115000	238464	441784	753664	1207224	2693944	3815424	*184*
185	2960	14985	47360	115625	239760	444185	757760	1213785	2708585	3836160	*185*
186	2976	15066	47616	116250	241056	446586	761856	1220346	2723226	3856896	*186*
187	2992	15147	47872	116875	242352	448987	765952	1226907	2737867	3877632	*187*
188	3008	15228	48128	117500	243648	451388	770048	1233468	2752508	3898368	*188*
189	3024	15309	48384	118125	244944	453789	774144	1240029	2767149	3919104	*189*
190	3040	15390	48640	118750	246240	456190	778240	1246590	2781790	3939840	*190*
191	3056	15471	48896	119375	247536	458591	782336	1253151	2796431	3960576	*191*
192	3072	15552	49152	120000	248832	460992	786432	1259712	2811072	3981312	*192*
193	3088	15633	49408	120625	250128	463393	790528	1266273	2825713	4002048	*193*
194	3104	15714	49664	121250	251424	465794	794624	1272834	2840354	4022784	*194*
195	3120	15795	49920	121875	252720	468195	798720	1279395	2854995	4043520	*195*
196	3136	15876	50176	122500	254016	470596	802816	1285956	2869636	4064256	*196*
197	3152	15957	50432	123125	255312	472997	806912	1292517	2884277	4084992	*197*
198	3168	16038	50688	123750	256608	475398	811508	1299078	2898918	4105728	*198*
199	3184	16119	50944	128375	257904	477799	815104	1305639	2913559	4126464	*199*
200	3200	16200	51200	125000	259200	480200	819200	1312200	2928200	4147200	*200*

TABLE L—(*continued*).

Ordinate 2—11. Frequency 201—250.

n	$x=2$	$x=3$	$x=4$	$x=5$	$x=6$	$x=7$	$x=8$	$x=9$	$x=11$	n
201	3216	16281	51456	125625	260496	482601	823296	1318761	2942841	*201*
202	3232	16362	51712	126250	261792	485002	827392	1325322	2957482	*202*
203	3248	16443	51968	126875	263088	487403	831488	1331883	2972123	*203*
204	3264	16524	52224	127500	264384	489804	835584	1338444	2986764	*204*
205	3280	16605	52480	128125	265680	492205	839680	1345005	3001405	*205*
206	3296	16686	52736	128750	266976	494606	843776	1351566	3016046	*206*
207	3312	16767	52992	129375	268272	497007	847872	1358127	3030687	*207*
208	3328	16848	53248	130000	269568	499408	851968	1364688	3045328	*208*
209	3344	16929	53504	130625	270864	501809	856064	1371249	3059969	*209*
210	3360	17010	53760	131250	272160	504210	860160	1377810	3074610	*210*
211	3376	17091	54016	131875	273456	506611	864256	1384371	3089251	*211*
212	3392	17172	54272	132500	274752	509012	868352	1390932	3103892	*212*
213	3408	17253	54528	133125	276048	511413	872448	1397493	3118533	*213*
214	3424	17334	54784	133750	277344	513814	876544	1404054	3133174	*214*
215	3440	17415	55040	134375	278640	516215	880640	1410615	3147815	*215*
216	3456	17496	55296	135000	279936	518616	884736	1417176	3162456	*216*
217	3472	17577	55552	135625	281232	521017	888832	1423737	3177097	*217*
218	3488	17658	55808	136250	282528	523418	892928	1430298	3191738	*218*
219	3504	17739	56064	136875	283824	525819	897024	1436859	3206379	*219*
220	3520	17820	56320	137500	285120	528220	901120	1443420	3221020	*220*
221	3536	17901	56576	138125	286416	530621	905216	1449981	3235661	*221*
222	3552	17982	56832	138750	287712	533022	909312	1456542	3250302	*222*
223	3568	18063	57088	139375	289008	535423	913408	1463103	3264943	*223*
224	3584	18144	57344	140000	290304	537824	917504	1469664	3279584	*224*
225	3600	18225	57600	140625	291600	540225	921600	1476225	3294225	*225*
226	3616	18306	57856	141250	292896	542626	925696	1482786	3308866	*226*
227	3632	18387	58112	141875	294192	545027	929792	1489347	3323507	*227*
228	3648	18468	58368	142500	295488	547428	933888	1495908	3338148	*228*
229	3664	18549	58624	143125	296784	549829	937984	1502469	3352789	*229*
230	3680	18630	58880	143750	298080	552230	942080	1509030	3367430	*230*
231	3696	18711	59136	144375	299376	554631	946176	1515591	3382071	*231*
232	3712	18792	59392	145000	300672	557032	950272	1522152	3396712	*232*
233	3728	18873	59648	145625	301968	559433	954368	1528713	3411353	*233*
234	3744	18954	59904	146250	303264	561834	958464	1535274	3425994	*234*
235	3760	19035	60160	146875	304560	564235	962560	1541835	3440635	*235*
236	3776	19116	60416	147500	305856	566636	966656	1548396	3455276	*236*
237	3792	19197	60672	148125	307152	569037	970752	1554957	3469917	*237*
238	3808	19278	60928	148750	308448	571438	974848	1561518	3484558	*238*
239	3824	19359	61184	149375	309744	573839	978944	1568079	3499199	*239*
240	3840	19440	61440	150000	311040	576240	983040	1574640	3513840	*240*
241	3856	19521	61696	150625	312336	578641	987136	1581201	3528481	*241*
242	3872	19602	61952	151250	313632	581042	991232	1587762	3543122	*242*
243	3888	19683	62208	151875	314928	583443	995328	1594323	3557763	*243*
244	3904	19764	62464	152500	316224	585844	999424	1600884	3572404	*244*
245	3920	19845	62720	153125	317520	588245	1003520	1607445	3587045	*245*
246	3936	19926	62976	153750	318816	590646	1007616	1614006	3601686	*246*
247	3952	20007	63232	154375	320112	593047	1011712	1620567	3616327	*247*
248	3968	20088	63488	155000	321408	595448	1015808	1627128	3630968	*248*
249	3984	20169	63744	155625	322704	597849	1019904	1633689	3645609	*249*
250	4000	20250	64000	156250	324000	600250	1024000	1640250	3660250	*250*

TABLE L—(*continued*).

Ordinate 2—9. Frequency 251—300.

n	$x=2$	$x=3$	$x=4$	$x=5$	$x=6$	$x=7$	$x=8$	$x=9$	n
251	4016	20331	64256	156875	325296	602651	1028096	1646811	*251*
252	4032	20412	64512	157500	326592	605052	1032192	1653372	*252*
253	4048	20493	64768	158125	327888	607453	1036288	1659933	*253*
254	4064	20574	65024	158750	329184	609854	1040384	1666494	*254*
255	4080	20655	65280	159375	330480	612255	1044480	1673055	*255*
256	4096	20736	65536	160000	331776	614656	1048576	1679616	*256*
257	4112	20817	65792	160625	333072	617057	1052672	1686177	*257*
258	4128	20898	66048	161250	334368	619458	1056768	1692738	*258*
259	4144	20979	66304	161875	335664	621859	1060864	1699299	*259*
260	4160	21060	66560	162500	336960	624260	1064960	1705860	*260*
261	4176	21141	66816	163125	338256	626661	1069056	1712421	*261*
262	4192	21222	67072	163750	339552	629062	1073152	1718982	*262*
263	4208	21303	67328	164375	340848	631463	1077248	1725543	*263*
264	4224	21384	67584	165000	342144	633864	1081344	1732104	*264*
265	4240	21465	67840	165625	343440	636265	1085440	1738665	*265*
266	4256	21546	68096	166250	344736	638666	1089536	1745226	*266*
267	4272	21627	68352	166875	346032	641067	1093632	1751787	*267*
268	4288	21708	68608	167500	347328	643468	1097728	1758348	*268*
269	4304	21789	68864	168125	348624	645869	1101824	1764909	*269*
270	4320	21870	69120	168750	349920	648270	1105920	1771470	*270*
271	4336	21951	69376	169375	351216	650671	1110016	1778031	*271*
272	4352	22032	69632	170000	352512	653072	1114112	1784592	*272*
273	4368	22113	69888	170625	353808	655473	1118208	1791153	*273*
274	4384	22194	70144	171250	355104	657874	1122304	1797714	*274*
275	4400	22275	70400	171875	356400	660275	1126400	1804275	*275*
276	4416	22356	70656	172500	357696	662676	1130496	1810836	*276*
277	4432	22437	70912	173125	358992	665077	1134592	1817397	*277*
278	4448	22518	71168	173750	360288	667478	1138688	1823958	*278*
279	4464	22599	71424	174375	361584	669879	1142784	1830519	*279*
280	4480	22680	71680	175000	362880	672280	1146880	1837080	*280*
281	4496	22761	71936	175625	364176	674681	1150976	1843641	*281*
282	4512	22842	72192	176250	365472	677082	1155072	1850202	*282*
283	4528	22923	72448	176875	366768	679483	1159168	1856763	*283*
284	4544	23004	72704	177500	368064	681884	1163264	1863324	*284*
285	4560	23085	72960	178125	369360	684285	1167360	1869885	*285*
286	4576	23166	73216	178750	370656	686686	1171456	1876446	*286*
287	4592	23247	73472	179375	371952	689087	1175552	1883007	*287*
288	4608	23328	73728	180000	373248	691488	1179648	1889568	*288*
289	4624	23409	73984	180625	374544	693889	1183744	1896129	*289*
290	4640	23490	74240	181250	375840	696290	1187840	1902690	*290*
291	4656	23571	74496	181875	377136	698691	1191936	1909251	*291*
292	4672	23652	74752	182500	378432	701092	1196032	1915812	*292*
293	4688	23733	75008	183125	379728	703493	1200128	1922373	*293*
294	4704	23814	75264	183750	381024	705894	1204224	1928934	*294*
295	4720	23895	75520	184375	382320	708295	1208320	1935495	*295*
296	4736	23976	75776	185000	383616	710696	1212416	1942056	*296*
297	4752	24057	76032	185625	384912	713097	1216512	1948617	*297*
298	4768	24138	76288	186250	386208	715498	1220608	1955178	*298*
299	4784	24219	76544	186875	387504	717899	1224704	1961739	*299*
300	4800	24300	76800	187500	388800	720300	1228800	1968300	*300*

TABLE L—(*continued*).

Ordinate 2—8. Frequency 301—350.

n	$x=2$	$x=3$	$x=4$	$x=5$	$x=6$	$x=7$	$x=8$	n
301	4816	24381	77056	188125	390096	722701	1232896	*301*
302	4832	24462	77312	188750	391392	725102	1236992	*302*
303	4848	24543	77568	189375	392688	727503	1241088	*303*
304	4864	24624	77824	190000	393984	729904	1245184	*304*
305	4880	24705	78080	190625	395280	732305	1249280	*305*
306	4896	24786	78336	191250	396576	734706	1253376	*306*
307	4912	24867	78592	191875	397872	737107	1257472	*307*
308	4928	24948	78848	192500	399168	739508	1261568	*308*
309	4944	25029	79104	193125	400464	741909	1265664	*309*
310	4960	25110	79360	193750	401760	744310	1269760	*310*
311	4976	25191	79616	194375	403056	746711	1273856	*311*
312	4992	25272	79872	195000	404352	749112	1277952	*312*
313	5008	25353	80128	195625	405648	751513	1282048	*313*
314	5024	25434	80384	196250	406944	753914	1286144	*314*
315	5040	25515	80640	196875	408240	756315	1290240	*315*
316	5056	25596	80896	197500	409536	758716	1294336	*316*
317	5072	25677	81152	198125	410832	761117	1298432	*317*
318	5088	25758	81408	198750	412128	763518	1302528	*318*
319	5104	25839	81664	199375	413424	765919	1306624	*319*
320	5120	25920	81920	200000	414720	768320	1310720	*320*
321	5136	26001	82176	200625	416016	770721	1314816	*321*
322	5152	26082	82432	201250	417312	773122	1318912	*322*
323	5168	26163	82688	201875	418608	775523	1323008	*323*
324	5184	26244	82944	202500	419904	777924	1327104	*324*
325	5200	26325	83200	203125	421200	780325	1331200	*325*
326	5216	26406	83456	203750	422496	782726	1335296	*326*
327	5232	26487	83712	204375	423792	785127	1339392	*327*
328	5248	26568	83968	205000	425088	787528	1343488	*328*
329	5264	26649	84224	205625	426384	789929	1347584	*329*
330	5280	26730	84480	206250	427680	792330	1351680	*330*
331	5296	26811	84736	206875	428976	794731	1355776	*331*
332	5312	26892	84992	207500	430272	797132	1359872	*332*
333	5328	26973	85248	208125	431568	799533	1363968	*333*
334	5344	27054	85504	208750	432864	801934	1368064	*334*
335	5360	27135	85760	209375	434160	804335	1372160	*335*
336	5376	27216	86016	210000	435456	806736	1376256	*336*
337	5392	27297	86272	210625	436752	809137	1380352	*337*
338	5408	27378	86528	211250	438048	811538	1384448	*338*
339	5424	27459	86784	211875	439344	813939	1388544	*339*
340	5440	27540	87040	212500	440640	816340	1392640	*340*
341	5456	27621	87296	213125	441936	818741	1396736	*341*
342	5472	27702	87552	213750	443232	821142	1400832	*342*
343	5488	27783	87808	214375	444528	823543	1404928	*343*
344	5504	27864	88064	215000	445824	825944	1409024	*344*
345	5520	27945	88320	215625	447120	828345	1413120	*345*
346	5536	28026	88576	216250	448416	830746	1417216	*346*
347	5552	28107	88832	216875	449712	833147	1421312	*347*
348	5568	28188	89088	217500	451008	835548	1425408	*348*
349	5584	28269	89344	218125	452304	837949	1429504	*349*
350	5600	28350	89600	218750	453600	840350	1433600	*350*

TABLE L—(*continued*).

Ordinate 2—7. Frequency 351—400.

n	$x=2$	$x=3$	$x=4$	$x=5$	$x=6$	$x=7$	n
351	5616	28431	89856	219375	454896	842751	*351*
352	5632	28512	90112	220000	456192	845152	*352*
353	5648	28593	90368	220625	457488	847553	*353*
354	5664	28674	90624	221250	458784	849954	*354*
355	5680	28755	90880	221875	460080	852355	*355*
356	5696	28836	91136	222500	461376	854756	*356*
357	5712	28917	91392	223125	462672	857157	*357*
358	5728	28998	91648	223750	463968	859558	*358*
359	5744	29079	91904	224375	465264	861959	*359*
360	5760	29160	92160	225000	466560	864360	*360*
361	5776	29241	92416	225625	467856	866761	*361*
362	5792	29322	92672	226250	469152	869162	*362*
363	5808	29403	92928	226875	470448	871563	*363*
364	5824	29484	93184	227500	471744	873964	*364*
365	5840	29565	93440	228125	473040	876365	*365*
366	5856	29646	93696	228750	474336	878766	*366*
367	5872	29727	93952	229375	475632	881167	*367*
368	5888	29808	94208	230000	476928	883568	*368*
369	5904	29889	94464	230625	478224	885969	*369*
370	5920	29970	94720	231250	479520	888370	*370*
371	5936	30051	94976	231875	480816	890771	*371*
372	5952	30132	95232	232500	482112	893172	*372*
373	5968	30213	95488	233125	483408	895573	*373*
374	5984	30294	95744	233750	484704	897974	*374*
375	6000	30375	96000	234375	486000	900375	*375*
376	6016	30456	96256	235000	487296	902776	*376*
377	6032	30537	96512	235625	488592	905177	*377*
378	6048	30618	96768	236250	489888	907578	*378*
379	6064	30699	97024	236875	491184	909979	*379*
380	6080	30780	97280	237500	492480	912380	*380*
381	6096	30861	97536	238125	493776	914781	*381*
382	6112	30942	97792	238750	495072	917182	*382*
383	6128	31023	98048	239375	496368	919583	*383*
384	6144	31104	98304	240000	497664	921984	*384*
385	6160	31185	98560	240625	498960	924385	*385*
386	6176	31266	98816	241250	500256	926786	*386*
387	6192	31347	99072	241875	501552	929187	*387*
388	6208	31428	99328	242500	502848	931588	*388*
389	6224	31509	99584	243125	504144	933989	*389*
390	6240	31590	99840	243750	505440	936390	*390*
391	6256	31671	100096	244375	506736	938791	*391*
392	6272	31752	100352	245000	508032	941192	*392*
393	6288	31833	100608	245625	509328	943593	*393*
394	6304	31914	100864	246250	510624	945994	*394*
395	6320	31995	101120	246875	511920	948395	*395*
396	6336	32076	101376	247500	513216	950796	*396*
397	6352	32157	101632	248125	514512	953197	*397*
398	6368	32238	101888	248750	515808	955598	*398*
399	6384	32319	102144	249375	517104	957999	*399*
400	6400	32400	102400	250000	518400	960400	*400*

TABLE LI. *Tables of $e^{-m}m^x/x!$: General Term of Poisson's Exponential Expansion ("Law of Small Numbers").*

x	m										x
	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0	
0	·904837	·818731	·740818	·670320	·606531	·548812	·496585	·449329	·406570	·367879	0
1	·090484	·163746	·222245	·268128	·303265	·329287	·347610	·359463	·365913	·367879	1
2	·004524	·016375	·033337	·053626	·075816	·098786	·121663	·143785	·164661	·183940	2
3	·000151	·001092	·003334	·007150	·012636	·019757	·028388	·038343	·049398	·061313	3
4	·000004	·000055	·000250	·000715	·001580	·002964	·004968	·007669	·011115	·015328	4
5	—	·000002	·000015	·000057	·000158	·000356	·000696	·001227	·002001	·003066	5
6	—	—	·000001	·000004	·000013	·000036	·000081	·000164	·000300	·000511	6
7	—	—	—	—	·000001	·000003	·000008	·000019	·000039	·000073	7
8	—	—	—	—	—	—	·000001	·000002	·000004	·000009	8
9	—	—	—	—	—	—	—	—	—	·000001	9
x	1·1	1·2	1·3	1·4	1·5	1·6	1·7	1·8	1·9	2·0	x
0	·332871	·301194	·272532	·246597	·223130	.201897	·182684	·165299	·149569	·135335	0
1	·366158	·361433	·354291	·345236	·334695	.323034	·310562	·297538	·284180	·270671	1
2	·201387	·216860	·230289	·241665	·251021	.258428	·263978	·267784	·269971	·270671	2
3	·073842	·086744	·099792	·112777	·125510	·137828	·149587	·160671	·170982	·180447	3
4	·020307	·026023	·032432	·039472	·047067	·055131	·063575	·072302	·081216	·090224	4
5	·004467	·006246	·008432	·011052	·014120	·017642	·021615	·026029	·030862	·036089	5
6	·000819	·001249	·001827	·002579	·003530	·004705	·006124	·007809	·009773	·012030	6
7	·000129	·000214	·000339	·000516	·000756	·001075	·001487	·002008	·002653	·003437	7
8	·000018	·000032	·000055	·000090	·000142	·000215	·000316	·000452	·000630	·000859	8
9	·000002	·000004	·000008	·000014	·000024	·000038	·000060	·000090	·000133	·000191	9
10	—	·000001	·000001	·000002	·000004	·000006	·000010	·000016	·000025	·000038	10
11	—	—	—	—	—	·000001	·000002	·000003	·000004	·000007	11
12	—	—	—	—	—	—	—	—	·000001	·000001	12
x	2·1	2·2	2·3	2·4	2·5	2·6	2·7	2·8	2·9	3·0	x
0	·122456	·110803	·100259	·090718	·082085	·074274	·067206	·060810	·055023	·049787	0
1	·257159	·243767	·230595	·217723	·205212	·193111	·181455	·170268	·159567	·149361	1
2	·270016	·268144	·265185	·261268	·256516	·251045	·244964	·238375	·231373	·224042	2
3	·189012	·196639	·203308	·209014	·213763	·217572	·220468	·222484	·223660	·224042	3
4	·099231	·108151	·116902	·125409	·133602	·141422	·148816	·155739	·162154	·168031	4
5	·041677	·047587	·053775	·060196	·066801	·073539	·080360	·087214	·094049	·100819	5
6	·014587	·017448	·020614	·024078	·027834	·031867	·036162	·040700	·045457	·050409	6
7	·004376	·005484	·006773	·008255	·009941	·011836	·013948	·016280	·018832	·021604	7
8	·001149	·001508	·001947	·002477	·003106	·003847	·004708	·005698	·006827	·008102	8
9	·000268	·000369	·000498	·000660	·000863	·001111	·001412	·001773	·002200	·002701	9
10	·000056	·000081	·000114	·000158	·000216	·000289	·000381	·000496	·000638	·000810	10
11	·000011	·000016	·000024	·000035	·000049	·000068	·000094	·000126	·000168	·000221	11
12	·000002	·000003	·000005	·000007	·000010	·000015	·000021	·000029	·000041	·000055	12
13	—	·000001	·000001	·000001	·000002	·000003	·000004	·000006	·000009	·000013	13
14	—	—	—	—	—	·000001	·000001	·000001	·000002	·000003	14
15	—	—	—	—	—	—	—	—	—	·000001	15

TABLE LI—(*continued*).

x	m										x
	3·1	3·2	3·3	3·4	3·5	3·6	3·7	3·8	3·9	4·0	
0	·045049	·040762	·036883	·033373	·030197	·027324	·024724	·022371	·020242	·018316	0
1	·139653	·130439	·121714	·113469	·105691	·098365	·091477	·085009	·078943	·073263	1
2	·216461	·208702	·200829	·192898	·184959	·177058	·169233	·161517	·153940	·146525	2
3	·223677	·222616	·220912	·218617	·215785	·212469	·208720	·204588	·200122	·195367	3
4	·173350	·178093	·182252	·185825	·188812	·191222	·193066	·194359	·195119	·195367	4
5	·107477	·113979	·120286	·126361	·132169	·137680	·142869	·147713	·152193	·156293	5
6	·055530	·060789	·066158	·071604	·077098	·082608	·088102	·093551	·098925	·104196	6
7	·024592	·027789	·031189	·034779	·038549	·042484	·046568	·050785	·055115	·059540	7
8	·009529	·011116	·012865	·014781	·016865	·019118	·021538	·024123	·026869	·029770	8
9	·003282	·003952	·004717	·005584	·006559	·007647	·008854	·010185	·011643	·013231	9
10	·001018	·001265	·001557	·001899	·002296	·002753	·003276	·003870	·004541	·005292	10
11	·000287	·000368	·000467	·000587	·000730	·000901	·001102	·001337	·001610	·001925	11
12	·000074	·000098	·000128	·000166	·000213	·000270	·000340	·000423	·000523	·000642	12
13	·000018	·000024	·000033	·000043	·000057	·000075	·000097	·000124	·000157	·000197	13
14	·000004	·000006	·000008	·000011	·000014	·000019	·000026	·000034	·000044	·000056	14
15	·000001	·000001	·000002	·000002	·000003	·000005	·000006	·000009	·000011	·000015	15
16	—	—	—	·000001	·000001	·000001	·000001	·000002	·000003	·000004	16
17	—	—	—	—	—	—	—	—	·000001	·000001	17

x	4·1	4·2	4·3	4·4	4·5	4·6	4·7	4·8	4·9	5·0	x
0	·016573	·014996	·013569	·012277	·011109	·010052	·009095	·008230	·007447	·006738	0
1	·067948	·062981	·058345	·054020	·049990	·046238	·042748	·039503	·036488	·033690	1
2	·139293	·132261	·125441	·118845	·112479	·106348	·100457	·094807	·089396	·084224	2
3	·190368	·185165	·179799	·174305	·168718	·163068	·157383	·151691	·146014	·140374	3
4	·195127	·194424	·193284	·191736	·189808	·187528	·184925	·182029	·178867	·175467	4
5	·160004	·163316	·166224	·168728	·170827	·172525	·173830	·174748	·175290	·175467	5
6	·109336	·114321	·119127	·123734	·128120	·132270	·136167	·139798	·143153	·146223	6
7	·064040	·068593	·073178	·077775	·082363	·086920	·091426	·095862	·100207	·104445	7
8	·032820	·036011	·039333	·042776	·046329	·049979	·053713	·057517	·061377	·065278	8
9	·014951	·016805	·018793	·020913	·023165	·025545	·028050	·030676	·033416	·036266	9
10	·006130	·007058	·008081	·009202	·010424	·011751	·013184	·014724	·016374	·018133	10
11	·002285	·002695	·003159	·003681	·004264	·004914	·005633	·006425	·007294	·008242	11
12	·000781	·000943	·001132	·001350	·001599	·001884	·002206	·002570	·002978	·003434	12
13	·000246	·000305	·000374	·000457	·000554	·000667	·000798	·000949	·001123	·001321	13
14	·000072	·000091	·000115	·000144	·000178	·000219	·000268	·000325	·000393	·000472	14
15	·000020	·000026	·000033	·000042	·000053	·000067	·000084	·000104	·000128	·000157	15
16	·000005	·000007	·000009	·000012	·000015	·000019	·000025	·000031	·000039	·000049	16
17	·000001	·000002	·000002	·000003	·000004	·000005	·000007	·000009	·000011	·000014	17
18	—	—	·000001	·000001	·000001	·000001	·000002	·000002	·000003	·000004	18
19	—	—	—	—	—	—	—	·000001	·000001	·000001	19

x	5·1	5·2	5·3	5·4	5·5	5·6	5·7	5·8	5·9	6·0	x
0	·006097	·005517	·004992	·004517	·004087	·003698	·003346	·003028	·002739	·002479	0
1	·031093	·028686	·026455	·024390	·022477	·020708	·019072	·017560	·016163	·014873	1
2	·079288	·074584	·070107	·065852	·061812	·057982	·054355	·050923	·047680	·044618	2
3	·134790	·129279	·123856	·118533	·113323	·108234	·103275	·098452	·093771	·089235	3

TABLE LI—(*continued*).

x	m										x
	5·1	5·2	5·3	5·4	5·5	5·6	5·7	5·8	5·9	6·0	
4	·171857	·168063	·164109	·160020	·155819	·151528	·147167	·142755	·138312	·133853	4
5	·175294	·174785	·173955	·172821	·171401	·169711	·167770	·165596	·163208	·160623	5
6	·149000	·151480	·153660	·155539	·157117	·158397	·159382	·160076	·160488	·160623	6
7	·108557	·112528	·116343	·119987	·123449	·126717	·129782	·132635	·135268	·137677	7
8	·069205	·073143	·077077	·080991	·084871	·088702	·092470	·096160	·099760	·103258	8
9	·039216	·042261	·045390	·048595	·051866	·055192	·058564	·061970	·065398	·068838	9
10	·020000	·021976	·024057	·026241	·028526	·030908	·033382	·035943	·038585	·041303	10
11	·009273	·010388	·011591	·012882	·014263	·015735	·017298	·018952	·020696	·022529	11
12	·003941	·004502	·005119	·005797	·006537	·007343	·008216	·009160	·010175	·011264	12
13	·001546	·001801	·002087	·002408	·002766	·003163	·003603	·004087	·004618	·005199	13
14	·000563	·000669	·000790	·000929	·001087	·001265	·001467	·001693	·001946	·002228	14
15	·000191	·000232	·000279	·000334	·000398	·000472	·000557	·000655	·000766	·000891	15
16	·000061	·000075	·000092	·000113	·000137	·000165	·000199	·000237	·000282	·000334	16
17	·000018	·000023	·000029	·000036	·000044	·000054	·000067	·000081	·000098	·000118	17
18	·000005	·000007	·000008	·000011	·000014	·000017	·000021	·000026	·000032	·000039	18
19	·000001	·000002	·000002	·000003	·000004	·000005	·000006	·000008	·000010	·000012	19
20	—	—	·000001	·000001	·000001	·000001	·000002	·000002	·000003	·000004	20
21	—	—	—	—	—	—	—	·000001	·000001	·000001	21
x	6·1	6·2	6·3	6·4	6·5	6·6	6·7	6·8	6·9	7·0	x
0	·002243	·002029	·001836	·001662	·001503	·001360	·001231	·001114	·001008	·000912	0
1	·013682	·012582	·011569	·010634	·009772	·008978	·008247	·007574	·006954	·006383	1
2	·041729	·039006	·036441	·034029	·031760	·029629	·027628	·025751	·023990	·022341	2
3	·084848	·080612	·076527	·072595	·068814	·065183	·061702	·058368	·055178	·052129	3
4	·129393	·124948	·120530	·116151	·111822	·107553	·103351	·099225	·095182	·091226	4
5	·157860	·154936	·151868	·148674	·145369	·141969	·138490	·134946	·131351	·127717	5
6	·160491	·160100	·159461	·158585	·157483	·156166	·154648	·152939	·151053	·149003	6
7	·139856	·141803	·143515	·144992	·146234	·147243	·148020	·148569	·148895	·149003	7
8	·106640	·109897	·113018	·115994	·118815	·121475	·123967	·126284	·128422	·130377	8
9	·072278	·075707	·079113	·082484	·085811	·089082	·092286	·095415	·098457	·101405	9
10	·044090	·046938	·049841	·052790	·055777	·058794	·061832	·064882	·067935	·070983	10
11	·024450	·026456	·028545	·030714	·032959	·035276	·037661	·040109	·042614	·045171	11
12	·012429	·013669	·014986	·016381	·017853	·019402	·021028	·022728	·024503	·026350	12
13	·005832	·006519	·007263	·008064	·008926	·009850	·010837	·011889	·013005	·014188	13
14	·002541	·002887	·003268	·003687	·004144	·004644	·005186	·005774	·006410	·007094	14
15	·001033	·001193	·001373	·001573	·001796	·002043	·002317	·002618	·002949	·003311	15
16	·000394	·000462	·000540	·000629	·000730	·000843	·000970	·001113	·001272	·001448	16
17	·000141	·000169	·000200	·000237	·000279	·000327	·000382	·000445	·000516	·000596	17
18	·000048	·000058	·000070	·000084	·000101	·000120	·000142	·000168	·000198	·000232	18
19	·000015	·000019	·000023	·000028	·000034	·000042	·000050	·000060	·000072	·000085	19
20	·000005	·000006	·000007	·000009	·000011	·000014	·000017	·000020	·000025	·000030	20
21	·000001	·000002	·000002	·000003	·000003	·000004	·000005	·000007	·000008	·000010	21
22	—	—	·000001	·000001	·000001	·000001	·000002	·000002	·000003	·000003	22
23	—	—	—	—	—	—	—	·000001	·000001	·000001	23

TABLE LI—(*continued*).

x	m										x
	7·1	7·2	7·3	7·4	7·5	7·6	7·7	7·8	7·9	8·0	
0	·000825	·000747	·000676	·000611	·000553	·000500	·000453	·000410	·000371	·000335	0
1	·005858	·005375	·004931	·004523	·004148	·003803	·003487	·003196	·002929	·002684	1
2	·020797	·019352	·018000	·016736	·015555	·014453	·013424	·012464	·011569	·010735	2
3	·049219	·046444	·043799	·041282	·038889	·036614	·034455	·032407	·030465	·028626	3
4	·087364	·083598	·079934	·076372	·072916	·069567	·066326	·063193	·060169	·057252	4
5	·124057	·120382	·116703	·113031	·109375	·105742	·102142	·098581	·095067	·091604	5
6	·146800	·144458	·141989	·139405	·136718	·133940	·131082	·128156	·125171	·122138	6
7	·148897	·148586	·148074	·147371	·146484	·145421	·144191	·142802	·141264	·139587	7
8	·132146	·133727	·135118	·136318	·137329	·138150	·138783	·139232	·139499	·139587	8
9	·104249	·106982	·109596	·112084	·114440	·116660	·118737	·120668	·122449	·124077	9
10	·074017	·077027	·080005	·082942	·085830	·088661	·091427	·094121	·096735	·099262	10
11	·047774	·050418	·053094	·055797	·058521	·061257	·063999	·066740	·069473	·072190	11
12	·028267	·030251	·032299	·034408	·036575	·038796	·041066	·043381	·045736	·048127	12
13	·015438	·016754	·018137	·019586	·021101	·022681	·024324	·026029	·027794	·029616	13
14	·007829	·008616	·009457	·010353	·011304	·012312	·013378	·014502	·015684	·016924	14
15	·003706	·004136	·004603	·005107	·005652	·006238	·006867	·007541	·008260	·009026	15
16	·001644	·001861	·002100	·002362	·002649	·002963	·003305	·003676	·004078	·004513	16
17	·000687	·000788	·000902	·001028	·001169	·001325	·001497	·001687	·001895	·002124	17
18	·000271	·000315	·000366	·000423	·000487	·000559	·000640	·000731	·000832	·000944	18
19	·000101	·000119	·000141	·000165	·000192	·000224	·000259	·000300	·000346	·000397	19
20	·000036	·000043	·000051	·000061	·000072	·000085	·000100	·000117	·000137	·000159	20
21	·000012	·000015	·000018	·000021	·000026	·000031	·000037	·000043	·000051	·000061	21
22	·000004	·000005	·000006	·000007	·000009	·000011	·000013	·000015	·000018	·000022	22
23	·000001	·000002	·000002	·000002	·000003	·000004	·000004	·000005	·000006	·000008	23
24	—	—	·000001	·000001	·000001	·000001	·000001	·000002	·000002	·000003	24
25	—	—	—	—	—	—	—	·000001	·000001	·000001	25
x	8·1	8·2	8·3	8·4	8·5	8·6	8·7	8·8	8·9	9·0	x
0	·000304	·000275	·000249	·000225	·000203	·000184	·000167	·000151	·000136	·000123	0
1	·002459	·002252	·002063	·001889	·001729	·001583	·001449	·001326	·001214	·001111	1
2	·009958	·009234	·008560	·007933	·007350	·006808	·006304	·005836	·005402	·004998	2
3	·026885	·025239	·023683	·022213	·020826	·019517	·018283	·017120	·016025	·014994	3
4	·054443	·051740	·049142	·046648	·044255	·041961	·039765	·037664	·035656	·033737	4
5	·088198	·084854	·081576	·078368	·075233	·072174	·069192	·066289	·063467	·060727	5
6	·119067	·115967	·112847	·109716	·106581	·103449	·100328	·097224	·094143	·091090	6
7	·137778	·135848	·133805	·131659	·129419	·127094	·124693	·122224	·119696	·117116	7
8	·139500	·139244	·138823	·138242	·137508	·136626	·135604	·134446	·133161	·131756	8
9	·125550	·126866	·128025	·129026	·129869	·130554	·131084	·131459	·131682	·131756	9
10	·101696	·104031	·106261	·108382	·110388	·112277	·114043	·115684	·117197	·118580	10
11	·074885	·077550	·080179	·082764	·085300	·087780	·090197	·092547	·094823	·097020	11
12	·050547	·052993	·055457	·057935	·060421	·062909	·065393	·067868	·070327	·072765	12
13	·031495	·033426	·035407	·037435	·039506	·041617	·043763	·045941	·048147	·050376	13
14	·018222	·019578	·020991	·022461	·023986	·025565	·027196	·028877	·030608	·032384	14
15	·009840	·010703	·011615	·012578	·013592	·014657	·015773	·016941	·018161	·019431	15
16	·004981	·005485	·006025	·006604	·007221	·007878	·008577	·009318	·010102	·010930	16
17	·002373	·002646	·002942	·003263	·003610	·003985	·004389	·004823	·005289	·005786	17
18	·001068	·001205	·001356	·001523	·001705	·001904	·002121	·002358	·002615	·002893	18
19	·000455	·000520	·000593	·000673	·000763	·000862	·000971	·001092	·001225	·001370	19
20	·000184	·000213	·000246	·000283	·000324	·000371	·000423	·000481	·000545	·000617	20

TABLE LI—(*continued*).

x	m										x
	8·1	8·2	8·3	8·4	8·5	8·6	8·7	8·8	8·9	9·0	
21	·000071	·000083	·000097	·000113	·000131	·000152	·000175	·000201	·000231	·000264	21
22	·000026	·000031	·000037	·000043	·000051	·000059	·000069	·000081	·000093	·000108	22
23	·000009	·000011	·000013	·000016	·000019	·000022	·000026	·000031	·000036	·000042	23
24	·000003	·000004	·000005	·000006	·000007	·000008	·000009	·000011	·000013	·000016	24
25	·000001	·000001	·000002	·000002	·000002	·000003	·000003	·000004	·000005	·000006	25
26	—	—	—	·000001	·000001	·000001	·000001	·000001	·000002	·000002	26
27	—	—	—	—	—	—	—	—	·000001	·000001	27
x	9·1	9·2	9·3	9·4	9·5	9·6	9·7	9·8	9·9	10·0	x
0	·000112	·000101	·000091	·000083	·000075	·000068	·000061	·000055	·000050	·000045	0
1	·001016	·000930	·000850	·000778	·000711	·000650	·000594	·000543	·000497	·000454	1
2	·004624	·004276	·003954	·003655	·003378	·003121	·002883	·002663	·002459	·002270	2
3	·014025	·013113	·012256	·011452	·010696	·009987	·009322	·008698	·008114	·007567	3
4	·031906	·030160	·028496	·026911	·025403	·023969	·022606	·021311	·020082	·018917	4
5	·058069	·055494	·053002	·050593	·048266	·046020	·043855	·041770	·039763	·037833	5
6	·088072	·085091	·082154	·079262	·076421	·073632	·070899	·068224	·065609	·063055	6
7	·114493	·111834	·109147	·106438	·103714	·100981	·098246	·095514	·092790	·090079	7
8	·130236	·128609	·126883	·125065	·123160	·121178	·119123	·117004	·114827	·112599	8
9	·131683	·131467	·131113	·130623	·130003	·129256	·128388	·127405	·126310	·125110	9
10	·119832	·120950	·121935	·122786	·123502	·124086	·124537	·124857	·125047	·125110	10
11	·099133	·101158	·103090	·104926	·106661	·108293	·109819	·111236	·112542	·113736	11
12	·075176	·077555	·079895	·082192	·084440	·086634	·088770	·090843	·092847	·094780	12
13	·052623	·054885	·057156	·059431	·061706	·063976	·066236	·068481	·070707	·072908	13
14	·034205	·036067	·037968	·039904	·041872	·043869	·045892	·047937	·050000	·052077	14
15	·020751	·022121	·023540	·025006	·026519	·028076	·029677	·031319	·033000	·034718	15
16	·011802	·012720	·013683	·014691	·015743	·016846	·017992	·019183	·020419	·021699	16
17	·006318	·006884	·007485	·008123	·008799	·009513	·010266	·011058	·011891	·012764	17
18	·003194	·003518	·003867	·004242	·004644	·005074	·005532	·006021	·006540	·007091	18
19	·001530	·001704	·001893	·002099	·002322	·002563	·002824	·003105	·003408	·003732	19
20	·000696	·000784	·000880	·000986	·001103	·001230	·001370	·001522	·001687	·001866	20
21	·000302	·000343	·000390	·000442	·000499	·000563	·000633	·000710	·000795	·000889	21
22	·000125	·000144	·000165	·000189	·000215	·000245	·000279	·000316	·000358	·000404	22
23	·000049	·000057	·000067	·000077	·000089	·000102	·000118	·000135	·000154	·000176	23
24	·000019	·000022	·000026	·000030	·000035	·000041	·000048	·000055	·000064	·000073	24
25	·000007	·000008	·000010	·000011	·000013	·000016	·000018	·000022	·000025	·000029	25
26	·000002	·000003	·000003	·000004	·000005	·000006	·000007	·000008	·000010	·000011	26
27	·000001	·000001	·000001	·000001	·000002	·000002	·000002	·000003	·000004	·000004	27
28	—	—	—	—	·000001	·000001	·000001	·000001	·000001	·000001	28
29	—	—	—	—	—	—	—	—	—	·000001	29
x	10·1	10·2	10·3	10·4	10·5	10·6	10·7	10·8	10·9	11·0	x
0	·000041	·000037	·000034	·000030	·000028	·000025	·000023	·000020	·000018	·000017	0
1	·000415	·000379	·000346	·000317	·000289	·000264	·000241	·000220	·000201	·000184	1
2	·002095	·001934	·001784	·001646	·001518	·001400	·001291	·001190	·001097	·001010	2
3	·007054	·006574	·006125	·005705	·005313	·004946	·004603	·004283	·003984	·003705	3

TABLE LI—(*continued*).

x	m										x
	10·1	10·2	10·3	10·4	10·5	10·6	10·7	10·8	10·9	11·0	
4	·017811	·016764	·015773	·014834	·013946	·013107	·012313	·011564	·010856	·010189	4
5	·035979	·034199	·032492	·030855	·029287	·027786	·026350	·024978	·023667	·022415	5
6	·060565	·058139	·055777	·053482	·051252	·049089	·046991	·044960	·042995	·041095	6
7	·087387	·084716	·082072	·079458	·076878	·074334	·071830	·069367	·066949	·064577	7
8	·110326	·108013	·105668	·103296	·100902	·098493	·096072	·093646	·091218	·088794	8
9	·123810	·122415	·120931	·119364	·117720	·116003	·114219	·112375	·110475	·108526	9
10	·125048	·124863	·124559	·124139	·123606	·122963	·122215	·121365	·120418	·119378	10
11	·114817	·115782	·116633	·117368	·117987	·118492	·118882	·119159	·119323	·119378	11
12	·096637	·098415	·100110	·101719	·103239	·104667	·106003	·107243	·108386	·109430	12
13	·075080	·077218	·079318	·081375	·083385	·085344	·087248	·089094	·090877	·092595	13
14	·054165	·056259	·058355	·060450	·062539	·064618	·066683	·068730	·070754	·072753	14
15	·036471	·038256	·040071	·041912	·043777	·045663	·047567	·049485	·051415	·053352	15
16	·023022	·024388	·025795	·027243	·028729	·030252	·031810	·033403	·035026	·036680	16
17	·013678	·014633	·015629	·016666	·017744	·018863	·020022	·021220	·022458	·023734	17
18	·007675	·008292	·008943	·009629	·010351	·011108	·011902	·012732	·013600	·014504	18
19	·004080	·004451	·004848	·005271	·005720	·006197	·006703	·007237	·007802	·008397	19
20	·002060	·002270	·002497	·002741	·003003	·003285	·003586	·003908	·004252	·004618	20
21	·000991	·001103	·001225	·001357	·001502	·001658	·001827	·002010	·002207	·002419	21
22	·000455	·000511	·000573	·000642	·000717	·000799	·000889	·000987	·001093	·001210	22
23	·000200	·000227	·000257	·000290	·000327	·000368	·000413	·000463	·000518	·000578	23
24	·000084	·000096	·000110	·000126	·000143	·000163	·000184	·000208	·000235	·000265	24
25	·000034	·000039	·000045	·000052	·000060	·000069	·000079	·000090	·000103	·000117	25
26	·000013	·000015	·000018	·000021	·000024	·000028	·000032	·000037	·000043	·000049	26
27	·000005	·000006	·000007	·000008	·000009	·000011	·000013	·000015	·000017	·000020	27
28	·000002	·000002	·000003	·000003	·000004	·000004	·000005	·000006	·000007	·000008	28
29	·000001	·000001	·000001	·000001	·000001	·000002	·000002	·000002	·000003	·000003	29
30	—	—	—	—	—	·000001	·000001	·000001	·000001	·000001	30
x	11·1	11·2	11·3	11·4	11·5	11·6	11·7	11·8	11·9	12·0	x
0	·000015	·000014	·000012	·000011	·000010	·000009	·000008	·000008	·000007	·000006	0
1	·000168	·000153	·000140	·000128	·000116	·000106	·000097	·000089	·000081	·000074	1
2	·000931	·000858	·000790	·000727	·000670	·000617	·000568	·000522	·000481	·000442	2
3	·003445	·003202	·002976	·002764	·002568	·002385	·002214	·002055	·001907	·001770	3
4	·009559	·008965	·008406	·007879	·007382	·006915	·006476	·006062	·005674	·005309	4
5	·021221	·020082	·018997	·017963	·016979	·016043	·015153	·014307	·013504	·012741	5
6	·039259	·037487	·035778	·034130	·032544	·031017	·029549	·028137	·026782	·025481	6
7	·062253	·059979	·057755	·055584	·053465	·051400	·049388	·047432	·045530	·043682	7
8	·086376	·083970	·081579	·079206	·076856	·074529	·072231	·069962	·067725	·065523	8
9	·106531	·104496	·102427	·100328	·098204	·096060	·093900	·091728	·089548	·087364	9
10	·118249	·117036	·115743	·114374	·112935	·111430	·109863	·108239	·106562	·104837	10
11	·119324	·119164	·118899	·118533	·118068	·117508	·116854	·116110	·115281	·114368	11
12	·110375	·111220	·111964	·112607	·113149	·113591	·113933	·114175	·114320	·114363	12
13	·094243	·095820	·097322	·098747	·100093	·101358	·102539	·103636	·104647	·105570	13
14	·074721	·076656	·078553	·080409	·082219	·083982	·085694	·087350	·088950	·090489	14
15	·055294	·057236	·059177	·061110	·063035	·064946	·066841	·068716	·070567	·072391	15
16	·038360	·040065	·041793	·043541	·045306	·047086	·048877	·050678	·052484	·054293	16
17	·025047	·026396	·027780	·029198	·030648	·032129	·033639	·035176	·036739	·038325	17
18	·015446	·016424	·017440	·018492	·019581	·020706	·021865	·023060	·024288	·025550	18
19	·009023	·009682	·010372	·011095	·011852	·012641	·013465	·014322	·015212	·016137	19

TABLE LI—(*continued*).

x	m										x
	11·1	11·2	11·3	11·4	11·5	11·6	11·7	11·8	11·9	12·0	
20	·005008	·005422	·005860	·006324	·006815	·007332	·007877	·008450	·009051	·009682	20
21	·002647	·002892	·003153	·003433	·003732	·004050	·004388	·004748	·005129	·005533	21
22	·001336	·001472	·001620	·001779	·001951	·002136	·002334	·002547	·002774	·003018	22
23	·000645	·000717	·000796	·000882	·000975	·001077	·001187	·001307	·001435	·001575	23
24	·000298	·000335	·000375	·000419	·000467	·000521	·000579	·000642	·000712	·000787	24
25	·000132	·000150	·000169	·000191	·000215	·000242	·000271	·000303	·000339	·000378	25
26	·000057	·000065	·000074	·000084	·000095	·000108	·000122	·000138	·000155	·000174	26
27	·000023	·000027	·000031	·000035	·000041	·000046	·000053	·000060	·000068	·000078	27
28	·000009	·000011	·000012	·000014	·000017	·000019	·000022	·000025	·000029	·000033	28
29	·000004	·000004	·000005	·000006	·000007	·000008	·000009	·000010	·000012	·000014	29
30	·000001	·000002	·000002	·000002	·000003	·000003	·000003	·000004	·000005	·000005	30
31	—	·000001	·000001	·000001	·000001	·000001	·000001	·000002	·000002	·000002	31
32	—	—	—	—	—	—	—	·000001	·000001	·000001	32
x	12·1	12·2	12·3	12·4	12·5	12·6	12·7	12·8	12·9	13·0	x
0	·000006	·000005	·000005	·000004	·000004	·000003	·000003	·000003	·000002	·000002	0
1	·000067	·000061	·000056	·000051	·000047	·000042	·000039	·000035	·000032	·000029	1
2	·000407	·000374	·000344	·000317	·000291	·000268	·000246	·000226	·000208	·000191	2
3	·001641	·001522	·001412	·001309	·001213	·001124	·001042	·000965	·000894	·000828	3
4	·004966	·004643	·004341	·004057	·003791	·003541	·003307	·003088	·002882	·002690	4
5	·012017	·011330	·010679	·010062	·009477	·008924	·008400	·007905	·007436	·006994	5
6	·024233	·023037	·021892	·020794	·019744	·018740	·017781	·016864	·015988	·015153	6
7	·041889	·040151	·038467	·036836	·035258	·033733	·032259	·030837	·029464	·028141	7
8	·063358	·061230	·059142	·057095	·055091	·053129	·051212	·049339	·047511	·045730	8
9	·085181	·083000	·080828	·078665	·076515	·074381	·072266	·070171	·068100	·066054	9
10	103069	·101261	·099418	·097544	·095644	·093720	·091777	·089819	·087849	·085870	10
11	·113376	·112308	·111168	·109959	·108686	·107352	·105961	·104516	·103023	·101483	11
12	·114321	·114180	·113947	·113624	·113215	·112720	·112142	·111484	·110749	·109940	12
13	·106406	·107153	·107811	·108380	·108860	·109251	·109554	·109769	·109897	·109940	13
14	·091965	·093376	·094729	·095994	·097197	·098326	·099381	·100360	·101263	·102087	14
15	·074185	·075946	·077670	·079355	·080997	·082594	·084143	·085641	·087086	·088475	15
16	·056103	·057909	·059709	·061500	·063279	·065043	·066788	·068513	·070213	·071886	16
17	·039932	·041558	·043201	·044859	·046529	·048208	·049895	·051586	·053279	·054972	17
18	·026843	·028167	029521	·030903	·032312	·033746	·035204	·036683	·038183	·039702	18
19	·017095	·018086	·019111	·020168	·021258	·022379	·023531	·024713	·025925	·027164	19
20	·010342	·011033	·011753	·012504	·013286	·014099	·014942	·015816	·016721	·017657	20
21	·005959	·006409	·006884	·007383	·007908	·008459	·009036	·009640	·010272	·010930	21
22	·003278	·003554	·003849	·004162	·004493	·004845	·005216	·005609	·006023	·006459	22
23	·001724	·001885	·002058	·002244	·002442	·002654	·002880	·003122	·003378	·003651	23
24	·000869	·000958	·001055	·001159	·001272	·001393	·001524	·001665	·001816	·001977	24
25	·000421	·000468	·000519	·000575	·000636	·000702	·000774	·000852	·000937	·001028	25
26	·000196	·000219	·000246	·000274	·000306	·000340	·000378	·000420	·000465	·000514	26
27	·000088	·000099	·000112	·000126	·000142	·000159	·000178	·000199	·000222	·000248	27
28	·000038	·000043	·000049	·000056	·000063	·000071	·000081	·000091	·000102	·000115	28
29	·000016	·000018	·000021	·000024	·000027	·000031	·000035	·000040	·000046	·000052	29
30	·000006	·000007	·000009	·000010	·000011	·000013	·000015	·000017	·000020	·000022	30
31	·000002	·000003	·000003	·000004	·000005	·000005	·000006	·000007	·000008	·000009	31
32	·000001	·000001	·000001	·000002	·000002	·000002	·000002	·000003	·000003	·000004	32
33	—	—	—	·000001	·000001	·000001	·000001	·000001	·000001	·000002	33
34	—	—	—	—	—	—	—	—	—	·000001	34

TABLE LI—(*continued*).

x	m										x
	13·1	13·2	13·3	13·4	13·5	13·6	13·7	13·8	13·9	14·0	
0	·000002	·000002	·000002	·000002	·000001	·000001	·000001	·000001	·000001	·000001	0
1	·000027	·000024	·000022	·000020	·000019	·000017	·000015	·000014	·000013	·000012	1
2	·000175	·000161	·000148	·000136	·000125	·000115	·000105	·000097	·000089	·000081	2
3	·000766	·000709	·000657	·000608	·000562	·000520	·000481	·000445	·000411	·000380	3
4	·002510	·002341	·002183	·002035	·001897	·001768	·001648	·001535	·001429	·001331	4
5	·006575	·006180	·005807	·005455	·005123	·004810	·004514	·004236	·003974	·003727	5
6	·014356	·013596	·012872	·012183	·011526	·010902	·010308	·009743	·009206	·008696	6
7	·026867	·025639	·024458	·023322	·022230	·021181	·020173	·019207	·018280	·017392	7
8	·043994	·042304	·040661	·039064	·037512	·036007	·034547	·033132	·031762	·030435	8
9	·064036	·062046	·060088	·058161	·056269	·054410	·052588	·050802	·049054	·047344	9
10	·083887	·081901	·079916	·077936	·075963	·073998	·072046	·070107	·068185	·066282	10
11	·099901	·098281	·096626	·094940	·093227	·091489	·089730	·087953	·086162	·084359	11
12	·109059	·108109	·107094	·106017	·104880	·103687	·102441	·101146	·099804	·098418	12
13	·109898	·109773	·109566	·109279	·108914	·108473	·107957	·107370	·106713	·105989	13
14	·102833	·103500	·104087	·104595	·105024	·105373	·105644	·105836	·105951	·105989	14
15	·089807	·091080	·092291	·093439	·094522	·095539	·096488	·097369	·098181	·098923	15
16	·073530	·075141	·076717	·078255	·079753	·081208	·082618	·083981	·085295	·086558	16
17	·056661	·058345	·060019	·061683	·063333	·064966	·066580	·068173	·069741	·071283	17
18	·041237	·042786	·044348	·045920	·047500	·049086	·050675	·052266	·053856	·055442	18
19	·028432	·029725	·031043	·032385	·033750	·035135	·036539	·037962	·039400	·040852	19
20	·018623	·019619	·020644	·021698	·022781	·023892	·025030	·026193	·027383	·028597	20
21	·011617	·012332	·013074	·013846	·014645	·015473	·016329	·017213	·018125	·019064	21
22	·006917	·007399	·007904	·008433	·008987	·009565	·010168	·010797	·011452	·012132	22
23	·003940	·004246	·004571	·004913	·005275	·005656	·006057	·006478	·006921	·007385	23
24	·002151	·002336	·002533	·002743	·002967	·003205	·003457	·003725	·004008	·004308	24
25	·001127	·001233	·001348	·001470	·001602	·001744	·001895	·002056	·002229	·002412	25
26	·000568	·000626	·000689	·000758	·000832	·000912	·000998	·001091	·001191	·001299	26
27	·000275	·000306	·000340	·000376	·000416	·000459	·000507	·000558	·000613	·000674	27
28	·000129	·000144	·000161	·000180	·000201	·000223	·000248	·000275	·000305	·000337	28
29	·000058	·000066	·000074	·000083	·000093	·000105	·000117	·000131	·000146	·000163	29
30	·000025	·000029	·000033	·000037	·000042	·000047	·000053	·000060	·000068	·000076	30
31	·000011	·000012	·000014	·000016	·000018	·000021	·000024	·000027	·000030	·000034	31
32	·000004	·000005	·000006	·000007	·000008	·000009	·000010	·000012	·000013	·000015	32
33	·000002	·000002	·000002	·000003	·000003	·000004	·000004	·000005	·000006	·000006	33
34	·000001	·000001	·000001	·000001	·000001	·000001	·000002	·000002	·000002	·000003	34
35	—	—	—	—	—	·000001	·000001	·000001	·000001	·000001	35

x	14·1	14·2	14·3	14·4	14·5	14·6	14·7	14·8	14·9	15·0	x
0	·000001	·000001	·000001	·000001	·000001	—	—	—	—	—	0
1	·000011	·000010	·000009	·000008	·000007	·000007	·000006	·000006	·000005	·000005	1
2	·000075	·000069	·000063	·000058	·000053	·000049	·000045	·000041	·000038	·000034	2
3	·000352	·000325	·000300	·000277	·000256	·000237	·000219	·000202	·000186	·000172	3
4	·001239	·001153	·001073	·000999	·000929	·000864	·000803	·000747	·000694	·000645	4
5	·003494	·003275	·003070	·002876	·002694	·002523	·002362	·002211	·002069	·001936	5
6	·008212	·007752	·007316	·006902	·006510	·006139	·005787	·005454	·005138	·004839	6
7	·016541	·015726	·014946	·014199	·013486	·012804	·012152	·011530	·010937	·010370	7
8	·029153	·027913	·026715	·025559	·024443	·023367	·022330	·021331	·020370	·019444	8
9	·045673	·044040	·042447	·040894	·039380	·037907	·036472	·035078	·033723	·032407	9
10	·064399	·062537	·060700	·058887	·057101	·055343	·053614	·051915	·050247	·048611	10
11	·082547	·080730	·078910	·077089	·075270	·073456	·071648	·069850	·068062	·066287	11

TABLE LI—(*continued*).

x	m										x
	14·1	*14·2*	*14·3*	*14·4*	*14·5*	*14·6*	*14·7*	*14·8*	*14·9*	*15·0*	
12	·096993	·095530	·094034	·092507	·090951	·089371	·087769	·086148	·084510	·082859	*12*
13	·105200	·104349	·103437	·102469	·101446	·100371	·099247	·098076	·096862	·095607	*13*
14	·105951	·105839	·105654	·105396	·105069	·104672	·104209	·103681	·103089	·102436	*14*
15	·099594	·100195	·100723	·101181	·101567	·101881	·102125	·102298	·102402	·102436	*15*
16	·087768	·088923	·090021	·091063	·092045	·092967	·093827	·094626	·095361	·096034	*16*
17	·072795	·074277	·075724	·077135	·078509	·079842	·081133	·082380	·083581	·084736	*17*
18	·057023	·058596	·060158	·061708	·063243	·064761	·066259	·067735	·069187	·070613	*18*
19	·042317	·043793	·045277	·046768	·048264	·049763	·051263	·052762	·054257	·055747	*19*
20	·029834	·031093	·032373	·033673	·034992	·036327	·037678	·039044	·040422	·041810	*20*
21	·020031	·021025	·022045	·023090	·024161	·025256	·026375	·027517	·028680	·029865	*21*
22	·012838	·013570	·014329	·015114	·015924	·016761	·017625	·018511	·019424	·020362	*22*
23	·007870	·008378	·008909	·009462	·010039	·010640	·011264	·011911	·012584	·013280	*23*
24	·004624	·004957	·005308	·005677	·006065	·006472	·006899	·007345	·007812	·008300	*24*
25	·002608	·002816	·003036	·003270	·003518	·003780	·004057	·004348	·004656	·004980	*25*
26	·001414	·001538	·001670	·001811	·001962	·002123	·002294	·002475	·002668	·002873	*26*
27	·000739	·000809	·000884	·000966	·001054	·001148	·001249	·001357	·001473	·001596	*27*
28	·000372	·000410	·000452	·000497	·000546	·000598	·000656	·000717	·000784	·000855	*28*
29	·000181	·000201	·000223	·000247	·000273	·000301	·000332	·000366	·000403	·000442	*29*
30	·000085	·000095	·000106	·000118	·000132	·000147	·000163	·000181	·000200	·000221	*30*
31	·000039	·000044	·000049	·000055	·000062	·000069	·000077	·000086	·000096	·000107	*31*
32	·000017	·000019	·000022	·000025	·000028	·000032	·000035	·000040	·000045	·000050	*32*
33	·000007	·000008	·000009	·000011	·000012	·000014	·000016	·000018	·000020	·000023	*33*
34	·000003	·000003	·000004	·000005	·000005	·000006	·000007	·000008	·000009	·000010	*34*
35	·000001	·000001	·000002	·000002	·000002	·000002	·000003	·000003	·000004	·000004	*35*
36	—	·000001	·000001	·000001	·000001	·000001	·000001	·000001	·000002	·000002	*36*
37	—	—	—	—	—	—	—	·000001	·000001	·000001	*37*

TABLE LII.

Table of Poisson-Exponential for Cell Frequencies **1** *to* **30.**

Cell Frequencies

	x	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
Per cent. occurrence of values differing by *x* or more in defect from Actual.	*22*										
	21										
	20										
	19										
	18										
	17										
	16										
	15										
	14										
	13										
	12										
	11										
	10										·005
	9									·012	·050
	8								·034	·123	·277
	7							·091	·302	·623	1·033
	6						·248	·730	1·375	2·123	2·925
	5					·674	1·735	2·964	4·238	5·496	6·708
	4				1·832	4·043	6·197	8·177	9·963	11·569	13·014
	3			4·979	9·158	12·465	15·120	17·299	19·124	20·678	22·022
	2		13·534	19·915	23·810	26·503	28·506	30·071	31·337	32·390	33·282
	1	36·788	40·601	42·319	43·347	44·049	44·568	44·971	45·296	45·565	45·793
	Actual	36·788	27·067	22·404	19·537	17·547	16·062	14·900	13·959	13·176	12·511
Per cent. occurrence of values differing by *x* or more in excess from Actual.	*1*	26·424	32·332	35·277	37·116	38·404	39·370	40·129	40·745	41·259	41·696
	2	8·030	14·288	18·474	21·487	23·782	25·602	27·091	28·338	29·401	30·323
	3	1·899	5·265	8·392	11·067	13·337	15·276	16·950	18·411	19·699	20·845
	4	·366	1·656	3·351	5·113	6·809	8·392	9·852	11·192	12·422	13·554
	5	·059	·453	1·191	2·136	3·183	4·262	5·335	6·380	7·385	8·346
	6	·008	·110	·380	·813	1·370	2·009	2·700	3·418	4·146	4·875
	7	·001	·024	·110	·284	·545	·883	1·281	1·726	2·203	2·705
	8	·000	·005	·029	·092	·202	·363	·572	·823	1·110	1·428
	9	—	·001	·007	·027	·070	·140	·241	·372	·532	·719
	10	—	·000	·002	·008	·023	·051	·096	·159	·242	·346
	11	—	—	·000	·002	·007	·018	·036	·065	·105	·160
	12	—	—	—	·001	·002	·006	·013	·025	·044	·071
	13	—	—	—	·000	·001	·002	·005	·009	·017	·030
	14	—	—	—	—	·000	·001	·002	·003	·007	·013
	15	—	—	—	—	—	·000	·001	·001	·002	·006
	16	—	—	—	—	—	—	·000	·000	·001	·002
	17	—	—	—	—	—	—	—	—	·000	·001
	18	—	—	—	—	—	—	—	—	—	·001
	19	—	—	—	—	—	—	—	—	—	·000
	20	—	—	—	—	—	—	—	—	—	—
	21	—	—	—	—	—	—	—	—	—	—
	22	—	—	—	—	—	—	—	—	—	—
	23	—	—	—	—	—	—	—	—	—	—
	24	—	—	—	—	—	—	—	—	—	—
	25	—	—	—	—	—	—	—	—	—	—
	26	—	—	—	—	—	—	—	—	—	—
	27	—	—	—	—	—	—	—	—	—	—
	28	—	—	—	—	—	—	—	—	—	—

TABLE LII—(*continued*).

Cell Frequencies

	x	11	12	13	14	15	16	17	18	19	20
Per cent. occurrence of values differing by x or more in defect from Actual.	22										
	21										
	20										—
	19									—	—
	18								—	—	—
	17							—	—	·000	·000
	16						—	·000	·000	·001	·002
	15					·000	·000	·001	·002	·004	·007
	14				·000	·001	·002	·004	·008	·015	·026
	13			·000	·001	·004	·009	·018	·032	·052	·078
	12		·001	·003	·009	·021	·040	·067	·104	·151	·209
	11	·002	·008	·022	·047	·086	·138	·206	·289	·387	·500
	10	·020	·052	·105	·181	·279	·401	·543	·706	·886	1·081
	9	·121	·229	·374	·553	·763	1·000	1·260	1·538	1·832	2·139
	8	·492	·760	1·073	1·423	1·800	2·199	2·612	3·037	3·467	3·901
	7	1·510	2·034	2·589	3·162	3·745	4·330	4·912	5·489	6·056	6·613
	6	3·752	4·582	5·403	6·206	6·985	7·740	8·467	9·167	9·840	10·486
	5	7·861	8·950	9·976	10·940	11·846	12·699	13·502	14·260	14·975	15·651
	4	14·319	15·503	16·581	17·568	18·475	19·312	20·087	20·808	21·479	22·107
	3	23·198	24·239	25·168	26·004	26·761	27·451	28·084	28·665	29·203	29·703
	2	34·051	34·723	35·317	35·846	36·322	36·753	37·146	37·505	37·836	38·142
	1	45·989	46·150	46·311	46·445	46·565	46·674	46·774	46·865	46·948	47·026
	Actual	11·938	11·437	10·994	10·599	10·244	9·922	9·629	9·360	9·112	8·884
Per cent. occurrence of values differing by x or more in excess from Actual.	1	42·073	42·404	42·695	42·956	43·191	43·404	43·597	43·776	43·939	44·091
	2	31·130	31·846	32·486	33·064	33·588	34·066	34·503	34·909	35·283	35·630
	3	21·871	22·798	23·639	24·408	25·114	25·765	26·367	26·928	27·451	27·939
	4	14·596	15·559	16·450	17·280	18·053	18·776	19·451	20·088	20·686	21·251
	5	9·261	10·129	10·953	11·736	12·478	13·184	13·852	14·491	15·099	15·677
	6	5·593	6·297	6·983	7·650	8·297	8·923	9·526	10·111	10·675	11·219
	7	3·219	3·742	4·266	4·791	5·311	5·825	6·329	6·826	7·313	7·789
	8	1·769	2·128	2·501	2·884	3·275	3·669	4·064	4·461	4·856	5·248
	9	·929	1·160	1·407	1·671	1·947	2·232	2·523	2·824	3·127	3·433
	10	·467	·607	·762	·933	1·117	1·312	1·516	1·732	1·954	2·182
	11	·225	·305	·396	·502	·619	·746	·882	1·030	1·185	1·348
	12	·104	·148	·201	·261	·331	·411	·497	·595	·699	·809
	13	·047	·069	·097	·131	·172	·219	·272	·333	·400	·473
	14	·020	·031	·046	·063	·086	·114	·144	·182	·223	·269
	15	·008	·014	·021	·030	·042	·057	·074	·096	·121	·149
	16	·003	·006	·009	·013	·020	·028	·036	·050	·064	·081
	17	·001	·002	·004	·006	·009	·014	·017	·025	·033	·042
	18	·001	·000	·002	·002	·004	·006	·008	·012	·017	·022
	19	·000	·000	·001	·001	·002	·003	·003	·006	·008	·011
	20	—	—	·001	·000	·001	·002	·002	·003	·004	·005
	21	—	—	·000	·000	·000	·001	·001	·002	·002	·003
	22	—	—	—	—	—	·000	·000	·001	·001	·001
	23	—	—	—	—	—	—	—	·000	·000	·001
	24	—	—	—	—	—	—	—	—	—	·000
	25	—	—	—	—	—	—	—	—	—	—
	26	—	—	—	—	—	—	—	—	—	—
	27	—	—	—	—	—	—	—	—	—	—
	28	—	—	—	—	—	—	—	—	—	—

TABLE LII—(*continued*).

Cell Frequencies

	x	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*	*30*
Per cent. occurrence of values differing by *x* or more in defect from Actual.	*22*		—	—	—	—	—	—	—	—	·000
	21	—	—	—	—	—	—	·000	·000	·000	·001
	20	—	—	—	—	·000	·000	·001	·001	·001	·002
	19	—	—	·000	·000	·001	·001	·002	·003	·004	·006
	18	·000	·000	·001	·001	·002	·004	·006	·009	·012	·017
	17	·001	·002	·003	·005	·008	·011	·016	·023	·031	·041
	16	·003	·006	·010	·015	·022	·031	·043	·056	·073	·092
	15	·012	·020	·030	·043	·059	·078	·102	·129	·160	·195
	14	·039	·058	·081	·109	·142	·180	·224	·273	·328	·387
	13	·111	·150	·198	·252	·314	·384	·460	·543	·632	·727
	12	·277	·355	·443	·540	·647	·762	·884	1·014	1·151	1·293
	11	·625	·763	·912	1·072	1·240	1·417	1·601	1·791	1·987	2·187
	10	1·290	1·512	1·743	1·983	2·229	2·482	2·739	3·000	3·263	3·528
	9	2·455	2·778	3·107	3·440	3·775	4·111	4·446	4·781	5·114	5·444
	8	4·336	4·769	5·200	5·626	6·048	6·463	6·872	7·274	7·669	8·057
	7	7·157	7·689	8·208	8·713	9·204	9·682	10·147	10·599	11·038	11·465
	6	11·107	11·704	12·277	12·827	13·358	13·867	14·357	14·830	15·285	15·724
	5	16·292	16·900	17·477	18·025	18·549	19·048	19·525	19·981	20·417	20·836
	4	22·696	23·250	23·771	24·263	24·730	25·172	25·591	25·990	26·371	26·734
	3	30·168	30·603	31·010	31·391	31·753	32·094	32·416	32·721	33·011	33·287
	2	38·426	38·691	38·938	39·168	39·387	39·593	39·786	39·970	40·143	40·308
	1	47·097	47·164	47·226	47·283	47·340	47·392	47·440	47·486	47·530	47·572
	Actual	8·671	8·473	8·288	8·115	7·952	7·799	7·654	7·517	7·387	7·264
Per cent. occurrence of values differing by *x* or more in excess from Actual.	*1*	44·232	44·363	44·485	44·603	44·708	44·810	44·906	44·997	45·083	45·165
	2	35·955	36·258	36·542	36·812	37·062	37·299	37·525	37·739	37·942	38·135
	3	28·397	28·828	29·235	29·620	29·982	30·326	30·653	30·965	31·262	31·546
	4	21·785	22·290	22·770	23·227	23·660	24·074	24·469	24·847	25·208	25·555
	5	16·230	16·758	17·264	17·748	18·211	18·655	19·083	19·493	19·888	20·269
	6	11·744	12·251	12·740	13·213	13·669	14·110	14·538	14·951	15·351	15·738
	7	8·254	8·709	9·153	9·585	10·007	10·418	10·819	11·210	11·591	11·962
	8	5·637	6·022	6·402	6·777	7·146	7·509	7·866	8·218	8·562	8·901
	9	3·742	4·052	4·362	4·670	4·978	5·284	5·588	5·890	6·188	6·484
	10	2·415	2·654	2·895	3·138	3·385	3·632	3·880	4·129	4·377	4·625
	11	1·517	1·692	1·873	2·057	2·246	2·438	2·633	2·831	3·030	3·230
	12	·927	1·051	1·181	1·315	1·456	1·599	1·747	1·899	2·053	2·210
	13	·552	·637	·727	·821	·921	1·025	1·134	1·247	1·362	1·481
	14	·320	·376	·437	·500	·570	·643	·720	·801	·885	·973
	15	·181	·217	·256	·298	·345	·394	·448	·504	·564	·626
	16	·100	·122	·147	·173	·204	·237	·272	·311	·352	·395
	17	·054	·067	·082	·098	·118	·139	·162	·188	·215	·245
	18	·028	·036	·045	·055	·067	·080	·095	·111	·129	·149
	19	·015	·019	·024	·030	·037	·045	·054	·065	·076	·089
	20	·007	·010	·013	·016	·020	·025	·031	·037	·044	·052
	21	·004	·005	·007	·009	·011	·014	·017	·021	·025	·030
	22	·002	·002	·004	·005	·006	·007	·009	·011	·014	·017
	23	·001	·001	·002	·003	·003	·004	·005	·006	·007	·010
	24	·001	·001	·001	·002	·002	·002	·003	·003	·004	·006
	25	·000	·000	·001	·001	·001	·001	·001	·002	·002	·003
	26	—	—	·000	·000	·000	·001	·001	·001	·001	·002
	27	—	—	—	—	—	·000	·000	·000	·000	·001
	28	—	—	—	—	—	—	—	—	—	·000

LENGTHS OF CIRCULAR ARCS

Deg.	Arc	Deg.	Arc	Deg.	Arc	′	Deg.	Arc	″	Deg.	Arc
1	·017 4533	61	1·064 6508	121	2·111 8484	1	·01667	·000 2909	1	·00028	·000 0048
2	·034 9066	62	1·082 1041	122	2·129 3017	2	·03333	·000 5818	2	·00056	·000 0097
3	·052 3599	63	1·099 5574	123	2·146 7550	3	·05000	·000 8727	3	·00083	·000 0145
4	·069 8132	64	1·117 0107	124	2·164 2083	4	·06667	·001 1636	4	·00111	·000 0194
5	·087 2665	65	1·134 4640	125	2·181 6616	5	·08333	·001 4544	5	·00139	·000 0242
6	·104 7198	66	1·151 9173	126	2·199 1149	6	·10000	·001 7453	6	·00167	·000 0291
7	·122 1730	67	1·169 3706	127	2·216 5682	7	·11667	·002 0362	7	·00194	·000 0339
8	·139 6263	68	1·186 8239	128	2·234 0214	8	·13333	·002 3271	8	·00222	·000 0388
9	·157 0796	69	1·204 2772	129	2·251 4747	9	·15000	·002 6180	9	·00250	·000 0436
10	·174 5329	70	1·221 7305	130	2·268 9280	10	·16667	·002 9089	10	·00278	·000 0485
11	·191 9862	71	1·239 1838	131	2·286 3813	11	·18333	·003 1998	11	·00306	·000 0533
12	·209 4395	72	1·256 6371	132	2·303 8346	12	·20000	·003 4907	12	·00333	·000 0582
13	·226 8928	73	1·274 0904	133	2·321 2879	13	·21667	·003 7815	13	·00361	·000 0630
14	·244 3461	74	1·291 5436	134	2·338 7412	14	·23333	·004 0724	14	·00389	·000 0679
15	·261 7994	75	1·308 9969	135	2·356 1945	15	·25000	·004 3633	15	·00417	·000 0727
16	·279 2527	76	1·326 4502	136	2·373 6478	16	·26667	·004 6542	16	·00444	·000 0776
17	·296 7060	77	1·343 9035	137	2·391 1011	17	·28333	·004 9451	17	·00472	·000 0824
18	·314 1593	78	1·361 3568	138	2·408 5544	18	·30000	·005 2360	18	·00500	·000 0873
19	·331 6126	79	1·378 8101	139	2·426 0077	19	·31667	·005 5269	19	·00528	·000 0921
20	·349 0659	80	1·396 2634	140	2·443 4610	20	·33333	·005 8178	20	·00556	·000 0970
21	·366 5191	81	1·413 7167	141	2·460 9142	21	·35000	·006 1087	21	·00583	·000 1018
22	·383 9724	82	1·431 1700	142	2·478 3675	22	·36667	·006 3995	22	·00611	·000 1067
23	·401 4257	83	1·448 6233	143	2·495 8208	23	·38333	·006 6904	23	·00639	·000 1115
24	·418 8790	84	1·466 0766	144	2·513 2741	24	·40000	·006 9813	24	·00667	·000 1164
25	·436 3323	85	1·483 5299	145	2·530 7274	25	·41667	·007 2722	25	·00694	·000 1212
26	·453 7856	86	1·500 9832	146	2·548 1807	26	·43333	·007 5631	26	·00722	·000 1261
27	·471 2389	87	1·518 4364	147	2·565 6340	27	·45000	·007 8540	27	·00750	·000 1309
28	·488 6922	88	1·535 8897	148	2·583 0873	28	·46667	·008 1449	28	·00778	·000 1357
29	·506 1455	89	1·553 3430	149	2·600 5406	29	·48333	·008 4358	29	·00806	·000 1406
30	·523 5988	90	1·570 7963	150	2·617 9939	30	·50000	·008 7266	30	·00833	·000 1454
31	·541 0521	91	1·588 2496	151	2·635 4472	31	·51667	·009 0175	31	·00861	·000 1503
32	·558 5054	92	1·605 7029	152	2·652 9005	32	·53333	·009 3084	32	·00889	·000 1551
33	·575 9587	93	1·623 1562	153	2·670 3538	33	·55000	·009 5993	33	·00917	·000 1600
34	·593 4119	94	1·640 6095	154	2·687 8070	34	·56667	·009 8902	34	·00944	·000 1648
35	·610 8652	95	1·658 0628	155	2·705 2603	35	·58333	·010 1811	35	·00972	·000 1697
36	·628 3185	96	1·675 5161	156	2·722 7136	36	·60000	·010 4720	36	·01000	·000 1745
37	·645 7718	97	1·692 9694	157	2·740 1669	37	·61667	·010 7629	37	·01028	·000 1794
38	·663 2251	98	1·710 4227	158	2·757 6202	38	·63333	·011 0538	38	·01056	·000 1842
39	·680 6784	99	1·727 8760	159	2·775 0735	39	·65000	·011 3446	39	·01083	·000 1891
40	·698 1317	100	1·745 3293	160	2·792 5268	40	·66667	·011 6355	40	·01111	·000 1939
41	·715 5850	101	1·762 7825	161	2·809 9801	41	·68333	·011 9264	41	·01139	·000 1988
42	·733 0383	102	1·780 2358	162	2·827 4334	42	·70000	·012 2173	42	·01167	·000 2036
43	·750 4916	103	1·797 6891	163	2·844 8867	43	·71667	·012 5082	43	·01194	·000 2085
44	·767 9449	104	1·815 1424	164	2·862 3400	44	·73333	·012 7991	44	·01222	·000 2133
45	·785 3982	105	1·832 5957	165	2·879 7933	45	·75000	·013 0900	45	·01250	·000 2182
46	·802 8515	106	1·850 0490	166	2·897 2466	46	·76667	·013 3809	46	·01278	·000 2230
47	·820 3047	107	1·867 5023	167	2·914 6999	47	·78333	·013 6717	47	·01306	·000 2279
48	·837 7580	108	1·884 9556	168	2·932 1531	48	·80000	·013 9626	48	0·1333	·000 2327
49	·855 2113	109	1·902 4089	169	2·949 6064	49	·81667	·014 2535	49	·01361	·000 2376
50	·872 6646	110	1·919 8622	170	2·967 0597	50	·83333	·014 5444	50	·01389	·000 2424
51	·890 1179	111	1·937 3155	171	2·984 5130	51	·85000	·014 8353	51	·01417	·000 2473
52	·907 5712	112	1·954 7688	172	3·001 9663	52	·86667	·015 1262	52	·01444	·000 2521
53	·925 0245	113	1·972 2221	173	3·019 4196	53	·88333	·015 4171	53	·01472	·000 2570
54	·942 4778	114	1·989 6753	174	3·036 8729	54	·90000	·015 7080	54	·01500	·000 2618
55	·959 9311	115	2·007 1286	175	3·054 3262	55	·91667	·015 9989	55	·01528	·000 2666
56	·977 3844	116	2·024 5819	176	3·071 7795	56	·93333	·016 2897	56	·01556	·000 2715
57	·994 8377	117	2·042 0352	177	3·089 2328	57	·95000	·016 5806	57	·01583	·000 2763
58	1·012 2910	118	2·059 4885	178	3·106 6861	58	·96667	·016 8715	58	·01611	·000 2812
59	1·029 7443	119	2·076 9418	179	3·124 1394	59	·98333	·017 1624	59	·01639	·000 2860
60	1·047 1976	120	2·094 3951	180	3·141 5927	60	1·00000	·017 4533	60	·01667	·000 2909

TABLE LIV. *The $G(r, \nu)$-Integrals.*

$\phi°$	$r=1$			$r=2$			
	$\log F(r, \nu)$	Δ	Δ^2	$\log F(r, \nu)$	$\log H(r, \nu)$	Δ	Δ^2
0	0·301 0300			0·196 1199	0·196 1199		
		309				192	
1	·301 0609		619	·196 2052	·196 1391		383
		928				575	
2	·301 1538		621	·196 4614	·196 1966		383
		1550				958	
3	·301 3087		625	·196 8890	·196 2924		382
		2175				1340	
4	·301 5262		630	·197 4890	·196 4264		381
		2805				1722	
5	·301 8067		636	·198 2627	·196 5985		380
		3441				2102	
6	·302 1508		645	·199 2118	·196 8087		379
		4086				2480	
7	·302 5594		654	·200 3385	·197 0567		377
		4740				2857	
8	·303 0335		666	·201 6452	·197 3424		375
		5406				3231	
9	·303 5741		679	·203 1349	·197 6655		373
		6085				3604	
10	·304 1825		693	·204 8110	·198 0260		370
		6778				3974	
11	·304 8603		710	·206 6774	·198 4234		367
		7488				4341	
12	·305 6091		728	·208 7382	·198 8574		364
		8216				4705	
13	·306 4307		749	·210 9985	·199 3280		360
		8964				5064	
14	·307 3271		771	·213 4631	·199 8344		356
		9735				5420	
15	·308 3006		796	·216 1383	·200 3764		352
		10532				5773	
16	·309 3538		822	·219 0303	·200 9537		347
		11353				6120	
17	·310 4892		853	·222 1462	·201 5657		343
		12206				6463	
18	·311 7098		885	·225 4936	·202 2120		338
		13091				6801	
19	·313 0189		919	·229 0807	·202 8921		332
		14011				7133	
20	·314 4200		958	·232 9167	·203 6054		327
		14969				7460	
21	·315 9169		999	·237 0114	·204 3514		320
		15968				7780	
22	·317 5137		1045	·241 3755	·205 1294		313
		17013				8093	
23	·319 2150		1093	·246 0203	·205 9387		307
		18106				8400	
24	·321 0256		1146	·250 9584	·206 7787		300
		19252				8700	
25	·322 9507		1204	·256 2034	·207 6487		291
		20456				8991	
26	·324 9963		1266	·261 7697	·208 5478		284
		21722				9275	
27	·327 1685		1334	·267 6733	·209 4753		275
		23055				9549	
28	·329 4740		1407	·273 9311	·210 4302		266
		24462				9816	
29	·331 9202		1486	·280 5618	·211 4118		256
		25948				1 0072	
30	·334 5150		1573	·287 5852	·212 4190		247
		27521				1 0319	
31	·337 2672		1667	·295 0232	·213 4509		236
		29188				1 0556	
32	·340 1860		1769	·302 8992	·214 5064		226
		30958				1 0782	
33	·343 2818		1881	·311 2388	·215 5846		213
		32838				1 0996	
34	·346 5656		2002	·320 0695	·216 6842		203
		34840				1 1199	
35	·350 0496		2134	·329 4214	·217 8041		191
		36974				1 1390	
36	·353 7469		2279	·339 3271	·218 9431		178
		39252				1 1569	
37	·357 6722		2436	·349 8221	·220 1000		165
		41689				1 1734	
38	·361 8410		2609	·360 9451	·221 2734		152
		44298				1 1886	
39	·366 2708		2799	·372 7382	·222 4621		138
		47096				1 2023	
40	·370 9305		3007	·385 2475	·223 6644		125
		50103				1 2148	
41	·375 9908		3235	·398 5232	·224 8791		109
		53338				1 2256	
42	·381 3246		3486	·412 6205	·226 1048		94
		56824				1 2350	
43	·387 0070		3764	·427 5995	·227 3397		77
		60588				1 2427	
44	·393 0658		4069	·443 5266	·228 5824		62
		64658				1 2489	
45	·399 5316			·460 4745	·229 8313		

TABLE LIV—(*continued*).

φ°	$r=3$				$r=4$				$r=5$			
	log $F(r,\nu)$	log $H(r,\nu)$	Δ	Δ²	log $F(r,\nu)$	log $H(r,\nu)$	Δ	Δ²	log $F(r,\nu)$	log $H(r,\nu)$	Δ	Δ²
0	0·124 9387	0·275 4537			0·071 1811	0·309 7418			0·028 0289	0·329 0589		
			137				105				84	
1	·125 0847	·275 4674		273	·071 3902	·309 7523		211	·028 3019	·329 0673		173
			410				316				257	
2	·125 5230	·275 5084		273	·072 0177	·309 7840		211	·029 1221	·329 0930		171
			683				527				428	
3	·126 2545	·275 5767		272	·073 0650	·309 8366		210	·030 4908	·329 1357		171
			955				737				598	
4	·127 2807	·275 6721		271	·074 5342	·309 9103		209	·032 4110	·329 1956		170
			1226				946				768	
5	·128 6039	·275 7948		270	·076 4285	·310 0049		208	·034 8865	·329 2723		169
			1496				1154				937	
6	·130 2270	·275 9444		269	·078 7517	·310 1204		207	·037 9224	·329 3661		168
			1766				1361				1105	
7	·132 1533	·276 1209		267	·081 5088	·310 2565		206	·041 5250	·329 4765		167
			2032				1567				1272	
8	·134 3870	·276 3242		266	·084 7055	·310 4132		204	·045 7016	·329 6037		165
			2298				1771				1437	
9	·136 9331	·276 5540		263	·088 3486	·310 5904		202	·050 4609	·329 7474		164
			2561				1973				1601	
10	·139 7969	·276 8101		261	·092 4458	·310 7877		201	·055 8130	·329 9075		162
			2822				2174				1763	
11	·142 9850	·277 0923		258	·097 0060	·311 0051		198	·061 7690	·330 0838		160
			3080				2372				1923	
12	·146 5043	·277 4003		256	·102 0399	·311 2423		196	·068 3415	·330 2761		158
			3336				2567				2081	
13	·150 3626	·277 7338		252	·107 5555	·311 4990		193	·075 5446	·330 4842		156
			3588				2760				2237	
14	·154 5688	·278 0926		249	·113 5680	·311 7751		190	·083 3937	·330 7079		153
			3837				2950				2390	
15	·159 1322	·278 4762		245	·120 0895	·312 0701		187	·091 9061	·330 9469		151
			4082				3137				2541	
16	·164 0636	·278 8844		241	·127 1349	·312 3838		184	·101 1002	·331 2010		148
			4323				3321				2689	
17	·169 3743	·279 3167		237	·134 7199	·312 7159		180	·110 9967	·331 4700		144
			4560				3501				2833	
18	·175 0768	·279 7726		233	·142 8621	·313 0660		176	·121 6176	·331 7532		142
			4793				3677				2975	
19	·181 1848	·280 2519		228	·151 5802	·313 4337		172	·132 9872	·332 0508		138
			5020				3849				3114	
20	·187 7130	·280 7539		223	·160 8948	·313 8186		168	·145 1317	·332 3621		135
			5243				4018				3249	
21	·194 6774	·281 2782		217	·170 8281	·314 2204		164	·158 0795	·332 6870		131
			5460				4181				3380	
22	·202 0955	·281 8243		212	·181 4042	·314 6385		160	·171 8614	·333 0250		127
			5673				4341				3507	
23	·209 9858	·282 3915		206	·192 6491	·315 0726		154	·186 5105	·333 3757		123
			5879				4495				3630	
24	·218 3688	·282 9794		200	·204 5907	·315 5222		150	·202 0627	·333 7387		119
			6079				4645				3750	
25	·227 2664	·283 5873		194	·217 2596	·315 9866		144	·218 5568	·334 1137		115
			6272				4789				3864	
26	·236 7023	·284 2145		188	·230 6885	·316 4655		138	·236 0346	·334 5001		110
			6460				4927				3975	
27	·246 7020	·284 8604		180	·244 9127	·316 9583		134	·254 5413	·334 8976		106
			6640				5062				4081	
28	·257 2933	·285 5244		173	·259 9707	·317 4644		127	·274 1255	·335 3057		101
			6813				5189				4182	
29	·268 5060	·286 2057		165	·275 9034	·317 9833		121	·294 8399	·335 7239		96
			6978				5310				4277	
30	·280 3725	·286 9035		158	·292 7555	·318 5143		116	·316 7413	·336 1516		91
			7136				5426				4368	
31	·292 9278	·287 6170		150	·310 5754	·319 0569		109	·339 8909	·336 5884		87
			7286				5535				4455	
32	·306 2096	·288 3456		141	·329 4149	·319 6104		102	·364 3553	·337 0339		80
			7427				5637				4535	
33	·320 2589	·289 0883		133	·349 3304	·320 1741		96	·390 2059	·337 4874		75
			7560				5733				4610	
34	·335 1201	·289 8443		124	·370 3832	·320 7474		89	·417 5203	·337 9485		70
			7684				5822				4680	
35	·350 8413	·290 6127		115	·392 6390	·321 3297		82	·446 3827	·338 4164		64
			7800				5904				4744	
36	·367 4747	·291 3927		106	·416 1697	·321 9201		75	·476 8841	·338 8908		58
			7905				5980				4802	
37	·385 0770	·292 1832		97	·441 0529	·322 5181		68	·509 1232	·339 3709		52
			8002				6047				4854	
38	·403 7099	·292 9834		87	·467 3733	·323 1228		60	·543 2072	·339 8563		46
			8089				6107				4900	
39	·423 4403	·293 7923		77	·495 2227	·323 7335		53	·579 2529	·340 3463		40
			8166				6160				4940	
40	·444 3416	·294 6089		67	·524 7011	·324 3495		45	·617 3872	·340 8404		34
			8232				6205				4975	
41	·466 4933	·295 4321		57	·555 9177	·324 9700		38	·657 7483	·341 3378		28
			8289				6243				5003	
42	·489 9829	·296 2610		46	·588 9916	·325 5943		30	·700 4872	·341 8381		22
			8335				6272				5024	
43	·514 9055	·297 0945		36	·624 0530	·326 2216		22	·745 7688	·342 3405		16
			8371				6295				5040	
44	·541 3658	·297 9316		23	·661 2446	·326 8510		12	·793 7739	·342 8446		9
			8394				6307				5049	
45	·569 4783	·298 7710			·700 7225	·327 4817			·844 6999	·343 3495		

TABLE LIV—(*continued*).

$\phi°$	$r=6$				$r=7$				$r=8$			
	$\log F(r,\nu)$	$\log H(r,\nu)$	Δ	Δ^2	$\log F(r,\nu)$	$\log H(r,\nu)$	Δ	Δ^2	$\log F(r,\nu)$	$\log H(r,\nu)$	Δ	Δ^2
0	$\bar{1}\cdot991\ 9999$	$0\cdot341\ 4849$			$\bar{1}\cdot961\ 0819$	$0\cdot350\ 1576$			$\bar{1}\cdot934\ 0080$	$0\cdot356\ 5570$		
			72				62				54	
1	$\cdot992\ 3379$	$\cdot341\ 4921$		144	$\cdot961\ 4851$	$\cdot350\ 1638$		124	$\cdot934\ 4765$	$\cdot356\ 5624$		109
			216				186				163	
2	$\cdot993\ 3526$	$\cdot341\ 5137$		144	$\cdot962\ 6953$	$\cdot350\ 1824$		124	$\cdot935\ 8831$	$\cdot356\ 5788$		109
			359				310				272	
3	$\cdot995\ 0459$	$\cdot341\ 5496$		143	$\cdot964\ 7151$	$\cdot350\ 2134$		123	$\cdot938\ 2304$	$\cdot356\ 6059$		108
			503				433				380	
4	$\cdot997\ 4213$	$\cdot341\ 5999$		143	$\cdot967\ 5483$	$\cdot350\ 2567$		123	$\cdot941\ 5232$	$\cdot356\ 6440$		108
			645				556				488	
5	$0\cdot000\ 4836$	$\cdot341\ 6644$		142	$\cdot971\ 2008$	$\cdot350\ 3123$		122	$\cdot945\ 7679$	$\cdot356\ 6928$		107
			787				678				596	
6	$\cdot004\ 2390$	$\cdot341\ 7432$		141	$\cdot975\ 6796$	$\cdot350\ 3801$		122	$\cdot950\ 9729$	$\cdot356\ 7524$		107
			928				800				702	
7	$\cdot008\ 6950$	$\cdot341\ 8360$		140	$\cdot980\ 9940$	$\cdot350\ 4601$		121	$\cdot957\ 1486$	$\cdot356\ 8226$		106
			1068				921				808	
8	$\cdot013\ 8607$	$\cdot341\ 9428$		139	$\cdot987\ 1544$	$\cdot350\ 5522$		120	$\cdot964\ 3073$	$\cdot356\ 9034$		105
			1207				1040				913	
9	$\cdot019\ 7468$	$\cdot342\ 0635$		137	$\cdot994\ 1735$	$\cdot350\ 6562$		118	$\cdot972\ 4634$	$\cdot356\ 9947$		104
			1345				1158				1017	
10	$\cdot026\ 3653$	$\cdot342\ 1980$		136	$0\cdot002\ 0658$	$\cdot350\ 7720$		117	$\cdot981\ 6333$	$\cdot357\ 0964$		103
			1481				1275				1120	
11	$\cdot033\ 7300$	$\cdot342\ 3461$		134	$\cdot010\ 8465$	$\cdot350\ 8995$		116	$\cdot991\ 8357$	$\cdot357\ 2083$		101
			1615				1391				1221	
12	$\cdot041\ 8562$	$\cdot342\ 5075$		133	$\cdot020\ 5347$	$\cdot351\ 0386$		114	$0\cdot003\ 0914$	$\cdot357\ 3304$		100
			1747				1505				1321	
13	$\cdot050\ 7609$	$\cdot342\ 6823$		131	$\cdot031\ 1503$	$\cdot351\ 1891$		112	$\cdot015\ 4237$	$\cdot357\ 4625$		98
			1878				1617				1419	
14	$\cdot060\ 4633$	$\cdot342\ 8701$		128	$\cdot042\ 7157$	$\cdot351\ 3508$		110	$\cdot028\ 8583$	$\cdot357\ 6044$		97
			2006				1727				1516	
15	$\cdot070\ 9839$	$\cdot343\ 0707$		127	$\cdot055\ 2551$	$\cdot351\ 5235$		109	$\cdot043\ 4233$	$\cdot357\ 7560$		95
			2133				1836				1611	
16	$\cdot082\ 3457$	$\cdot343\ 2839$		123	$\cdot068\ 7956$	$\cdot351\ 7071$		106	$\cdot059\ 1498$	$\cdot357\ 9170$		93
			2256				1942				1704	
17	$\cdot094\ 5734$	$\cdot343\ 5095$		121	$\cdot083\ 3665$	$\cdot351\ 9012$		104	$\cdot076\ 0714$	$\cdot358\ 0874$		91
			2377				2046				1795	
18	$\cdot107\ 6941$	$\cdot343\ 7472$		119	$\cdot098\ 9997$	$\cdot352\ 1058$		102	$\cdot094\ 2250$	$\cdot358\ 2669$		89
			2496				2148				1884	
19	$\cdot121\ 7375$	$\cdot343\ 9968$		115	$\cdot115\ 7298$	$\cdot352\ 3206$		99	$\cdot113\ 6505$	$\cdot358\ 4553$		87
			2611				2247				1971	
20	$\cdot136\ 7352$	$\cdot344\ 2579$		112	$\cdot133\ 5946$	$\cdot352\ 5452$		97	$\cdot134\ 3912$	$\cdot358\ 6524$		84
			2723				2343				2055	
21	$\cdot152\ 7219$	$\cdot344\ 5302$		109	$\cdot152\ 6347$	$\cdot352\ 7795$		94	$\cdot156\ 4939$	$\cdot358\ 8579$		82
			2833				2437				2137	
22	$\cdot169\ 7350$	$\cdot344\ 8135$		106	$\cdot172\ 8941$	$\cdot353\ 0232$		91	$\cdot180\ 0093$	$\cdot359\ 0716$		80
			2939				2528				2217	
23	$\cdot187\ 8149$	$\cdot345\ 1074$		103	$\cdot194\ 4206$	$\cdot353\ 2760$		88	$\cdot204\ 9923$	$\cdot359\ 2933$		77
			3041				2615				2293	
24	$\cdot207\ 0053$	$\cdot345\ 4115$		99	$\cdot217\ 2653$	$\cdot353\ 5375$		85	$\cdot231\ 5019$	$\cdot359\ 5226$		74
			3141				2700				2368	
25	$\cdot227\ 3532$	$\cdot345\ 7256$		95	$\cdot241\ 4839$	$\cdot353\ 8075$		82	$\cdot259\ 6019$	$\cdot359\ 7594$		71
			3236				2782				2439	
26	$\cdot248\ 9095$	$\cdot346\ 0492$		92	$\cdot267\ 1362$	$\cdot354\ 0857$		79	$\cdot289\ 3613$	$\cdot360\ 0033$		68
			3328				2860				2507	
27	$\cdot271\ 7291$	$\cdot346\ 3819$		88	$\cdot294\ 2868$	$\cdot354\ 3717$		75	$\cdot320\ 8543$	$\cdot360\ 2540$		65
			3415				2935				2572	
28	$\cdot295\ 8713$	$\cdot346\ 7235$		84	$\cdot323\ 0053$	$\cdot354\ 6652$		72	$\cdot354\ 1610$	$\cdot360\ 5112$		62
			3499				3007				2635	
29	$\cdot321\ 3998$	$\cdot347\ 0734$		78	$\cdot353\ 3670$	$\cdot354\ 9659$		67	$\cdot389\ 3678$	$\cdot360\ 7747$		59
			3577				3074				2694	
30	$\cdot348\ 3836$	$\cdot347\ 4311$		76	$\cdot385\ 4529$	$\cdot355\ 2732$		64	$\cdot426\ 5682$	$\cdot361\ 0441$		56
			3653				3138				2750	
31	$\cdot376\ 8974$	$\cdot347\ 7965$		71	$\cdot419\ 3506$	$\cdot355\ 5871$		60	$\cdot465\ 8626$	$\cdot361\ 3191$		52
			3724				3199				2802	
32	$\cdot407\ 0214$	$\cdot348\ 1689$		66	$\cdot455\ 1549$	$\cdot355\ 9070$		56	$\cdot507\ 3601$	$\cdot361\ 5993$		49
			3791				3255				2851	
33	$\cdot438\ 8428$	$\cdot348\ 5480$		62	$\cdot492\ 9680$	$\cdot356\ 2324$		53	$\cdot551\ 1780$	$\cdot361\ 8844$		46
			3852				3308				2897	
34	$\cdot472\ 4556$	$\cdot348\ 9332$		57	$\cdot532\ 9005$	$\cdot356\ 5632$		49	$\cdot597\ 4436$	$\cdot362\ 1741$		42
			3910				3356				2939	
35	$\cdot507\ 9618$	$\cdot349\ 3242$		52	$\cdot575\ 0721$	$\cdot356\ 8988$		44	$\cdot646\ 2944$	$\cdot362\ 4681$		39
			3962				3400				2978	
36	$\cdot545\ 4718$	$\cdot349\ 7204$		47	$\cdot619\ 6127$	$\cdot357\ 2388$		40	$\cdot697\ 8795$	$\cdot362\ 7659$		35
			4009				3441				3013	
37	$\cdot585\ 1052$	$\cdot350\ 1213$		43	$\cdot666\ 6629$	$\cdot357\ 5829$		36	$\cdot752\ 3605$	$\cdot363\ 0671$		31
			4052				3477				3044	
38	$\cdot626\ 9922$	$\cdot350\ 5265$		38	$\cdot716\ 3753$	$\cdot357\ 9306$		32	$\cdot809\ 9127$	$\cdot363\ 3715$		28
			4090				3509				3072	
39	$\cdot671\ 2739$	$\cdot350\ 9354$		33	$\cdot768\ 9159$	$\cdot358\ 2814$		28	$\cdot870\ 7267$	$\cdot363\ 6787$		24
			4122				3536				3096	
40	$\cdot718\ 1040$	$\cdot351\ 3477$		28	$\cdot824\ 4653$	$\cdot358\ 6350$		23	$\cdot935\ 0097$	$\cdot363\ 9883$		20
			4150				3559				3116	
41	$\cdot767\ 6502$	$\cdot351\ 7627$		22	$\cdot883\ 2199$	$\cdot358\ 9910$		19	$1\cdot002\ 9876$	$\cdot364\ 2998$		16
			4172				3578				3132	
42	$\cdot820\ 0951$	$\cdot352\ 1799$		17	$\cdot945\ 3944$	$\cdot359\ 3488$		14	$\cdot074\ 9063$	$\cdot364\ 6130$		12
			4190				3592				3144	
43	$\cdot875\ 6383$	$\cdot352\ 5989$		12	$1\cdot011\ 2229$	$\cdot359\ 7080$		10	$\cdot151\ 0352$	$\cdot364\ 9274$		9
			4202				3602				3153	
44	$\cdot934\ 4983$	$\cdot353\ 0191$		7	$\cdot080\ 9618$	$\cdot360\ 0682$		6	$\cdot231\ 6680$	$\cdot365\ 2427$		5
			4209				3608				3157	
45	$\cdot996\ 9145$	$\cdot353\ 4399$			$\cdot154\ 8920$	$\cdot360\ 4290$			$\cdot317\ 1271$	$\cdot365\ 5584$		

TABLE LIV—(*continued*).

φ°	r=9				r=10				r=11			
	log F (r, ν)	log H (r, ν)	Δ	Δ²	log F (r, ν)	log H (r, ν)	Δ	Δ²	log F (r, ν)	log H (r, ν)	Δ	Δ²
0	$\bar{1}$·909 9294	0·361 4744			$\bar{1}$·888 2505	0·365 3717			$\bar{1}$·868 5367	0·368 5367		
			49				44				40	
1	·910 4635	·361 4793		97	·888 8502	·365 3761		87	·869 2023	·368 5407		80
			146				131				120	
2	·912 0669	·361 4938		97	·890 6508	·365 3892		87	·871 2002	·368 5527		79
			242				219				199	
3	·914 7427	·361 5180		97	·893 6556	·365 4111		87	·874 5345	·368 5725		79
			339				306				278	
4	·918 4961	·361 5520		96	·897 8705	·365 4416		87	·879 2114	·368 6004		79
			435				392				357	
5	·923 3346	·361 5955		96	·903 3037	·365 4808		86	·885 2402	·368 6361		78
			531				479				435	
6	·929 2675	·361 6485		95	·909 9658	·365 5287		86	·892 6324	·368 6796		78
			626				564				514	
7	·936 3067	·361 7111		94	·917 8700	·365 5851		85	·901 4027	·368 7310		77
			720				649				591	
8	·944 4661	·361 7831		94	·927 0317	·365 6500		84	·911 5681	·368 7900		77
			813				733				668	
9	·953 7619	·361 8644		92	·937 4692	·365 7233		83	·923 1487	·368 8568		76
			906				817				743	
10	·964 2128	·361 9550		92	·949 2033	·365 8050		82	·936 1674	·368 9311		75
			997				899				818	
11	·975 8398	·362 0547		90	·962 2575	·365 8949		82	·950 6505	·369 0129		74
			1088				981				892	
12	·988 6667	·362 1635		89	·976 6581	·365 9929		80	·966 6268	·369 1022		73
			1177				1061				965	
13	0·002 7197	·362 2812		88	·992 4345	·366 0990		79	·984 1288	·369 1987		72
			1264				1139				1037	
14	·018 0278	·362 4075		86	0·009 6193	·366 2129		78	0·003 1923	·369 3024		71
			1350				1217				1108	
15	·034 6230	·362 5426		84	·028 2479	·366 3346		76	·023 8567	·369 4132		69
			1435				1293				1177	
16	·052 5403	·362 6860		83	·048 3595	·366 4639		75	·046 1651	·369 5308		68
			1518				1368				1245	
17	·071 8179	·362 8378		81	·069 9966	·366 6007		73	·070 1646	·369 6553		66
			1599				1441				1311	
18	·092 4974	·362 9976		79	·093 2058	·366 7447		72	·095 9063	·369 7864		65
			1678				1512				1376	
19	·114 6239	·363 1654		77	·118 0374	·366 8960		69	·123 4460	·369 9240		63
			1755				1581				1439	
20	·138 2465	·363 3409		75	·144 5461	·367 0541		68	·152 8439	·370 0679		62
			1830				1649				1500	
21	·163 4180	·363 5239		73	·172 7910	·367 2190		66	·184 1654	·370 2179		60
			1903				1714				1560	
22	·190 1960	·363 7142		71	·202 8360	·367 3904		64	·217 4809	·370 3739		58
			1973				1778				1618	
23	·218 6422	·363 9115		68	·234 7504	·367 5682		61	·252 8669	·370 5357		56
			2042				1839				1673	
24	·248 8237	·364 1157		66	·268 6087	·367 7522		59	·290 4057	·370 7030		54
			2107				1899				1727	
25	·280 8124	·364 3264		63	·304 4913	·367 9420		57	·330 1857	·370 8757		52
			2171				1956				1779	
26	·314 6864	·364 5435		61	·342 4851	·368 1376		55	·372 3033	·371 0536		50
			2231				2010				1829	
27	·350 5295	·364 7666		58	·382 6838	·368 3386		52	·416 8615	·371 2365		47
			2290				2062				1876	
28	·388 4323	·364 9955		55	·425 1882	·368 5448		50	·463 9718	·371 4241		45
			2345				2112				1921	
29	·428 4925	·365 2300		52	·470 1076	·368 7559		47	·513 7544	·371 6162		43
			2397				2159				1964	
30	·470 8156	·365 4697		50	·517 5593	·368 9718		44	·566 3390	·371 8125		40
			2447				2203				2004	
31	·515 5153	·365 7144		47	·567 6703	·369 1922		42	·621 8657	·372 0129		38
			2493				2245				2042	
32	·562 7146	·365 9636		44	·620 5776	·369 4167		39	·680 4854	·372 2171		36
			2537				2284				2078	
33	·612 5463	·366 2173		40	·676 4293	·369 6451		36	·742 3617	·372 4249		33
			2577				2321				2110	
34	·665 1541	·366 4750		37	·735 3855	·369 8772		33	·807 6710	·372 6359		30
			2615				2354				2141	
35	·720 6932	·366 7365		34	·797 6194	·370 1126		31	·876 6045	·372 8500		28
			2649				2385				2169	
36	·779 3322	·367 0013		31	·863 3188	·370 3510		28	·949 3690	·373 0669		25
			2680				2412				2194	
37	·841 2534	·367 2693		28	·932 6869	·370 5923		25	1·026 1888	·373 2862		23
			2707				2437				2216	
38	·906 6549	·367 5400		24	1·005 9444	·370 8360		22	·107 3073	·373 5078		20
			2731				2459				2236	
39	·975 7519	·367 8131		21	·083 3313	·371 0819		19	·192 9888	·373 7314		17
			2752				2478				2253	
40	1·048 7784	·368 0884		18	·165 1080	·371 3297		16	·283 5209	·373 9567		14
			2770				2494				2267	
41	·125 9892	·368 3654		14	·251 5588	·371 5790		13	·379 2165	·374 1834		12
			2784				2506				2279	
42	·207 6624	·368 6438		11	·342 9931	·371 8296		10	·480 4171	·374 4113		9
			2795				2516				2287	
43	·294 1013	·368 9233		7	·439 7491	·372 0813		6	·587 4953	·374 6400		6
			2803				2522				2293	
44	·385 6379	·369 2036		4	·542 1966	·372 3335		4	·700 8587	·374 8693		3
			2806				2526				2296	
45	·482 6360	·369 4842			·650 7407	·372 5861			·820 9540	·375 0990		

TABLE LIV—(*continued*).

ϕ°	$r=12$				$r=13$				$r=14$			
	$\log F(r,\nu)$	$\log H(r,\nu)$	Δ	Δ^2	$\log F(r,\nu)$	$\log H(r,\nu)$	Δ	Δ^2	$\log F(r,\nu)$	$\log H(r,\nu)$	Δ	Δ^2
0	$\bar{1}\cdot850\ 4619$	0·371 1582			$\bar{1}\cdot833\ 7746$	0·373 3653			$\bar{1}\cdot818\ 2772$	0·375 2489		
			36				34				31	
1	·851 1933	·371 1619		73	·834 5719	·373 3686		68	·819 1404	·375 2520		63
			110				101				94	
2	·853 3889	·371 1729		73	·836 9652	·373 3787		67	·821 7316	·375 2614		63
			183				169				157	
3	·857 0528	·371 1912		73	·840 9592	·373 3956		67	·826 0558	·375 2771		62
			255				236				219	
4	·862 1923	·371 2166		72	·846 5615	·373 4192		67	·832 1212	·375 2990		62
			328				303				281	
5	·868 8171	·371 2494		72	·853 7829	·373 4494		67	·839 9395	·375 3271		62
			400				369				343	
6	·876 9402	·371 2893		71	·862 6374	·373 4864		66	·849 5258	·375 3614		61
			471				435				404	
7	·886 5774	·371 3364		71	·873 1421	·373 5299		66	·860 8985	·375 4019		61
			542				501				465	
8	·897 7474	·371 3907		70	·885 3174	·373 5800		65	·874 0798	·375 4484		60
			613				566				526	
9	·910 4722	·371 4519		69	·899 1873	·373 6365		65	·889 0954	·375 5010		60
			682				630				585	
10	·924 7770	·371 5201		69	·914 7789	·373 6995		64	·905 9746	·375 5595		59
			751				694				644	
11	·940 6901	·371 5952		68	·932 1233	·373 7689		63	·924 7510	·375 6239		58
			819				756				703	
12	·958 2436	·371 6771		67	·951 2549	·373 8445		62	·945 4618	·375 6942		57
			886				818				760	
13	·977 4727	·371 7656		66	·972 2124	·373 9263		61	·968 1485	·375 7702		56
			951				879				816	
14	·998 4167	·371 8608		65	·995 0382	·374 0142		60	·992 8571	·375 8518		55
			1016				938				872	
15	0·021 1187	·371 9624		64	0·019 7792	·374 1081		59	0·019 6382	·375 9390		54
			1080				997				926	
16	·045 6258	·372 0703		62	·046 4865	·374 2078		58	·048 5469	·376 0317		53
			1142				1055				980	
17	·071 9896	·372 1845		61	·075 2161	·374 3132		56	·079 6435	·376 1297		52
			1203				1111				1032	
18	·100 2660	·372 3048		59	·106 0288	·374 4243		55	·112 9939	·376 2329		51
			1262				1166				1083	
19	·130 5159	·372 4310		58	·138 9908	·374 5409		53	·148 6693	·376 3412		50
			1320				1219				1133	
20	·162 8054	·372 5630		56	·174 1737	·374 6628		52	·186 7470	·376 4544		48
			1376				1271				1181	
21	·197 2059	·372 7006		55	·211 6551	·374 7899		51	·227 3107	·376 5725		47
			1431				1322				1228	
22	·233 7945	·372 8437		53	·251 5187	·374 9221		49	·270 4509	·376 6952		45
			1484				1370				1273	
23	·272 6547	·372 9921		51	·293 8552	·375 0591		47	·316 2653	·376 8225		44
			1535				1417				1317	
24	·313 8765	·373 1456		49	·338 7623	·375 2008		46	·364 8594	·376 9542		42
			1584				1464				1359	
25	·357 5571	·373 3040		47	·386 3455	·375 3472		43	·416 3469	·377 0901		40
			1632				1506				1399	
26	·403 8013	·373 4672		45	·436 7186	·375 4978		42	·470 8506	·377 2301		39
			1677				1549				1439	
27	·452 7221	·373 6349		43	·490 0042	·375 6527		40	·528 5029	·377 3739		37
			1720				1589				1476	
28	·504 4412	·373 8069		41	·546 3346	·375 8115		38	·589 4465	·377 5215		35
			1762				1627				1511	
29	·559 0903	·373 9831		39	·605 8526	·375 9742		36	·653 8352	·377 6726		33
			1801				1663				1544	
30	·616 8111	·374 1631		37	·668 7120	·376 1405		34	·721 8353	·377 8270		32
			1838				1697				1576	
31	·677 7567	·374 3469		35	·735 0790	·376 3102		32	·793 6258	·377 9846		30
			1873				1729				1606	
32	·742 0923	·374 5341		33	·805 1331	·376 4831		30	·869 4003	·378 1452		28
			1905				1759				1634	
33	·809 9965	·374 7246		30	·879 0680	·376 6589		28	·949 3679	·378 3085		26
			1935				1787				1659	
34	·881 6625	·374 9182		28	·957 0932	·376 8376		26	1·033 7545	·378 4745		24
			1963				1812				1683	
35	·957 2989	·375 1145		25	1·039 4354	·377 0188		23	·122 8047	·378 6428		22
			1988				1836				1705	
36	1·037 1322	·375 3133		23	·126 3402	·377 2024		21	·216 7831	·378 8133		19
			2011				1857				1724	
37	·121 4073	·375 5144		21	·218 0735	·377 3881		19	·315 9768	·378 9857		18
			2032				1876				1742	
38	·210 3905	·375 7176		18	·314 9240	·377 5757		17	·420 6970	·379 1599		15
			2050				1893				1757	
39	·304 3704	·375 9226		16	·417 2053	·377 7649		14	·531 2818	·379 3356		13
			2065				1907				1771	
40	·403 6615	·376 1291		13	·525 2853	·377 9556		12	·648 0989	·379 5127		11
			2078				1919				1782	
41	·508 6058	·376 3370		11	·639 4542	·378 1475		10	·771 5487	·379 6909		9
			2089				1928				1791	
42	·619 5764	·376 5458		8	·760 1977	·378 3403		7	·902 0674	·379 8699		7
			2097				1936				1797	
43	·736 9806	·376 7555		5	·887 9308	·378 5338		5	2·040 1318	·380 0497		5
			2102				1941				1802	
44	·861 2638	·376 9657		3	2·023 1367	·378 7279		2	·186 2627	·380 2299		2
			2105				1943				1804	
45	·992 9140	·377 1762			·166 3448	·378 9222			·341 0309	·380 4103		

TABLE LIV—(*continued*).

$\phi°$	$r=15$				$r=16$				$r=17$			
	log $F(r,\nu)$	log $H(r,\nu)$	Δ	Δ^2	log $F(r,\nu)$	log $H(r,\nu)$	Δ	Δ^2	log $F(r,\nu)$	log $H(r,\nu)$	Δ	Δ^2
0	$\bar{1}$·803 8114	0·376 8754			$\bar{1}$·790 2485	0·378 2941			$\bar{1}$·777 4825	0·379 5425		
			29				28				26	
1	·804 7405	·376 8783		59	·791 2436	·378 2969		55	·778 5436	·379 5450		52
			88				82				78	
2	·807 5297	·376 8871		58	·794 2308	·378 3051		55	·781 7289	·379 5528		52
			146				137				129	
3	·812 1842	·376 9018		58	·799 2158	·378 3188		55	·787 0445	·379 5657		51
			205				192				181	
4	·818 7130	·376 9222		58	·806 2081	·378 3380		54	·794 5004	·379 5838		51
			262				246				232	
5	·827 1285	·376 9485		58	·815 2211	·378 3626		54	·804 1110	·379 6070		51
			321				300				283	
6	·837 4469	·376 9805		57	·826 2719	·378 3927		54	·815 8945	·379 6352		51
			378				354				333	
7	·849 6881	·377 0183		57	·839 3819	·378 4281		53	·829 8736	·379 6686		50
			435				408				384	
8	·863 8758	·377 0617		56	·854 5764	·378 4688		53	·846 0751	·379 7070		50
			491				460				433	
9	·880 0376	·377 1108		56	·871 8848	·378 5149		52	·864 5306	·379 7503		49
			547				513				483	
10	·898 2051	·377 1655		55	·891 3410	·378 5661		52	·885 2758	·379 7986		49
			602				564				531	
11	·918 4141	·377 2256		54	·912 9831	·378 6226		51	·908 3516	·379 8517		48
			656				615				579	
12	·940 7047	·377 2912		54	·936 8541	·378 6841		50	·933 8035	·379 9096		47
			710				666				627	
13	·965 1215	·377 3622		53	·963 0016	·378 7507		49	·961 6821	·379 9723		47
			762				715				673	
14	·991 7138	·377 4384		52	·991 4782	·378 8222		49	·992 0436	·380 0396		46
			814				764				719	
15	0·020 5357	·377 5199		51	0·022 3418	·378 8985		48	0·024 9495	·380 1115		45
			865				811				764	
16	·051 6467	·377 6064		50	·055 6559	·378 9796		47	·060 4672	·380 1878		44
			915				858				808	
17	·085 1115	·377 6979		49	·091 4895	·379 0654		46	·098 6704	·380 2686		43
			964				904				851	
18	·121 0005	·377 7942		48	·129 9181	·379 1558		45	·139 6392	·380 3537		42
			1011				948				893	
19	·159 3904	·377 8953		46	·171 0234	·379 2506		43	·183 4606	·380 4430		41
			1057				992				933	
20	·200 3641	·378 0011		45	·214 8939	·379 3498		42	·230 2288	·380 5363		40
			1102				1034				973	
21	·244 0113	·378 1113		44	·261 6258	·379 4532		41	·280 0460	·380 6336		39
			1146				1075				1012	
22	·290 4293	·378 2259		43	·311 3226	·379 5606		40	·333 0225	·380 7348		37
			1189				1114				1049	
23	·339 7229	·378 3448		41	·364 0964	·379 6721		38	·389 2774	·380 8397		36
			1229				1153				1085	
24	·392 0053	·378 4677		39	·420 0681	·379 7874		37	·448 9393	·380 9482		35
			1269				1190				1120	
25	·447 3985	·378 5946		38	·479 3681	·379 9063		35	·512 1471	·381 0602		33
			1307				1225				1153	
26	·506 0343	·378 7252		36	·542 1372	·380 0289		34	·579 0504	·381 1756		32
			1343				1259				1185	
27	·568 0547	·378 8595		34	·608 5269	·380 1548		33	·649 8104	·381 2941		31
			1377				1292				1216	
28	·633 6129	·378 9973		33	·678 7007	·380 2839		31	·724 6013	·381 4157		29
			1411				1323				1245	
29	·702 8740	·379 1383		31	·752 8356	·380 4162		29	·803 6106	·381 5402		28
			1442				1352				1273	
30	·776 0162	·379 2825		30	·831 1213	·380 5514		28	·887 0408	·381 6674		26
			1471				1380				1299	
31	·853 2318	·379 4296		28	·913 7633	·380 6893		26	·975 1103	·381 7973		24
			1499				1405				1323	
32	·934 7285	·379 5795		26	1·000 9834	·380 8299		24	1·068 0549	·381 9296		23
			1525				1430				1346	
33	1·020 7304	·379 7320		24	·093 0211	·380 9729		23	·166 1294	·382 0642		21
			1549				1452				1367	
34	·111 4801	·379 8869		22	·190 1352	·381 1181		21	·269 6092	·382 2009		20
			1571				1473				1387	
35	·207 2399	·380 0440		20	·292 6060	·381 2654		19	·378 7922	·382 3395		18
			1591				1492				1404	
36	·308 2938	·380 2031		18	·400 7368	·381 4146		17	·494 0009	·382 4800		16
			1610				1509				1420	
37	·414 9495	·380 3641		17	·514 8561	·381 5655		15	·615 5849	·382 6220		14
			1626				1525				1435	
38	·527 5412	·380 5267		14	·635 3206	·381 7180		13	·743 9235	·382 7655		13
			1640				1538				1448	
39	·646 4313	·380 6907		12	·762 5175	·381 8718		12	·879 4285	·382 9102		11
			1653				1549				1458	
40	·772 0144	·380 8560		11	·896 8681	·382 0267		10	2·022 5477	·383 0561		9
			1663				1559				1468	
41	·904 7199	·381 0223		8	2·038 8307	·382 1826		8	·173 7686	·383 2028		7
			1671				1567				1475	
42	2·045 0157	·381 1894		6	·188 9051	·382 3393		6	·333 6229	·383 3503		6
			1678				1573				1480	
43	·193 4131	·381 3572		4	·347 6370	·382 4966		4	·502 6906	·383 4984		4
			1682				1577				1484	
44	·350 4709	·381 5254		2	·515 6232	·382 6543		2	·681 6063	·383 6468		2
			1684				1579				1486	
45	·516 8011	·381 6938			·693 5170	·382 8122			·871 0649	·383 7953		

TABLE LIV—(*continued*).

$\varphi°$	$r=18$				$r=19$				$r=20$			
	$\log F(r,\nu)$	$\log H(r,\nu)$	Δ	$\Delta°$	$\log F(r,\nu)$	$\log H(r,\nu)$	Δ	Δ^2	$\log F(r,\nu)$	$\log H(r,\nu)$	Δ	Δ^2
0	$\bar{1}\cdot765\ 4249$	$0\cdot380\ 6494$			$\bar{1}\cdot754\ 0014$	$0\cdot381\ 6376$			$\bar{1}\cdot743\ 1485$	$0\cdot382\ 5253$		
			24				23				22	
1	·766 5520	·380 6518		49	·755 1945	·381 6399		46	·744 4077	·382 5275		44
			73				70				66	
2	·769 9355	·380 6591		49	·758 7762	·381 6469		46	·748 1876	·382 5341		44
			122				116				110	
3	·775 5818	·380 6713		49	·764 7532	·381 6584		46	·754 4955	·382 5451		44
			171				162				154	
4	·783 5015	·380 6884		48	·773 1369	·381 6746		46	·763 3431	·382 5605		44
			219				208				197	
5	·793 7099	·380 7103		48	·783 9431	·381 6954		46	·774 7474	·382 5802		43
			267				253				241	
6	·806 2262	·380 7370		48	·797 1925	·381 7207		45	·788 7299	·382 6042		43
			315				299				284	
7	·821 0746	·380 7685		47	·812 9104	·381 7505		45	·805 3174	·382 6326		43
			362				343				327	
8	·838 2835	·380 8048		47	·831 1269	·381 7849		45	·824 5417	·382 6652		42
			409				388				369	
9	·857 8863	·380 8457		47	·851 8772	·381 8237		44	·846 4398	·382 7021		42
			456				432				411	
10	·879 9209	·380 8913		46	·875 2015	·381 8669		44	·871 0541	·382 7432		41
			502				476				452	
11	·904 4307	·380 9415		45	·901 1455	·381 9144		43	·898 4326	·382 7883		41
			547				519				493	
12	·931 4638	·380 9962		45	·929 7603	·381 9663		42	·928 6293	·382 8376		40
			592				561				533	
13	·961 0741	·381 0554		44	·961 1026	·382 0224		42	·961 7039	·382 8909		40
			636				603				573	
14	·993 3209	·381 1190		43	·995 2351	·382 0826		41	·997 7224	·382 9482		39
			679				643				611	
15	$0\cdot028\ 2696$	·381 1869		42	$0\cdot032\ 2269$	·382 1470		40	$0\cdot036\ 7574$	·383 0093		39
			722				684				650	
16	·065 9915	·381 2591		42	·072 1535	·382 2154		39	·078 8894	·383 0743		37
			763				723				687	
17	·106 5648	·381 3354		41	·115 0974	·382 2877		38	·124 2043	·383 1430		37
			804				761				724	
18	·150 0744	·381 4157		40	·161 1483	·382 3638		38	·172 7969	·383 2153		36
			843				799				759	
19	·196 6125	·381 5001		39	·210 4036	·382 4437		37	·224 7699	·383 2912		35
			882				836				794	
20	·246 2790	·381 5882		38	·262 9690	·382 5273		36	·280 2346	·383 3706		34
			919				871				828	
21	·299 1823	·381 6802		36	·318 9588	·382 6144		35	·339 3115	·383 4534		33
			956				906				860	
22	·355 4391	·381 7757		35	·378 4966	·382 7049		33	·402 1306	·383 5394		32
			991				939				892	
23	·415 1758	·381 8749		34	·441 7157	·382 7988		32	·468 8327	·383 6286		31
			1025				971				923	
24	·478 5288	·381 9774		33	·508 7603	·382 8960		31	·539 5695	·383 7209		30
			1058				1002				952	
25	·545 6451	·382 0832		31	·579 7858	·382 9962		30	·614 5047	·383 8162		28
			1089				1032				981	
26	·616 6833	·382 1921		30	·654 9597	·383 0994		29	·693 8148	·383 9142		27
			1120				1061				1008	
27	·691 8145	·382 3041		29	·734 4627	·383 2055		27	·777 6902	·384 0150		26
			1148				1088				1034	
28	·771 2230	·382 4189		27	·818 4896	·383 3143		26	·866 3362	·384 1184		25
			1176				1114				1059	
29	·855 1078	·382 5365		26	·907 2506	·383 4257		25	·959 9740	·384 2243		23
			1202				1139				1082	
30	·943 6834	·382 6567		25	$1\cdot000\ 9723$	·383 5396		23	$1\cdot058\ 8424$	·384 3325		22
			1227				1162				1104	
31	$1\cdot037\ 1813$	·382 7794		23	·099 8993	·383 6558		22	·163 1992	·384 4429		21
			1249				1184				1125	
32	·135 8513	·382 9043		22	·204 2956	·383 7742		20	·273 3224	·384 5554		19
			1271				1204				1144	
33	·239 9636	·383 0314		20	·314 4464	·383 8946		19	·389 5124	·384 6698		18
			1291				1223				1162	
34	·349 8099	·383 1605		19	·430 6601	·384 0170		17	·512 0941	·384 7860		17
			1310				1241				1179	
35	·465 7060	·383 2915		17	·553 2702	·384 1411		16	·641 4188	·384 9039		15
			1326				1257				1194	
36	·587 9937	·383 4241		15	·682 6377	·384 2667		14	·777 8668	·385 0233		14
			1342				1271				1208	
37	·717 0435	·383 5583		14	·819 1539	·384 3938		13	·921 8503	·385 1440		12
			1355				1284				1220	
38	·853 2571	·383 6938		12	·963 2435	·384 5222		11	$2\cdot073\ 8165$	·385 2660		11
			1367				1295				1231	
39	·997 0711	·383 8305		10	$2\cdot115\ 3673$	·384 6518		10	·234 2509	·385 3891		9
			1377				1305				1240	
40	$2\cdot148\ 9600$	·383 9683		9	·276 0267	·384 7823		8	·403 6815	·385 5130		8
			1386				1313				1247	
41	·309 4403	·384 1069		7	·445 7673	·384 9136		7	·582 6831	·385 6378		6
			1393				1320				1254	
42	·479 0754	·384 2462		5	·625 1841	·385 0455		5	·771 8822	·385 7632		5
			1398				1324				1258	
43	·658 4799	·384 3860		4	·814 9263	·385 1780		3	·971 9629	·385 8890		3
			1402				1328				1261	
44	·848 3263	·384 5262		2	$3\cdot015\ 7041$	·385 3108		2	$3\cdot183\ 6730$	·386 0151		2
			1403				1330				1263	
45	$3\cdot049\ 3507$	·384 6665			·228 2952	·385 4437			·407 8314	·386 1414		

TABLE LIV—(*continued*).

φ°	r=21				r=22				r=23			
	log F (r, ν)	log H (r, ν)	Δ	Δ²	log F (r, ν)	log H (r, ν)	Δ	Δ²	log F (r, ν)	log H (r, ν)	Δ	Δ²
0	$\bar{1}$·732 8121	0·383 3271			$\bar{1}$·722 9451	0·384 0548			$\bar{1}$·713 5069	0·384 7182		
			21				20				19	
1	734 1352	·383 3292		42	·724 3364	·384 0568		40	·714 9643	·384 7202		38
			63				60				57	
2	·738 1155	·383 3354		42	·728 5130	·384 0628		40	·719 3391	·384 7259		38
			105				100				96	
3	·744 7542	·383 3459		42	·735 4826	·384 0728		40	·726 6397	·384 7355		38
			146				140				134	
4	·754 0660	·383 3606		41	·745 2584	·384 0867		40	·736 8798	·384 7488		38
			188				179				172	
5	·766 0683	·383 3793		41	·757 8590	·384 1047		39	·750 0786	·384 7660		38
			229				219				209	
6	·780 7641	·383 4022		41	·773 3082	·384 1266		39	·766 2613	·384 7869		37
			270				258				247	
7	·798 2414	·383 4293		41	·791 6354	·384 1523		39	·785 4585	·384 8116		37
			311				297				284	
8	·818 4736	·383 4604		40	·812 8757	·384 1820		38	·807 7070	·384 8400		37
			351				335				321	
9	·841 5196	·383 4955		40	·837 0698	·384 2155		38	·833 0493	·384 8721		36
			391				373				357	
10	·867 4241	·383 5346		39	·864 2646	·384 2529		38	·861 5346	·384 9078		36
			430				411				393	
11	·896 2374	·383 5776		39	·894 5129	·384 2940		37	·893 2180	·384 9471		36
			469				448				429	
12	·928 0162	·383 6245		38	·927 8740	·384 3388		37	·928 1617	·384 9899		35
			508				485				464	
13	·962 8233	·383 6753		38	·964 4139	·384 3872		36	·966 4346	·385 0363		34
			545				521				498	
14	0·000 7283	·383 7299		37	0·004 2054	·384 4393		35	0·008 1129	·385 0861		34
			582				556				532	
15	·041 8074	·383 7881		36	·047 3287	·384 4949		35	·053 2804	·385 1393		33
			619				591				565	
16	·086 1444	·383 8500		36	·093 8713	·384 5540		34	·102 0289	·385 1958		33
			654				625				598	
17	·133 8306	·383 9154		35	·143 9291	·384 6164		33	·154 4586	·385 2556		32
			689				658				629	
18	·184 9653	·383 9843		34	·197 6061	·384 6822		32	·210 6783	·385 3185		31
			723				690				660	
19	·239 6563	·384 0566		33	·255 0156	·384 7513		31	·270 8064	·385 3845		30
			756				722				691	
20	·298 0208	·384 1322		32	·316 2801	·384 8235		31	·334 9713	·385 4536		29
			788				753				720	
21	·360 1851	·384 2111		31	·381 5322	·384 8987		30	·403 3116	·385 5256		28
			820				782				748	
22	·426 2861	·384 2930		30	·450 9155	·384 9770		29	·475 9774	·385 6004		28
			850				811				776	
23	·496 4716	·384 3780		29	·524 5848	·385 0581		28	·553 1308	·385 6780		27
			879				839				803	
24	·570 9011	·384 4659		28	·602 7073	·385 1420		27	·634 9466	·385 7583		26
			907				866				828	
25	·649 7464	·384 5566		27	·685 4632	·385 2286		26	·721 6136	·385 8411		25
			934				892				853	
26	·733 1933	·384 6500		26	·773 0472	·385 3178		25	·813 3351	·385 9264		24
			960				916				877	
27	·821 4416	·384 7460		25	·865 6689	·385 4094		24	·910 3304	·386 0141		23
			985				940				899	
28	·914 7070	·384 8445		23	·963 5543	·385 5034		22	1·012 8362	·386 1040		22
			1008				962				921	
29	1·013 2222	·384 9453		22	1·066 9472	·385 5996		21	·121 1074	·386 1961		20
			1031				984				941	
30	·117 2380	·385 0484		21	·176 1108	·385 6980		20	·235 4191	·386 2902		19
			1052				1004				960	
31	·227 0250	·385 1535		20	·291 3286	·385 7984		19	·356 0681	·386 3862		18
			1071				1023				978	
32	·342 8756	·385 2607		18	·412 9072	·385 9007		18	·483 3750	·386 4840		17
			1090				1040				995	
33	·465 1055	·385 3696		17	·541 1773	·386 0047		16	·617 6859	·386 5836		16
			1107				1057				1011	
34	·594 0558	·385 4803		16	·676 4967	·386 1104		15	·759 3748	·386 6846		14
			1123				1072				1025	
35	·730 0957	·385 5926		14	·819 2523	·386 2175		14	·908 8466	·386 7871		13
			1137				1085				1038	
36	·873 6248	·385 7063		13	·969 8630	·386 3261		12	2·066 5394	·386 8910		12
			1150				1098				1050	
37	2·025 0761	·385 8213		12	2·128 7827	·386 4359		11	·232 9279	·386 9960		11
			1162				1109				1061	
38	·184 9195	·385 9375		10	·296 5039	·386 5468		10	·408 5272	·387 1021		10
			1172				1119				1070	
39	·353 6651	·386 0547		9	·473 5612	·386 6586		8	·593 8968	·387 2091		8
			1181				1127				1078	
40	·531 8676	·386 1728		7	·660 5361	·386 7713		7	·789 6446	·387 3169		7
			1188				1134				1085	
41	·720 1308	·386 2916		6	·858 0615	·386 8848		6	·996 4325	·387 4254		5
			1194				1140				1090	
42	·919 1130	·386 4110		4	3·066 8272	·386 9987		4	3·214 9823	·387 5344		4
			1198				1144				1094	
43	3·129 5327	·386 5308		3	·287 5865	·387 1131		3	·446 0817	·387 6438		3
			1201				1147				1097	
44	·352 1756	·386 6510		2	·521 1629	·387 2278		1	·690 5919	·387 7535		1
			1203				1148				1098	
45	·587 9021	·386 7712			·768 4580	·387 3426			·949 4561	·387 8633		

TABLE LIV—(*continued*).

$\phi°$	$r=24$				$r=25$				$r=26$			
	$\log F(r,\nu)$	$\log H(r,\nu)$	Δ	Δ^2	$\log F(r,\nu)$	$\log H(r,\nu)$	Δ	Δ^2	$\log F(r,\nu)$	$\log H(r,\nu)$	Δ	Δ^2
0	$\bar{1}\cdot704\ 4618$	$0\cdot385\ 3256$			$\bar{1}\cdot695\ 7781$	$0\cdot385\ 8838$			$\bar{1}\cdot687\ 4284$	$0\cdot386\ 3984$		
			18				18				17	
1	·705 9852	·385 3275		37	·697 3569	·385 8855		35	·689 0840	·386 4001		34
			55				53				51	
2	·710 5584	·385 3330		37	·702 1393	·385 8908		35	·694 0540	·386 4052		34
			92				88				85	
3	·718 1899	·385 3421		36	·710 1019	·385 8996		35	·702 3476	·386 4136		34
			128				123				118	
4	·728 8942	·385 3549		36	·721 2704	·385 9119		35	·713 9805	·386 4254		34
			164				158				152	
5	·742 6914	·385 3714		36	·735 6660	·385 9277		35	·728 9746	·386 4406		33
			201				193				185	
6	·759 6077	·385 3915		36	·753 3158	·385 9470		35	·747 3581	·386 4591		33
			236				227				218	
7	·779 6750	·385 4151		36	·774 2534	·385 9697		34	·769 1658	·386 4810		33
			272				261				251	
8	·802 9317	·385 4423		35	·798 5185	·385 9958		34	·794 4395	·386 5061		33
			307				295				284	
9	·829 4225	·385 4730		35	·826 1578	·386 0253		34	·823 2272	·386 5345		32
			342				329				316	
10	·859 1983	·385 5073		34	·857 2243	·386 0582		33	·855 5847	·386 5661		32
			377				362				348	
11	·892 3170	·385 5450		34	·891 7784	·386 0943		33	·891 5743	·386 6009		31
			411				394				379	
12	·928 8434	·385 5860		34	·929 8876	·386 1338		32	·931 2665	·386 6388		31
			444				427				411	
13	·968 8495	·385 6305		33	·971 6270	·386 1764		32	·974 7393	·386 6798		30
			477				458				440	
14	$0\cdot012\ 4148$	·385 6782		32	$0\cdot017\ 0795$	·386 2223		31	$0\cdot022\ 0791$	·386 7239		30
			510				489				471	
15	·059 6268	·385 7292		32	·066 3362	·386 2712		31	·073 3806	·386 7710		29
			542				520				500	
16	·110 5814	·385 7834		31	·119 4971	·386 3232		30	·128 7480	·386 8210		29
			573				550				529	
17	·165 3831	·385 8406		30	·176 6711	·386 3782		29	·188 2945	·386 8738		28
			603				579				557	
18	·224 1458	·385 9009		30	·237 9768	·386 4361		29	·252 1435	·386 9295		27
			633				608				584	
19	·286 9928	·385 9642		29	·303 5430	·386 4969		28	·320 4290	·386 9880		27
			662				635				611	
20	·354 0583	·386 0304		28	·373 5093	·386 5604		27	·393 2963	·387 0491		26
			690				662				637	
21	·425 4870	·386 0994		27	·448 0267	·386 6267		26	·470 9025	·387 1128		25
			717				689				662	
22	·501 4356	·386 1711		26	·527 2584	·386 6955		25	·553 4176	·387 1790		24
			744				714				687	
23	·582 0734	·386 2455		26	·611 3809	·386 7669		25	·641 0250	·387 2477		24
			769				739				710	
24	·667 5830	·386 3225		25	·700 5843	·386 8408		24	·733 9226	·387 3187		23
			794				762				733	
25	·758 1612	·386 4018		24	·795 0741	·386 9170		23	·832 3242	·387 3920		22
			818				785				755	
26	·854 0205	·386 4836		23	·895 0715	·386 9955		22	·936 4600	·387 4674		21
			840				807				776	
27	·955 3899	·386 5676		22	1·000 8153	·387 0762		21	1·046 5783	·387 5450		20
			862				827				796	
28	1·062 5163	·386 6538		21	·112 5627	·387 1589		20	·162 9470	·387 6246		19
			882				847				815	
29	·175 6661	·386 7420		19	·230 5913	·387 2436		19	·285 8547	·387 7060		18
			902				866				833	
30	·295 1263	·386 8322		18	·355 2004	·387 3302		18	·415 6129	·387 7893		17
			920				883				850	
31	·421 2069	·386 9242		17	·486 7129	·387 4185		17	·552 5576	·387 8742		16
			938				900				865	
32	·554 2426	·387 0180		16	·625 4776	·387 5085		16	·697 0516	·387 9608		15
			954				916				880	
33	·694 5945	·387 1134		15	·771 8709	·387 6001		14	·849 4867	·388 0488		14
			969				930				894	
34	·842 6534	·387 2102		14	·926 3001	·387 6931		13	2·010 2864	·388 1382		13
			982				943				907	
35	·998 8417	·387 3085		13	2·089 2053	·387 7874		12	·179 9088	·388 2289		12
			995				955				919	
36	2·163 6169	·387 4080		11	·261 0633	·387 8829		11	·358 8499	·388 3208		11
			1006				966				929	
37	·337 4747	·387 5086		10	·442 3906	·387 9795		10	·547 6471	·388 4137		9
			1017				976				938	
38	·520 9526	·387 6103		9	·633 7476	·388 0771		9	·746 8833	·388 5075		8
			1026				985				947	
39	·714 6347	·387 7128		8	·835 7426	·388 1756		7	·957 1916	·388 6022		7
			1033				992				954	
40	·919 1558	·387 8162		6	3·049 0373	·388 2748		6	3·179 2602	·388 6975		6
			1040				998				960	
41	3·135 2068	·387 9201		5	·274 3517	·388 3746		5	·413 8384	·388 7935		5
			1045				1003				964	
42	·363 5411	·388 0246		3	·512 4708	·388 4749		4	·661 7426	·388 8900		4
			1048				1007				968	
43	·604 9809	·388 1295		3	·764 2515	·388 5756		2	·923 8647	·388 9868		2
			1051				1009				970	
44	·860 4254	·388 2346		1	4·030 6307	·388 6765		1	4·201 1786	·389 0838		1
			1052				1010				972	
45	4·130 8591	·388 3398			·312 6342	·388 7775			·494 7524	·389 1810		

TABLE LIV—(*continued*).

$\phi°$	$r=27$				$r=28$				$r=29$			
	$\log F(r,\nu)$	$\log H(r,\nu)$	Δ	Δ^2	$\log F(r,\nu)$	$\log H(r,\nu)$	Δ	Δ^2	$\log F(r,\nu)$	$\log H(r,\nu)$	Δ	Δ^2
0	$\bar{1}\cdot679\ 3877$	$0\cdot386\ 8744$			$\bar{1}\cdot671\ 6341$	$0\cdot387\ 3160$			$\bar{1}\cdot664\ 1478$	$0\cdot387\ 7268$		
			16				16				15	
1	·681 1094	·386 8760		33	·673 4219	·387 3176		31	·666 0017	·387 7283		30
			49				47				46	
2	·686 2778	·386 8809		33	·678 7887	·387 3223		31	·671 5669	·387 7328		30
			81				79				76	
3	·694 9025	·386 8891		32	·687 7445	·387 3301		31	·680 8538	·387 7404		30
			114				110				106	
4	·706 9998	·386 9005		32	·700 3062	·387 3411		31	·693 8799	·387 7510		30
			146				141				136	
5	·722 5923	·386 9151		32	·716 4973	·387 3552		31	·710 6695	·387 7647		30
			178				172				166	
6	·741 7096	·386 9329		32	·736 3483	·387 3724		31	·731 2544	·387 7813		30
			210				203				196	
7	·764 3877	·386 9539		32	·759 8968	·387 3927		30	·755 6734	·387 8008		30
			242				233				225	
8	·790 6698	·386 9781		31	·787 1876	·387 4160		30	·783 9728	·387 8234		29
			273				264				254	
9	·820 6063	·387 0055		31	·818 2728	·387 4424		30	·816 2068	·387 8488		29
			304				293				283	
10	·854 2546	·387 0359		31	·853 2121	·387 4717		30	·852 4372	·387 8772		29
			335				323				312	
11	·891 6799	·387 0694		30	·892 0731	·387 5041		29	·892 7340	·387 9084		28
			365				352				340	
12	·932 9552	·387 1059		30	·934 9315	·387 5393		29	·937 1757	·387 9424		28
			395				381				368	
13	·978 1615	·387 1454		29	·981 8715	·387 5774		28	·985 8495	·387 9792		27
			424				409				395	
14	0·027 3887	·387 1879		29	0·032 9863	·387 6183		28	0·038 8519	·388 0187		27
			453				437				422	
15	·080 7353	·387 2332		28	·088 3780	·387 6620		27	·096 2888	·388 0609		26
			482				464				448	
16	·138 3092	·387 2814		28	·148 1586	·387 7084		27	·158 2763	·388 1057		26
			509				491				474	
17	·200 2283	·387 3323		27	·212 4505	·387 7576		26	·224 9410	·388 1531		25
			536				517				499	
18	·266 6208	·387 3859		26	·281 3865	·387 8093		25	·296 4208	·388 2031		25
			563				543				524	
19	·337 6258	·387 4422		26	·355 1112	·387 8635		25	·372 8653	·388 2555		24
			588				567				548	
20	·413 3944	·387 5010		25	·433 7811	·387 9203		24	·454 4367	·388 3103		23
			613				592				571	
21	·494 0896	·387 5624		24	·517 5657	·387 9794		23	·541 3106	·388 3674		23
			638				615				594	
22	·579 8882	·387 6261		24	·606 6479	·388 0409		23	·633 6767	·388 4268		22
			661				638				616	
23	·670 9806	·387 6923		23	·701 2256	·388 1047		22	·731 7398	·388 4883		21
			684				660				637	
24	·767 5727	·387 7607		22	·801 5122	·388 1706		21	·835 7212	·388 5520		20
			706				681				657	
25	·869 8863	·387 8312		21	·907 7380	·388 2387		20	·945 8594	·388 6177		20
			727				701				677	
26	·978 1616	·387 9039		20	1·020 1512	·388 3088		19	1·062 4115	·388 6854		19
			747				720				696	
27	1·092 6538	·387 9786		19	·139 0194	·388 3808		18	·185 6549	·388 7550		17
			766				739				713	
28	·213 6439	·388 0552		18	·264 6312	·388 4547		18	·315 8886	·388 8263		17
			784				756				730	
29	·341 4310	·388 1337		17	·397 2979	·388 5303		17	·453 4351	·388 8993		16
			802				773				747	
30	·476 3386	·388 2138		16	·537 3550	·388 6077		16	·598 6420	·388 9740		15
			818				789				762	
31	·618 7157	·388 2956		15	·685 1648	·388 6865		15	·751 8846	·389 0501		14
			833				804				776	
32	·768 9393	·388 3790		14	·841 1182	·388 7669		14	·913 5680	·389 1277		13
			848				818				789	
33	·927 4164	·388 4638		13	2·005 6375	·388 8487		13	2·084 1297	·389 2067		12
			861				830				802	
34	2·094 5869	·388 5499		12	·179 1790	·388 9317		12	·264 0426	·389 2868		11
			873				842				813	
35	·270 9268	·388 6372		11	·362 2366	·389 0159		11	·453 8181	·389 3682		10
			885				853				824	
36	·456 9512	·388 7257		10	·555 3447	·389 1012		10	·654 0100	·389 4505		9
			895				863				833	
37	·653 2186	·388 8151		9	·759 0825	·389 1875		9	·865 2184	·389 5338		8
			904				872				841	
38	·860 3344	·388 9055		8	·974 0781	·389 2746		7	3·088 0940	·389 6179		7
			912				879				849	
39	3·078 9562	·388 9967		7	3·201 0137	·389 3625		7	·323 3436	·389 7028		6
			918				886				855	
40	·309 7991	·389 0885		6	·440 6310	·389 4511		5	·571 7357	·389 7883		5
			924				891				860	
41	·553 6412	·389 1809		5	·693 7374	·389 5402		4	·834 1065	·389 8744		4
			929				896				865	
42	·811 3309	·389 2738		3	·961 2127	·389 6298		3	4·111 3678	·389 9608		3
			932				899				868	
43	4·083 7948	·389 3670		2	4·244 0178	·389 7197		2	·404 5148	·390 0476		2
			934				901				870	
44	·372 0436	·389 4604		1	·543 2028	·389 8098		1	·714 6355	·390 1346		1
			936				902				871	
45	·677 1878	·389 5540			·859 9176	·389 9000			5·042 9213	·390 2217		

TABLE LIV—(*continued*).

$\phi°$	$r=30$				$r=31$				$r=32$			
	$\log F(r,\nu)$	$\log H(r,\nu)$	Δ	Δ^2	$\log F(r,\nu)$	$\log H(r,\nu)$	Δ	Δ^2	$\log F(r,\nu)$	$\log H(r,\nu)$	Δ	Δ^2
0	$\bar{1}\cdot656\,9109$	$0\cdot388\,1099$			$\bar{1}\cdot649\,9073$	$0\cdot388\,4679$			$\bar{1}\cdot643\,1226$	$0\cdot388\,8034$		
			15				14				14	
1	·658 8309	·388 1113		29	·651 8935	·388 4694		28	·645 1748	·388 8048		28
			44				43				41	
2	·664 5945	·388 1157		29	·657 8555	·388 4736		28	·651 3354	·388 8089		27
			73				71				69	
3	·674 2126	·388 1231		29	·667 8048	·388 4808		28	·661 6158	·388 8158		27
			103				99				96	
4	·687 7031	·388 1333		29	·681 7598	·388 4906		28	·676 0352	·388 8254		27
			132				127				123	
5	·705 0914	·388 1465		29	·699 7466	·388 5034		28	·694 6208	·388 8378		27
			161				155				151	
6	·726 4101	·388 1625		29	·721 7993	·388 5189		28	·717 4073	·388 8528		27
			189				183				177	
7	·751 6995	·388 1815		29	·747 9592	·388 5372		28	·744 4379	·388 8706		27
			218				211				204	
8	·781 0077	·388 2032		28	·778 2762	·388 5583		27	·775 7637	·388 8910		26
			246				238				231	
9	·814 3906	·388 2278		28	·812 8080	·388 5822		27	·811 4444	·388 9140		26
			274				265				257	
10	·851 9121	·388 2552		28	·851 6207	·388 6086		27	·851 5483	·388 9397		26
			302				292				283	
11	·893 6447	·388 2854		27	·894 7893	·388 6378		26	·896 1530	·388 9680		26
			329				318				308	
12	·939 6698	·388 3183		27	·942 3977	·388 6696		26	·945 3449	·388 9988		25
			356				344				333	
13	·990 0775	·388 3538		26	·994 5394	·388 7041		26	·999 2205	·389 0322		25
			382				370				358	
14	0·044 9676	·388 3920		26	0·051 3173	·388 7410		25	0·057 8864	·389 0680		24
			408				395				383	
15	·104 4498	·388 4328		25	·112 8450	·388 7805		25	·121 4595	·389 1062		24
			433				419				406	
16	·168 6443	·388 4762		25	·179 2464	·388 8225		24	·190 0681	·389 1469		23
			458				444				430	
17	·237 6820	·388 5220		24	·250 6573	·388 8668		23	·263 8522	·389 1899		23
			483				467				453	
18	·311 7056	·388 5703		24	·327 2249	·388 9136		23	·342 9638	·389 2351		22
			507				490				475	
19	·390 8701	·388 6209		23	·409 1093	·388 9626		22	·427 5684	·389 2826		22
			530				513				497	
20	·475 3432	·388 6739		22	·496 4842	·389 0138		22	·517 8452	·389 3323		21
			552				534				518	
21	·565 3065	·388 7291		22	·589 5372	·389 0673		21	·613 9879	·389 3840		20
			574				556				538	
22	·660 9566	·388 7865		21	·688 4714	·389 1228		20	·716 2063	·389 4379		20
			595				576				558	
23	·762 5053	·388 8460		20	·793 5058	·389 1804		20	·824 7266	·389 4937		19
			616				596				577	
24	·870 1816	·388 9076		20	·904 8771	·389 2400		19	·939 7929	·389 5514		19
			635				615				596	
25	·984 2323	·388 9711		19	1·022 8405	·389 3015		18	1·061 6692	·389 6109		18
			654				633				613	
26	1·104 9235	·389 0366		18	·147 6710	·389 3648		18	·190 6391	·389 6723		17
			672				650				630	
27	·232 5423	·389 1038		17	·279 6653	·389 4299		17	·327 0091	·389 7353		16
			690				667				646	
28	·367 3981	·389 1727		16	·419 1433	·389 4966		16	·471 1094	·389 8000		15
			706				683				662	
29	·509 8245	·389 2433		16	·566 4498	·389 5649		15	·623 2962	·389 8661		15
			722				698				677	
30	·660 1814	·389 3155		15	·721 9568	·389 6348		14	·783 9534	·389 9338		14
			736				713				690	
31	·818·8570	·389 3891		14	·886 0657	·389 7060		13	·953 4956	·390 0028		13
			750				726				703	
32	·986 2707	·389 4642		13	2·059 2096	·389 7786		13	2·132 3701	·390 0732		12
			763				738				715	
33	2·162 8749	·389 5405		12	·241 8567	·389 8525		12	·321 0601	·390 1447		11
			775				750				727	
34	·349 1593	·389 6180		11	·434 5127	·389 9275		11	·520 0879	·390 2174		10
			786				761				737	
35	·545 6529	·389 6966		10	·637 7246	·390 0035		10	·730 0183	·390 2911		9
			796				770				746	
36	·752 9288	·389 7762		9	·852 0847	·390 0806		9	·951 4628	·390 3657		9
			805				779				755	
37	·971 6080	·389 8567		8	3·078 2349	·390 1585		8	3·185 0841	·390 4412		8
			813				787				763	
38	3·202 3639	·389 9380		7	·316 8712	·390 2372		7	·431 6009	·390 5174		7
			820				794				769	
39	·445 9277	·390 0201		6	·568 7494	·390 3166		6	·691 7938	·390 5944		6
			827				800				775	
40	·703 0947	·390 1028		5	·834 6915	·390 3966		5	·966 5111	·390 6719		5
			832				805				780	
41	·974 7302	·390 1859		4	4·115 5919	·390 4771		4	4·256 6765	·390 7498		4
			836				809				784	
42	4·261 7776	·390 2695		3	·412 4256	·390 5580		3	·563 2967	·390 8282		3
			839				812				787	
43	·565 2668	·390 3534		2	·726 2571	·390 6392		2	·887 4707	·390 9069		2
			841				814				789	
44	·886 3234	·390 4375		1	5·058 2499	·390 7206		1	5·230 3999	·390 9857		1
			842				815				789	
45	5·226 1804	·390 5217			·409 6782	·390 8021			·593 3997	·391 0646		

TABLE LIV—(*continued*).

ϕ°	$r=33$				$r=34$				$r=35$			
	$\log F(r,\nu)$	$\log H(r,\nu)$	Δ	Δ^2	$\log F(r,\nu)$	$\log H(r,\nu)$	Δ	Δ^2	$\log F(r,\nu)$	$\log H(r,\nu)$	Δ	Δ^2
0	$\bar{1}\cdot636\ 5434$	$0\cdot389\ 1183$			$\bar{1}\cdot630\ 1576$	$0\cdot389\ 4146$			$\bar{1}\cdot623\ 9542$	$0\cdot389\ 6937$		
			13				13				13	
1	·638 6617	·389 1197		27	·632 3421	·389 4159		26	·626 2048	·389 6949		25
			40				39				38	
2	·645 0208	·389 1237		27	·638 8996	·389 4198		26	·632 9608	·389 6987		25
			67				65				63	
3	·655 6323	·389 1304		26	·649 8424	·389 4262		26	·644 2348	·389 7050		25
			93				91				88	
4	·670 5163	·389 1397		26	·665 1908	·389 4353		26	·660 0478	·389 7138		25
			120				116				113	
5	·689 7005	·389 1516		26	·684 9738	·389 4469		26	·680 4295	·389 7251		25
			146				142				138	
6	·713 2210	·389 1662		26	·709 2283	·389 4611		25	·705 4180	·389 7389		25
			172				167				162	
7	·741 1222	·389 1835		26	·738 0001	·389 4778		25	·735 0604	·389 7551		24
			198				192				187	
8	·773 4568	·389 2033		26	·771 3436	·389 4970		25	·769 4129	·389 7738		24
			224				217				211	
9	·810 2865	·389 2256		25	·809 3225	·389 5187		25	·808 5407	·389 7948		24
			249				242				235	
10	·851 6818	·389 2505		25	·852 0090	·389 5429		24	·852 5188	·389 8183		24
			274				266				259	
11	·897 7225	·389 2780		25	·899 4858	·389 5695		24	·901 4317	·389 8442		23
			299				290				282	
12	·948 4980	·389 3078		24	·951 8449	·389 5985		24	·955 3744	·389 8724		23
			323				314				305	
13	0·004 1077	·389 3402		24	0·009 1887	·389 6299		23	0·014 4525	·389 9029		23
			347				337				328	
14	·064 6615	·389 3749		24	·071 6306	·389 6636		23	·078 7824	·389 9356		22
			371				360				350	
15	·130 2802	·389 4120		23	·139 2949	·389 6996		23	·148 4925	·389 9706		22
			394				382				372	
16	·201 0960	·389 4514		23	·212 3180	·389 7379		22	·223 7229	·390 0078		21
			417				405				393	
17	·277 2533	·389 4931		22	·290 8487	·389 7783		21	·304 6270	·390 0471		21
			439				426				414	
18	·358 9091	·389 5370		22	·375 0487	·389 8209		21	·391 3713	·390 0884		20
			461				447				434	
19	·446 2340	·389 5830		21	·465 0938	·389 8656		20	·484 1369	·390 1319		20
			482				467				454	
20	·539 4127	·389 6312		20	·561 1746	·389 9123		20	·583 1198	·390 1773		19
			502				487				473	
21	·638 6453	·389 6814		20	·663 4971	·389 9611		19	·688 5323	·390 2246		19
			522				507				492	
22	·744 1480	·389 7336		19	·772 2842	·390 0117		19	·800 6039	·390 2738		18
			541				525				510	
23	·856 1542	·389 7877		19	·887 7765	·390 0643		18	·919 5823	·390 3248		17
			560				543				528	
24	·974 9160	·389 8437		18	1·010 2336	·390 1186		17	1·045 7350	·390 3776		17
			578				561				545	
25	1·100 7050	·389 9014		17	·139 9357	·390 1746		17	·179 3502	·390 4321		16
			595				577				561	
26	·233 8145	·389 9609		17	·277 1848	·390 2324		16	·320 7390	·390 4881		16
			611				593				576	
27	·374 5602	·390 0220		16	·422 3065	·390 2917		15	·470 2366	·390 5458		15
			627				609				591	
28	·523 2830	·390 0847		15	·575 6518	·390 3525		14	·628 2047	·390 6049		14
			642				623				605	
29	·680 3501	·390 1489		14	·737 5995	·390 4148		14	·795 0329	·390 6654		13
			656				637				619	
30	·846 1578	·390 2145		13	·908 5576	·390 4785		13	·971 1417	·390 7273		13
			669				650				631	
31	2·021 1335	·390 2815		13	2·088 9669	·390 5435		12	2·156 9847	·390 7904		12
			682				662				643	
32	·205 7387	·390 3497		12	·279 3029	·390 6097		11	·353 0516	·390 8547		11
			693				673				654	
33	·400 4717	·390 4190		11	·480 0791	·390 6770		11	·559 8711	·390 9201		10
			705				684				664	
34	·605 8714	·390 4895		10	·691 8509	·390 7454		10	·778 0150	·390 9865		9
			715				694				674	
35	·822 5204	·390 5610		9	·915 2186	·390 8148		9	3·008 1016	·391 0539		9
			724				703				682	
36	3·051 0494	·390 6333		8	3·150 8322	·390 8850		8	·250 8000	·391 1222		8
			732				711				690	
37	·292 1420	·390 7065		7	·399 3963	·390 9561		7	·506 8356	·391 1912		7
			739				718				697	
38	·546 5396	·390 7805		6	·661 6747	·391 0278		6	·776 9950	·391 2609		6
			746				724				703	
39	·815 0471	·390 8551		6	·938 4971	·391 1002		5	4·062 1324	·391 3312		5
			751				729				709	
40	4·098 5399	·390 9302		5	4·230 7654	·391 1732		4	·363 1764	·391 4021		4
			756				734				713	
41	·397 9704	·391 0058		4	·539 4613	·391 2466		4	·681 1377	·391 4734		4
			760				738				716	
42	·714 3773	·391 0818		3	·865 6549	·391 3203		3	5·017 1183	·391 5450		3
			763				740				719	
43	5·048 3939	·391 1581		2	5·210 5144	·391 3943		2	·372 3207	·391 6169		2
			765				742				721	
44	·402 7596	·391 2345		1	·575 3166	·391 4686		1	·748 0596	·391 6890		1
			766				743				722	
45	·777 3311	·391 3111			·961 4600	·391 5429			6·145 7749	·391 7612		

TABLE LIV—(*continued*).

$\phi°$	$r=36$				$r=37$				$r=38$			
	$\log F(r,\nu)$	$\log H(r,\nu)$	Δ	Δ^2	$\log F(r,\nu)$	$\log H(r,\nu)$	Δ	Δ^2	$\log F(r,\nu)$	$\log H(r,\nu)$	Δ	Δ^2
0	$\bar{1}\cdot617\ 9231$	$0\cdot389\ 9572$			$\bar{1}\cdot612\ 0550$	$\cdot0390\ 2063$			$\bar{1}\cdot606\ 3413$	$0.390\ 4421$		
			12				12				12	
1	$\cdot620\ 2399$	$\cdot389\ 9584$		24	$\cdot614\ 4378$	$\cdot390\ 2074$		24	$\cdot608\ 7902$	$\cdot390\ 4433$		23
			37				36				35	
2	$\cdot627\ 1943$	$\cdot389\ 9620$		24	$\cdot621\ 5908$	$\cdot390\ 2110$		24	$\cdot616\ 1417$	$\cdot390\ 4468$		23
			61				60				58	
3	$\cdot638\ 7995$	$\cdot389\ 9682$		24	$\cdot633\ 5272$	$\cdot390\ 2170$		24	$\cdot628\ 4093$	$\cdot390\ 4526$		23
			85				83				81	
4	$\cdot655\ 0771$	$\cdot389\ 9767$		24	$\cdot650\ 2694$	$\cdot390\ 2253$		24	$\cdot645\ 6161$	$\cdot390\ 4607$		23
			110				107				104	
5	$\cdot676\ 0575$	$\cdot389\ 9877$		24	$\cdot671\ 8485$	$\cdot390\ 2360$		23	$\cdot667\ 7940$	$\cdot390\ 4711$		23
			134				130				127	
6	$\cdot701\ 7800$	$\cdot390\ 0011$		24	$\cdot698\ 3051$	$\cdot390\ 2490$		23	$\cdot694\ 9847$	$\cdot390\ 4837$		23
			158				154				149	
7	$\cdot732\ 2932$	$\cdot390\ 0168$		24	$\cdot729\ 6890$	$\cdot390\ 2643$		23	$\cdot727\ 2393$	$\cdot390\ 4987$		23
			182				177				172	
8	$\cdot767\ 6546$	$\cdot390\ 0350$		24	$\cdot766\ 0594$	$\cdot390\ 2820$		23	$\cdot764\ 6187$	$\cdot390\ 5159$		22
			205				200				194	
9	$\cdot807\ 9316$	$\cdot390\ 0555$		23	$\cdot807\ 4855$	$\cdot390\ 3020$		23	$\cdot807\ 1940$	$\cdot390\ 5353$		22
			228				222				216	
10	$\cdot853\ 2010$	$\cdot390\ 0783$		23	$\cdot854\ 0464$	$\cdot390\ 3242$		22	$\cdot855\ 0464$	$\cdot390\ 5569$		22
			251				245				238	
11	$\cdot903\ 5502$	$\cdot390\ 1035$		23	$\cdot905\ 8318$	$\cdot390\ 3486$		22	$\cdot908\ 2680$	$\cdot390\ 5808$		22
			274				267				260	
12	$\cdot959\ 0766$	$\cdot390\ 1309$		22	$\cdot962\ 9420$	$\cdot390\ 3753$		22	$\cdot966\ 9620$	$\cdot390\ 6067$		21
			296				288				281	
13	$0\cdot019\ 8889$	$\cdot390\ 1605$		22	$0\cdot025\ 4886$	$\cdot390\ 4041$		21	$0\cdot031\ 2429$	$\cdot390\ 6348$		21
			318				310				302	
14	$\cdot086\ 1070$	$\cdot390\ 1924$		22	$\cdot093\ 5948$	$\cdot390\ 4351$		21	$\cdot101\ 2374$	$\cdot390\ 6650$		20
			340				331				322	
15	$\cdot157\ 8628$	$\cdot390\ 2264$		21	$\cdot167\ 3965$	$\cdot390\ 4682$		21	$\cdot177\ 0849$	$\cdot390\ 6972$		20
			361				352				342	
16	$\cdot235\ 3007$	$\cdot390\ 2625$		21	$\cdot247\ 0418$	$\cdot390\ 5034$		20	$\cdot258\ 9378$	$\cdot390\ 7314$		20
			382				372				362	
17	$\cdot318\ 5782$	$\cdot390\ 3007$		20	$\cdot332\ 6929$	$\cdot390\ 5405$		20	$\cdot346\ 9624$	$\cdot390\ 7676$		19
			402				391				381	
18	$\cdot407\ 8669$	$\cdot390\ 3409$		20	$\cdot424\ 5260$	$\cdot390\ 5797$		19	$\cdot441\ 3400$	$\cdot390\ 8057$		19
			422				411				400	
19	$\cdot503\ 3529$	$\cdot390\ 3832$		19	$\cdot522\ 7325$	$\cdot390\ 6208$		19	$\cdot542\ 2671$	$\cdot390\ 8457$		18
			441				430				418	
20	$\cdot605\ 2380$	$\cdot390\ 4273$		19	$\cdot627\ 5199$	$\cdot390\ 6637$		18	$\cdot649\ 9569$	$\cdot390\ 8876$		18
			460				448				436	
21	$\cdot713\ 7407$	$\cdot390\ 4733$		18	$\cdot739\ 1128$	$\cdot390\ 7085$		18	$\cdot764\ 6400$	$\cdot390\ 9312$		17
			478				466				453	
22	$\cdot829\ 0968$	$\cdot390\ 5212$		18	$\cdot857\ 7536$	$\cdot390\ 7551$		17	$\cdot886\ 5655$	$\cdot390\ 9765$		17
			496				483				470	
23	$\cdot951\ 5615$	$\cdot390\ 5708$		17	$\cdot983\ 7045$	$\cdot390\ 8033$		16	$1\cdot016\ 0028$	$\cdot391\ 0235$		16
			513				499				486	
24	$1\cdot081\ 4096$	$\cdot390\ 6221$		16	$1\cdot117\ 2483$	$\cdot390\ 8532$		16	$\cdot153\ 2423$	$\cdot391\ 0721$		16
			529				515				502	
25	$\cdot218\ 9381$	$\cdot390\ 6750$		16	$\cdot258\ 6900$	$\cdot390\ 9048$		15	$\cdot298\ 5974$	$\cdot391\ 1223$		15
			545				530				517	
26	$\cdot364\ 4667$	$\cdot390\ 7296$		15	$\cdot408\ 3585$	$\cdot390\ 9578$		15	$\cdot452\ 4059$	$\cdot391\ 1739$		14
			560				545				531	
27	$\cdot518\ 3405$	$\cdot390\ 7856$		14	$\cdot566\ 6085$	$\cdot391\ 0123$		14	$\cdot615\ 0321$	$\cdot391\ 2270$		14
			575				559				545	
28	$\cdot680\ 9313$	$\cdot390\ 8431$		14	$\cdot733\ 8222$	$\cdot391\ 0682$		13	$\cdot786\ 8688$	$\cdot391\ 2815$		13
			588				573				557	
29	$\cdot852\ 6402$	$\cdot390\ 9019$		13	$\cdot910\ 4119$	$\cdot391\ 1255$		13	$\cdot968\ 3394$	$\cdot391\ 3372$		12
			601				585				570	
30	$2\cdot033\ 8997$	$\cdot390\ 9621$		12	$2\cdot096\ 8223$	$\cdot391\ 1840$		12	$2\cdot159\ 9006$	$\cdot391\ 3942$		12
			614				597				581	
31	$\cdot225\ 1766$	$\cdot391\ 0234$		12	$\cdot293\ 5330$	$\cdot391\ 2437$		11	$\cdot362\ 0454$	$\cdot391\ 4523$		11
			625				608				592	
32	$\cdot426\ 9744$	$\cdot391\ 0859$		11	$\cdot501\ 0620$	$\cdot391\ 3046$		10	$\cdot575\ 3055$	$\cdot391\ 5115$		10
			636				619				603	
33	$\cdot639\ 8374$	$\cdot391\ 1495$		10	$\cdot719\ 9685$	$\cdot391\ 3664$		9	$\cdot800\ 2556$	$\cdot391\ 5718$		9
			646				628				612	
34	$\cdot864\ 3536$	$\cdot391\ 2141$		9	$\cdot950\ 8571$	$\cdot391\ 4293$		9	$3\cdot037\ 5167$	$\cdot391\ 6330$		9
			655				637				621	
35	$3\cdot101\ 1591$	$\cdot391\ 2796$		8	$3\cdot194\ 3816$	$\cdot391\ 4930$		8	$\cdot287\ 7604$	$\cdot391\ 6950$		8
			664				646				629	
36	$\cdot350\ 9424$	$\cdot391\ 3460$		8	$\cdot451\ 2499$	$\cdot391\ 5576$		7	$\cdot551\ 7138$	$\cdot391\ 7579$		7
			671				653				636	
37	$\cdot614\ 4497$	$\cdot391\ 4131$		7	$\cdot722\ 2290$	$\cdot391\ 6229$		6	$\cdot830\ 1647$	$\cdot391\ 8215$		6
			678				659				642	
38	$\cdot892\ 4902$	$\cdot391\ 4809$		6	$4\cdot008\ 1507$	$\cdot391\ 6888$		6	$4\cdot123\ 9677$	$\cdot391\ 8857$		6
			684				665				648	
39	$4\cdot185\ 9427$	$\cdot391\ 5492$		5	$\cdot309\ 9183$	$\cdot391\ 7553$		5	$\cdot434\ 0507$	$\cdot391\ 9505$		5
			689				670				653	
40	$\cdot495\ 7624$	$\cdot391\ 6181$		4	$\cdot628\ 5140$	$\cdot391\ 8224$		4	$\cdot761\ 4223$	$\cdot392\ 0157$		4
			693				674				657	
41	$\cdot822\ 9894$	$\cdot391\ 6874$		3	$\cdot965\ 0066$	$\cdot391\ 8898$		3	$5\cdot107\ 1807$	$\cdot392\ 0814$		3
			697				678				660	
42	$5\cdot168\ 7569$	$\cdot391\ 7571$		3	$5\cdot320\ 5613$	$\cdot391\ 9576$		3	$\cdot472\ 5225$	$\cdot392\ 1474$		2
			699				680				662	
43	$\cdot534\ 3023$	$\cdot391\ 8270$		2	$\cdot696\ 4499$	$\cdot392\ 0256$		2	$\cdot858\ 7544$	$\cdot392\ 2136$		2
			701				682				664	
44	$\cdot920\ 9782$	$\cdot391\ 8971$		1	$6\cdot094\ 0627$	$\cdot392\ 0938$		1	$6\cdot267\ 3043$	$\cdot392\ 2800$		1
			702				683				665	
45	$6\cdot330\ 2655$	$\cdot391\ 9673$			$\cdot514\ 9222$	$\cdot392\ 1621$			$\cdot699\ 7361$	$\cdot392\ 3465$		

TABLE LIV—(*continued*).

$\phi°$	$r=39$				$r=40$				$r=41$			
	$\log F(r,\nu)$	$\log H(r,\nu)$	Δ	Δ^2	$\log F(r,\nu)$	$\log H(r,\nu)$	Δ	Δ^2	$\log F(r,\nu)$	$\log H(r,\nu)$	Δ	Δ^2
0	$\bar{1}\cdot600\ 7740$	$0\cdot390\ 6658$	11		$\bar{1}\cdot595\ 3459$	$0\cdot390\ 8782$	11		$\bar{1}\cdot590\ 0501$	$0\cdot391\ 0801$	11	
1	603 2891	·390 6669	34	23	·597 9271	·390 8793	33	22	·592 6975	·391 0812	32	21
2	·610 8391	·390 6703	56	23	·605 6756	·390 8826	55	22	·600 6445	·391 0844	54	21
3	·623 4380	·390 6760	79	23	·618 6058	·390 8881	77	22	·613 9059	·391 0898	75	21
4	·641 1093	·390 6838	101	22	·636 7416	·390 8958	99	22	·632 5064	·391 0973	96	21
5	·663 8860	·390 6940		22	·660 1171	·390 9057		22	·656 4807	·391 1069		21
6	·691 8107	·390 7063	124	22	·688 7760	·390 9177	120	22	·685 8737	·391 1187	118	21
7	·724 9361	·390 7209	146	22	·722 7721	·390 9319	142	21	·720 7406	·391 1325	139	21
8	·763 3246	·390 7377	168	22	·762 1697	·390 9483	163	21	·761 1472	·391 1485	159	21
9	·807 0490	·390 7566	189	22	·807 0434	·390 9667	185	21	·807 1702	·391 1665	180	20
10	·856 1930	·390 7777	211	21	·857 4789	·390 9873	206	21	·858 8973	·391 1865	200	20
11	·910 8510	·390 8009	232	21	·913 5732	·391 0099	226	20	·916 4280	·391 2086	221	20
12	·971 1287	·390 8262	253	21	·975 4348	·391 0346	247	20	·979 8734	·391 2327	241	20
13	0·037 1440	·390 8535	274	20	0·043 1845	·391 0612	267	20	0·049 3576	·391 2587	260	19
14	·109 0268	·390 8829	294	20	·116 9556	·391 0899	287	19	·125 0171	·391 2867	280	19
15	·186 9202	·390 9143	314	20	·196 8949	·391 1205	306	19	·207 0023	·391 3165	299	19
16	·270 9806	·390 9477	334	19	·283 1630	·391 1530	325	19	·295 4780	·391 3483	317	18
17	·361 3789	·390 9829	353	19	·375 9350	·391 1874	344	18	·390 6238	·391 3818	335	18
18	·458 3010	·391 0201	371	18	·475 4017	·391 2236	362	18	·492 6351	·391 4172	353	17
19	·561 9488	·391 0591	390	18	·581 7701	·391 2616	380	17	·601 7243	·391 4542	371	17
20	·672 5410	·391 0998	408	17	·695 2648	·391 3014	397	17	·718 1215	·391 4930	388	16
21	·790 3143	·391 1423	425	17	·816 1285	·391 3428	414	16	·842 0755	·391 5334	404	16
22	·915 5247	·391 1865	442	16	·944 6237	·391 3859	431	16	·973 8556	·391 5754	420	16
23	1·048 4484	·391 2323	458	16	1·081 0339	·391 4305	447	16	1·113 7524	·391 6190	436	15
24	·189 3837	·391 2796	474	15	·225 6650	·391 4767	462	15	·262 0794	·391 6640	451	14
25	·338 6522	·391 3285	489	14	·378 8470	·391 5243	477	14	·419 1750	·391 7105	465	14
26	·496 6007	·391 3788	503	14	·540 9357	·391 5734	491	14	·585 4038	·391 7584	479	13
27	·663 6033	·391 4306	517	13	·712 3146	·391 6238	504	13	·761 1593	·391 8076	492	13
28	·840 0630	·391 4836	530	13	·893 3974	·391 6756	517	12	·946 8652	·391 8581	505	12
29	2·026 4145	·391 5379	543	12	2·084 6300	·391 7285	530	12	2·142 9789	·391 9097	517	11
30	·223 1268	·391 5935	555	11	·286 4933	·391 7827	541	11	·349 9933	·391 9626	528	11
31	·430 7056	·391 6501	566	11	·499 5062	·391 8379	552	10	·568 4405	·392 0164	539	10
32	·649 6970	·391 7078	577	10	·724 2290	·391 8942	563	10	·798 8947	·392 0713	549	9
33	·880 6908	·391 7665	587	9	·961 2666	·391 9514	572	9	3·041 9761	·392 1272	558	9
34	3·124 3244	·391 8261	596	8	3·211 2729	·392 0095	581	8	·298 3551	·392 1839	567	8
35	·381 2874	·391 8866	605	8	·474 9551	·392 0685	590	8	·568 7567	·392 2414	575	8
36	·652 3259	·391 9479	612	7	·753 0789	·392 1282	597	7	·853 9658	·392 2997	583	7
37	·938 2488	·392 0098	619	6	4·046 4738	·392 1886	604	6	4·154 8329	·392 3586	589	6
38	4·239 9332	·392 0724	626	5	·356 0397	·392 2496	610	5	·472 2803	·392 4181	595	5
39	·558 3315	·392 1355	631	5	·682 7535	·392 3112	615	5	·807 3096	·392 4782	600	5
40	·894 4793	·392 1991	636	4	5·027 6774	·392 3732	620	4	5·161 0098	·392 5386	605	4
41	5·249 5035	·392 2631	640	3	·391 9676	·392 4355	624	3	·534 5660	·392 5995	609	3
42	·624 6328	·392 3274	643	2	·776 8842	·392 4982	627	2	·929 2701	·392 6607	612	2
43	6·021 2079	·392 3919	645	2	6·183 8028	·392 5612	629	1	6·346 5322	·392 7221	614	1
44	·440 6950	·392 4566	647	1	·614 2272	·392 6242	630	0	·787 8938	·392 7836	615	1
45	·884 6991	·392 5213	648		7·069 8037	·392 6874	631		7·255 0429	·392 8452	616	

TABLE LIV—(*continued*).

$\phi°$	$r=42$				$r=43$				$r=44$			
	$\log F(r,\nu)$	$\log H(r,\nu)$	Δ	Δ^2	$\log F(r,\nu)$	$\log H(r,\nu)$	Δ	Δ^2	$\log F(r,\nu)$	$\log H(r,\nu)$	Δ	Δ^2
0	$\bar{1}\cdot584\ 8804$	$0\cdot391\ 2724$			$\bar{1}\cdot579\ 8310$	$0\cdot391\ 4556$			$\bar{1}\cdot574\ 8962$	$0\cdot391\ 6305$		
			10				10				10	
1	$\cdot587\ 5939$	$\cdot391\ 2734$		21	$\cdot582\ 6106$	$\cdot391\ 4566$		21	$\cdot577\ 7420$	$\cdot391\ 6315$		20
			31				31				30	
2	$\cdot595\ 7394$	$\cdot391\ 2766$		21	$\cdot590\ 9546$	$\cdot391\ 4597$		20	$\cdot586\ 2845$	$\cdot391\ 6345$		20
			52				51				50	
3	$\cdot609\ 3321$	$\cdot391\ 2818$		21	$\cdot604\ 8785$	$\cdot391\ 4648$		20	$\cdot600\ 5397$	$\cdot391\ 6395$		20
			73				72				70	
4	$\cdot628\ 3972$	$\cdot391\ 2891$		21	$\cdot624\ 4083$	$\cdot391\ 4720$		20	$\cdot620\ 5341$	$\cdot391\ 6465$		20
			94				92				90	
5	$\cdot652\ 9704$	$\cdot391\ 2985$		21	$\cdot649\ 5803$	$\cdot391\ 4812$		20	$\cdot646\ 3049$	$\cdot391\ 6554$		20
			115				112				109	
6	$\cdot683\ 0975$	$\cdot391\ 3100$		21	$\cdot680\ 4416$	$\cdot391\ 4924$		20	$\cdot677\ 9004$	$\cdot391\ 6664$		20
			135				132				129	
7	$\cdot718\ 8352$	$\cdot391\ 3235$		20	$\cdot717\ 0501$	$\cdot391\ 5056$		20	$\cdot715\ 3798$	$\cdot391\ 6793$		20
			156				152				149	
8	$\cdot760\ 2510$	$\cdot391\ 3391$		20	$\cdot759\ 4750$	$\cdot391\ 5208$		20	$\cdot758\ 8138$	$\cdot391\ 6942$		19
			176				172				168	
9	$\cdot807\ 4232$	$\cdot391\ 3567$		20	$\cdot807\ 7965$	$\cdot391\ 5380$		19	$\cdot808\ 2846$	$\cdot391\ 7109$		19
			196				191				187	
10	$\cdot860\ 4419$	$\cdot391\ 3763$		20	$\cdot862\ 1068$	$\cdot391\ 5571$		19	$\cdot863\ 8866$	$\cdot391\ 7296$		19
			216				210				206	
11	$\cdot919\ 4089$	$\cdot391\ 3978$		19	$\cdot922\ 5103$	$\cdot391\ 5781$		19	$\cdot925\ 7265$	$\cdot391\ 7502$		19
			235				230				224	
12	$\cdot984\ 4383$	$\cdot391\ 4213$		19	$\cdot989\ 1236$	$\cdot391\ 6011$		19	$\cdot993\ 9238$	$\cdot391\ 7726$		18
			254				248				243	
13	$0\cdot055\ 6569$	$\cdot391\ 4467$		19	$0\cdot062\ 0767$	$\cdot391\ 6259$		19	$0\cdot068\ 6113$	$\cdot391\ 7769$		18
			273				267				261	
14	$\cdot133\ 2048$	$\cdot391\ 4740$		19	$\cdot141\ 5130$	$\cdot391\ 6526$		18	$\cdot149\ 9361$	$\cdot391\ 8229$		18
			292				285				278	
15	$\cdot217\ 2361$	$\cdot391\ 5032$		18	$\cdot227\ 5903$	$\cdot391\ 6810$		18	$\cdot238\ 0595$	$\cdot391\ 8508$		17
			310				303				296	
16	$\cdot307\ 9195$	$\cdot391\ 5341$		18	$\cdot320\ 4814$	$\cdot391\ 7113$		17	$\cdot333\ 1584$	$\cdot391\ 8803$		17
			328				320				313	
17	$\cdot405\ 4390$	$\cdot391\ 5669$		17	$\cdot420\ 3748$	$\cdot391\ 7433$		17	$\cdot435\ 4256$	$\cdot391\ 9116$		17
			345				337				329	
18	$\cdot509\ 9950$	$\cdot391\ 6014$		17	$\cdot527\ 4755$	$\cdot391\ 7770$		17	$\cdot545\ 0710$	$\cdot391\ 9445$		16
			362				353				346	
19	$\cdot621\ 8050$	$\cdot391\ 6376$		16	$\cdot642\ 0063$	$\cdot391\ 8123$		16	$\cdot662\ 3227$	$\cdot391\ 9791$		16
			378				370				361	
20	$\cdot741\ 1047$	$\cdot391\ 6754$		16	$\cdot764\ 2086$	$\cdot391\ 8493$		16	$\cdot787\ 4277$	$\cdot392\ 0152$		15
			395				385				377	
21	$\cdot868\ 1492$	$\cdot391\ 7149$		16	$\cdot894\ 3438$	$\cdot391\ 8878$		15	$\cdot920\ 6532$	$\cdot392\ 0528$		15
			410				401				391	
22	$1\cdot003\ 2143$	$\cdot391\ 7559$		15	$1\cdot032\ 6937$	$\cdot391\ 9279$		15	$1\cdot062\ 2883$	$\cdot392\ 0920$		14
			425				415				406	
23	$\cdot146\ 5976$	$\cdot391\ 7984$		15	$\cdot179\ 5637$	$\cdot391\ 9694$		14	$\cdot212\ 6450$	$\cdot392\ 1326$		14
			440				430				420	
24	$\cdot298\ 6206$	$\cdot391\ 8424$		14	$\cdot335\ 2826$	$\cdot392\ 0124$		14	$\cdot372\ 0600$	$\cdot392\ 1746$		13
			454				443				433	
25	$\cdot459\ 6298$	$\cdot391\ 8878$		13	$\cdot500\ 2054$	$\cdot392\ 0567$		13	$\cdot540\ 8966$	$\cdot392\ 2179$		13
			467				457				446	
26	$\cdot629\ 9989$	$\cdot391\ 9345$		13	$\cdot674\ 7149$	$\cdot392\ 1024$		13	$\cdot719\ 5463$	$\cdot392\ 2625$		12
			480				469				459	
27	$\cdot810\ 1308$	$\cdot391\ 9826$		12	$\cdot859\ 2234$	$\cdot392\ 1493$		12	$\cdot908\ 4315$	$\cdot392\ 3084$		12
			493				481				470	
28	$2\cdot000\ 4600$	$\cdot392\ 0318$		12	$2\cdot054\ 1759$	$\cdot392\ 1974$		12	$2\cdot108\ 0073$	$\cdot392\ 3554$		11
			504				493				482	
29	$\cdot201\ 4548$	$\cdot392\ 0823$		11	$\cdot260\ 0519$	$\cdot392\ 2467$		11	$\cdot318\ 7645$	$\cdot392\ 4035$		11
			516				504				492	
30	$\cdot413\ 6204$	$\cdot392\ 1338$		10	$\cdot477\ 3687$	$\cdot392\ 2970$		10	$\cdot541\ 2327$	$\cdot392\ 4528$		10
			526				514				502	
31	$\cdot637\ 5019$	$\cdot392\ 1864$		10	$\cdot706\ 6845$	$\cdot392\ 3484$		10	$\cdot775\ 9829$	$\cdot392\ 5030$		9
			536				523				512	
32	$\cdot873\ 6876$	$\cdot392\ 2400$		9	$\cdot948\ 6018$	$\cdot392\ 4007$		9	$3\cdot023\ 6317$	$\cdot392\ 5541$		9
			545				533				520	
33	$3\cdot122\ 8129$	$\cdot392\ 2945$		9	$3\cdot203\ 7711$	$\cdot392\ 4540$		8	$\cdot284\ 8450$	$\cdot392\ 6062$		8
			554				541				528	
34	$\cdot385\ 5647$	$\cdot392\ 3499$		8	$\cdot472\ 8957$	$\cdot392\ 5081$		8	$\cdot560\ 3426$	$\cdot392\ 6590$		8
			562				548				536	
35	$\cdot662\ 6857$	$\cdot392\ 4060$		7	$\cdot756\ 7363$	$\cdot392\ 5629$		7	$\cdot850\ 9027$	$\cdot392\ 7126$		7
			569				556				543	
36	$\cdot954\ 9802$	$\cdot392\ 4629$		7	$4\cdot056\ 1162$	$\cdot392\ 6185$		6	$4\cdot157\ 3682$	$\cdot392\ 7669$		6
			575				562				549	
37	$4\cdot263\ 3195$	$\cdot392\ 5204$		6	$\cdot371\ 9277$	$\cdot392\ 6747$		6	$\cdot480\ 6520$	$\cdot392\ 8218$		6
			581				567				555	
38	$\cdot588\ 6485$	$\cdot392\ 5785$		5	$\cdot705\ 1384$	$\cdot392\ 7314$		5	$\cdot821\ 7444$	$\cdot392\ 8773$		5
			586				572				559	
39	$\cdot931\ 9935$	$\cdot392\ 6371$		4	$5\cdot056\ 7991$	$\cdot392\ 7886$		4	$5\cdot181\ 7208$	$\cdot392\ 9332$		4
			590				577				564	
40	$5\cdot294\ 4700$	$\cdot392\ 6962$		4	$\cdot428\ 0520$	$\cdot392\ 8463$		3	$\cdot561\ 7502$	$\cdot392\ 9896$		4
			594				580				567	
41	$\cdot677\ 2923$	$\cdot392\ 7556$		3	$\cdot820\ 1404$	$\cdot392\ 9044$		3	$\cdot963\ 1049$	$\cdot393\ 0463$		3
			597				583				570	
42	$6\cdot081\ 7839$	$\cdot392\ 8153$		2	$6\cdot234\ 4197$	$\cdot392\ 9627$		2	$6\cdot387\ 1719$	$\cdot393\ 1033$		2
			599				585				572	
43	$\cdot509\ 3896$	$\cdot392\ 8752$		1	$\cdot672\ 3690$	$\cdot393\ 0212$		2	$\cdot835\ 4649$	$\cdot393\ 1605$		1
			601				587				573	
44	$\cdot961\ 6887$	$\cdot392\ 9353$		1	$7\cdot135\ 6055$	$\cdot393\ 0799$		1	$7\cdot309\ 6389$	$\cdot393\ 2178$		1
			601				587				574	
45	$7\cdot440\ 4103$	$\cdot392\ 9954$			$\cdot625\ 8999$	$\cdot393\ 1386$			$\cdot811\ 5060$	$\cdot393\ 2752$		

TABLE LIV—(*continued*).

ϕ°	$r=45$				$r=46$				$r=47$			
	$\log F(r,\nu)$	$\log H(r,\nu)$	Δ	Δ^2	$\log F(r,\nu)$	$\log H(r,\nu)$	Δ	Δ^2	$\log F(r,\nu)$	$\log H(r,\nu)$	Δ	Δ^2
0	$\bar{1}\cdot570\ 0711$	$0\cdot391\ 7975$			$\bar{1}\cdot565\ 3509$	$0\cdot391\ 9572$			$\bar{1}\cdot560\ 7311$	$0\cdot392\ 1100$		
			10				10				9	
1	·572 9830	·391 7984		20	·568 3289	·391 9581		19	·563 7753	·392 1110		19
			29				29				28	
2	·581 7241	·391 8014		20	·577 2685	·391 9610		19	·572 9134	·392 1138		19
			49				48				47	
3	·596 3106	·391 8063		19	·592 1863	·391 9658		19	·588 1625	·392 1185		19
			68				67				66	
4	·616 7696	·391 8131		19	·613 1100	·391 9725		19	·609 5508	·392 1250		19
			88				86				84	
5	·643 1393	·391 8219		19	·640 0785	·391 9811		19	·637 1182	·392 1334		18
			107				105				103	
6	·675 4689	·391 8326		19	·673 1423	·391 9915		19	·670 9162	·392 1437		18
			126				124				121	
7	·713 8192	·391 8452		19	·712 3634	·392 0039		19	·711 0082	·392 1557		18
			145				142				139	
8	·758 2623	·391 8598		19	·757 8157	·392 0181		18	·757 4696	·392 1697		18
			164				161				157	
9	·808 8824	·391 8762		19	·809 5852	·392 0342		18	·810 3885	·392 1854		18
			183				179				175	
10	·865 7761	·391 8944		18	·867 7706	·392 0520		18	·869 8656	·392 2029		18
			201				197				193	
11	·929 0524	·391 9145		18	·932 4834	·392 0717		18	·936 0149	·392 2221		17
			219				215				210	
12	·998 8337	·391 9365		18	0·003 8487	·392 0932		18	0·008 9642	·392 2431		17
			237				232				227	
13	0·075 2558	·391 9602		18	·082 0054	·392 1164		17	·088 8555	·392 2658		17
			255				249				244	
14	·158 4691	·391 9857		17	·167 1071	·392 1413		17	·175 8458	·392 2902		17
			272				266				261	
15	·248 6386	·392 0129		17	·259 3228	·392 1679		17	·270 1076	·392 3163		16
			289				283				277	
16	·345 9453	·392 0418		17	·358 8372	·392 1962		16	·371 8299	·392 3440		16
			306				299				293	
17	·450 5863	·392 0724		16	·465 8522	·392 2261		16	·481 2188	·392 3732		16
			322				315				308	
18	·562 7765	·392 1046		16	·580 5873	·392 2576		16	·598 4987	·392 4041		15
			338				331				323	
19	·682 7492	·392 1383		15	·703 2808	·392 2906		15	·723 9132	·392 4364		15
			353				346				338	
20	·810 7568	·392 1737		15	·834 1911	·392 3252		15	·857 7263	·392 4702		14
			368				360				353	
21	·947 0729	·392 2105		15	·973 5979	·392 3612		14	1·000 2236	·392 5055		14
			383				375				367	
22	1·091 9931	·392 2488		14	1·121 8031	·392 3987		14	·151 7141	·392 5421		13
			397				388				380	
23	·245 8365	·392 2885		14	·279 1333	·392 4375		13	·312 5311	·392 5801		13
			411				402				393	
24	·408 9476	·392 3295		13	·445 9406	·392 4776		13	·483 0345	·392 6194		13
			424				414				406	
25	·581 6979	·392 3719		13	·622 6047	·392 5191		12	·663 6124	·392 6600		12
			436				427				418	
26	·764 4880	·392 4155		12	·809 5352	·392 5618		12	·854 6834	·392 7018		12
			448				438				429	
27	·957 7499	·392 4603		12	2·007 1738	·392 6056		11	2·056 6988	·392 7447		11
			460				450				440	
28	2·161 9490	·392 5063		11	·215 9964	·392 6506		11	·270 1448	·392 7887		11
			471				461				451	
29	·377 5876	·392 5534		10	·436 5163	·392 6967		10	·495 5462	·392 8338		10
			481				471				461	
30	·605 2071	·392 6015		10	·669 2873	·392 7437		10	·733 4685	·392 8799		9
			491				480				470	
31	·845 3918	·392 6506		9	·914 9064	·392 7918		9	·984 5222	·392 9269		9
			500				489				479	
32	3·098 7723	·392 7006		9	3·174 0186	·392 8407		8	3·249 3662	·392 9748		8
			509				498				487	
33	·366 0297	·392 7515		8	·447 3201	·392 8905		8	·528 7119	·393 0235		8
			517				505				495	
34	·647 9002	·392 8032		7	·735 5636	·392 9410		7	·823 3284	·393 0729		7
			524				513				502	
35	·945 1800	·392 8556		7	4·039 5631	·392 9923		6	4·134 0476	·393 1231		6
			531				519				508	
36	4·258 7310	·392 9087		6	·360 1998	·393 0442		6	·461 7700	·393 1740		6
			537				525				514	
37	·589 4872	·392 9624		5	·698 4283	·393 0967		5	·807 4710	·393 2254		5
			542				530				519	
38	·938 4614	·393 0166		5	5·055 2844	·393 1498		5	5·172 2090	·393 2773		5
			547				535				524	
39	5·306 7534	·393 0713		4	·431 8925	·393 2033		4	·557 1330	·393 3297		4
			551				539				528	
40	·695 5595	·393 1264		3	·829 4749	·393 2572		3	·963 4920	·393 3824		3
			555				543				531	
41	6·106 1805	·393 1818		3	6·249 3623	·393 3115		3	6·392 6458	·393 4355		3
			557				545				534	
42	·540 0352	·393 2376		2	·693 0048	·393 3660		2	·846 0761	·393 4889		2
			559				547				536	
43	·998 6720	·393 2935		1	7·161 9854	·393 4207		1	7·325 4006	·393 5424		1
			561				548				537	
44	7·483 7836	·393 3496		1	·658 0347	·393 4755		1	·832 3876	·393 5961		0
			561				549				537	
45	·997 2234	·393 4057			8·183 0474	·393 5304			8·368 9732	·393 6498		

TABLE LIV—(*continued*).

$\phi°$	$r=48$				$r=49$				$r=50$			
	log $F(r, \nu)$	log $H(r, \nu)$	Δ	Δ^2	log $F(r, \nu)$	log $H(r, \nu)$	Δ	Δ^2	log $F(r, \nu)$	log $H(r, \nu)$	Δ	Δ^2
0	$\bar{1}$·556 2075	0·392 2565			$\bar{1}$·551 7763	0·392 3969			$\bar{1}$·547 4336	0·392 5316		
			9				9				9	
1	·559 3178	·392 2574	28	18	·554 9527	·392 3978	27	18	·550 6762	·392 5325	26	18
2	·568 6545	·392 2601	46	18	·564 4879	·392 4005	45	18	·560 4099	·392 5352	44	18
3	·584 2348	·392 2647	64	18	·580 3995	·392 4050	63	18	·576 6529	·392 5396	62	18
4	·606 0878	·392 2711	82	18	·602 7172	·392 4113	81	18	·599 4352	·392 5457	79	17
5	·634 2541	·392 2794			·631 4824	·392 4193			·628 7993	·392 5536		17
			100				98				96	
6	·668 7863	·392 2894	118	18	·666 7488	·392 4291	116	18	·664 7999	·392 5633	114	17
7	·709 7492	·392 3012	136	18	·708 5826	·392 4407	133	17	·707 5046	·392 5746	131	17
8	·757 2198	·392 3149	154	18	·757 0624	·392 4541	151	17	·756 9936	·392 5877	148	17
9	·811 2881	·392 3302	171	17	·812 2801	·392 4692	168	17	·813 3607	·392 6025	164	17
10	·872 0569	·392 3474		17	·874 3406	·392 4859		17	·876 7129	·392 6189		17
			189				185				181	
11	·939 6427	·392 3662	206	17	·943 3629	·392 5044	201	17	·947 1718	·392 6370	197	16
12	0·014 1760	·392 3868	222	17	0·019 4803	·392 5245	218	17	0·024 8733	·392 6568	213	16
13	·095 8020	·392 4090	239	17	·102 8409	·392 5463	234	16	·109 9685	·392 6781	229	16
14	·184 6808	·392 4329	255	16	·193 6083	·392 5697	250	16	·202 6245	·392 7010	245	16
15	·280 9888	·392 4584		16	·291 9625	·392 5947		16	·303 0249	·392 7255		15
			271				266				260	
16	·384 9189	·392 4855	287	16	·398 1005	·392 6213	281	15	·411 3709	·392 7516	275	15
17	·496 6818	·392 5142	302	15	·512 2374	·392 6493	296	15	·527 8818	·392 7791	290	15
18	·616 5066	·392 5444	317	15	·634 6070	·392 6789	310	14	·652 7964	·392 8080	304	14
19	·744 6421	·392 5760	331	15	·765 4636	·392 7099	324	14	·786 3739	·392 8384	318	14
20	·881 3579	·392 6092		14	·905 0822	·392 7424		14	·928 8954	·392 8702		14
			345				338				331	
21	1·026 9460	·392 6437	359	14	1·053 7610	·392 7762	352	13	1·080 6649	·392 9034	345	14
22	·181 7216	·392 6796	372	13	·211 8218	·392 8114	365	13	·242 0110	·392 9378	357	13
23	·346 0254	·392 7168	385	13	·379 6125	·392 8478	377	12	·413 2886	·392 9736	369	12
24	·520 2250	·392 7553	397	12	·557 5084	·392 8855	389	12	·594 8807	·393 0105	381	12
25	·704 7169	·392 7950		12	·745 9141	·392 9244		12	·787 2004	·393 0486		11
			409				401				393	
26	·899 9284	·392 8359	420	11	·945 2662	·392 9645	412	11	·990 6930	·393 0879	404	11
27	2·106 3205	·392 8779	431	11	2·156 0351	·393 0057	422	11	2·205 8389	·393 1282	414	10
28	·324 3901	·392 9210	441	10	·378 7283	·393 0479	432	10	·433 1556	·393 1696	424	10
29	·554 6729	·392 9652	451	10	·613 8925	·393 0911	442	10	·673 2014	·393 2120	433	9
30	·797 7467	·393 0103		9	·862 1178	·393 1353		9	·926 5783	·393 2553		9
			460				451				442	
31	3·054 2350	·393 0563	469	9	3·124 0408	·393 1804	459	8	3·193 9359	·393 2995	450	8
32	·324 8107	·393 1032	477	8	·400 3485	·393 2263	467	8	·475 9753	·393 3445	458	8
33	·610 2007	·393 1509	485	8	·691 7826	·393 2731	474	7	·773 4538	·393 3903	465	7
34	·911 1903	·393 1993	491	7	·999 1454	·393 3205	481	7	4·087 1898	·393 4368	472	7
35	4·228 6293	·393 2485		6	4·323 3042	·393 3686		6	·418 0685	·393 4840		6
			498				488				478	
36	·563 4374	·393 2982	503	6	·665 1980	·393 4174	493	6	·767 0481	·393 5318	483	5
37	·916 6109	·393 3486	508	5	5·025 8441	·393 4667	498	5	5·135 1668	·393 5801	488	5
38	5·289 2309	·393 3994	513	4	·406 3461	·393 5165	502	4	·523 5508	·393 6289	492	4
39	·682 4708	·393 4507	517	4	·807 9020	·393 5667	506	4	·933 4228	·393 6781	496	4
40	6·097 6055	·393 5024		3	6·231 8144	·393 6174		3	6·366 1119	·393 7277		3
			520				509				499	
41	·536 0267	·393 5543	522	2	·679 5011	·393 6683	512	3	·823 0651	·393 7776	502	2
42	·999 2449	·393 6066	524	2	7·152 5073	·393 7195	514	2	7·305 8593	·393 8278	504	2
43	7·488 9134	·393 6590	526	1	·652 5197	·393 7708	515	1	·816 2157	·393 8781	505	1
44	8·006 8381	·393 7116	526	1	8·181 3822	·393 8223	516	1	8·356 0160	·393 9286	505	1
45	·554 9966	·393 7642			·741 1137	·393 8739			·927 3206	·393 9791		

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